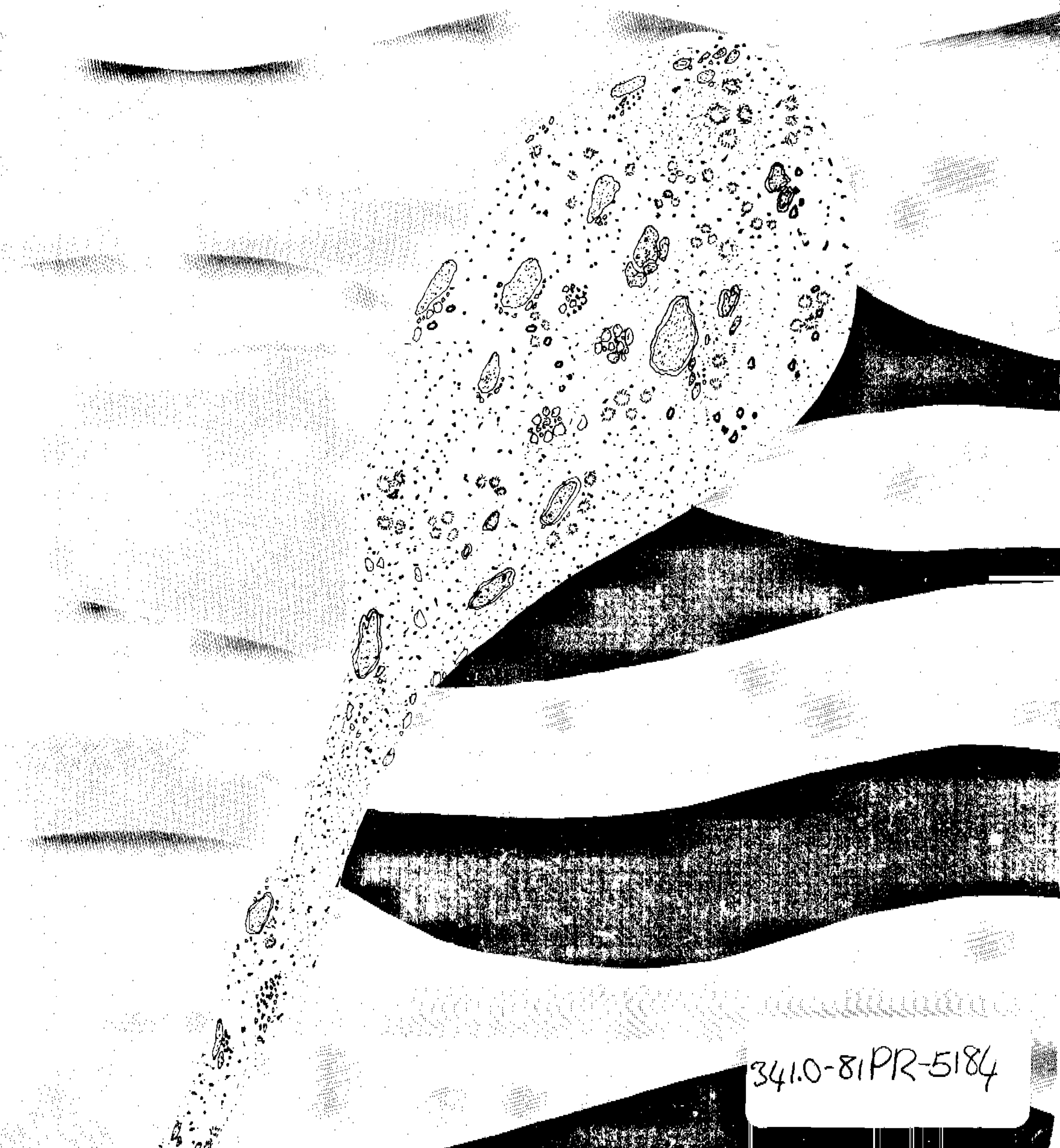
PRINCIPLES OF WASTEWATER TREATMENT

VOL.1 BIOLOGICAL PROCESSES

BY

E. A. R. OUANO



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VOL: 1 Biological Processes

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PREFACE

"Out of sight" was a common approach to wastewater problem even in early civilizations. People involved in the disposal, cleansing and treatment of wastewater were often looked down or relegated to low caste. In fact as late as the 1950's, a common joke in the university campus to sanitary engineering students was "No mother reared up a son to become a sanitary engineer!"

The ecological damage wrought by uncontrolled industrialization created a panic in the late 1950's in the industrialized countries. All of a sudden sanitary or environmental engineers were in demand. The cadre of trained and experienced scientists and engineers were very small to cope up with the demand. A large number of individuals from other disciplines became overnight experts. In 1978, environmental protection is blamed as a major cause of inflation and loss of productivity in U.S.A.

In order for the profession to retain the momentum of the 60's, it has to segregate the facts from the myth. Pollution control must have a very sound scientific basis to become cost effective. This book is an attempt to introduce the graduate student in environmental engineering to the scientific principles of wastewater treatment. A critical review of the currently accepted practices and principles of wastewater treatment is made.

In writing this book, I presume the reader have completed a course on environmental management, steam pollution and an introductory course in wastewater treatment. Previous exposure to chemical process control, instrumentation and design principles would be helpful but not necessary. I tried to conform with the IAWPR recommended notation when applicable. However, in writing this book I have to make extensive reference to chemical engineering textbooks and in some instance I maintained the standard chemical notation which I feel would help the reader should he require more extensive discussion on biochemical process design principles.

Part I is an introduction to the principles of dimensional analysis, statistics, and biochemical kinetics. Rather than putting important hydraulics, kinetics, and unit operations data in the Appendix, those are utilized as sample problems in the first three chapters. I hope this would facilitate cross referencing to other texts on those subjects for further explanations.

Part II is involved in the process description, exposition of principles, concepts, and design procedures. A critical review of the application and limitations of currently accepted design equations and criteria is made.

In writing this book, I tried my best to develop the discussion from the basic fundamentals. However, it is difficult to appreciate the fundamentals without discussing its significance to the later chapters. I hope the reader will appreciate the continuous cycle between the development and application of the fundamental principles.

In closing I would like to extend my appreciation to my wife Rosalie for editing the original draft, Elizabeth Q. Ramos for typing the manuscript, N. Guevarra, R. Ituralde, T. Alcala, and A. M. Perez for the drafting work. This work could not have been made possible without the support from the National Science Development Board of the Philippines. The assistance from Minister Melecio S. Magno, Deputy Minister Segundo V. Roxas of NSDB and their staff are gratefully acknowledged.

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CHAPTER I

SIMILITUDE

Pilot plant operation is often employed to estimate the expected performance and efficiency of a proposed waste treatment process. The kinetic constants and design parameters for a given type of wastewater are also obtained from pilot plant operation. In fact, almost 99% of all data published in journals on wastewater treatment process performance are from bench scale and pilot plant operations. While ideally, a pilot plant should not be smaller than 1/10th of the proposed prototype, nevertheless, most of the so-called design equations reported in literature today were developed using data from scale units of this size or even smaller.

Aside from the size of the pilot units, certain proper dimensionless groups which affect scale up, geometric similarity and process kinetics are likewise rarely considered by researchers. For these reasons, it is not surprising to find very large discrepancies among the different design equations reported in literature. For example, Ordon (1968) found the recommended trickling filter size using design equations commonly accepted in practice today to vary by as much as 1300% or a factor of 13. Utility of those design equations has become meaningless due to the wide variation of recommended equipment size.

Statistical test for verification of hypothesis is also often ignored. Respectable environmental engineers feel there is an urgency to produce design data so that a rigorous statistical test will be a waste of time or an unnecessary interference (Mckinney, 1978). Perhaps, this attitude stems out from the policies of some educational institutions to judge a professor's output by the number of research papers and reports published regardless of its contributions to the state of knowledge. In fact, research papers whose hypotheses are "confirmed" without rigorous statistical test only create confusion and retard future development.

For example, a curve on the controlling mechanism of substrate degradation has been fitted through three experimental points and the conclusions obtained have been quoted in more than a dozen research papers to support other researchers' theories. From the statistical point of view, two straight lines plotted through three points are meaningless.

This chapter provides introductory materials for the reader to appreciate the significance of the different dimensionless groups for experimental design, their derivation and use in design. Some of the design parameters currently used for scale up were developed from intuition and past experience which could be applicable only to a limited range of linearization or linear approximation. These would result to large deviations when applied to a wider range of variation due to lack of scientific basis. Hence, it is important for the reader to grasp the significance of proper derivation and interpretation of the different scale-up parameters.

GEOMETRIC SIMILARITY

Geometric similarity implies that the ratio of the length, width and depth between the model and prototype must be constant. Hence, for a three dimensional space, X, Y and Z with subscripts m and p denoting model and prototype respectively, geometric similarity specifies:

$$\frac{X_m}{X_p} = \frac{Y_m}{Y_p} = \frac{Z_m}{Z_p} = K \qquad (1-1)$$

where K is a constant. The ratios of geometric properties such as cross sectional area, A_c, surface area, A_r, and volume V could be expressed as:

$$\frac{A_{c,m}}{A_{c,p}} = \frac{X_m Z_m}{X_p Z_p} = K^2 \qquad (1-2)$$

$$\frac{A_{r,m}}{A_{r,p}} = \frac{X_m Y_m}{X_p Y_p} = K^2 \qquad (1-3)$$

and

$$\frac{V_m}{V_p} = \frac{X_m Y_m Z_m}{X_p Y_p Z_p} = K^3 \qquad (1-4)$$

However, in some instances, if geometric similarity is strictly applied the model would be very difficult to construct, as illustrated in the example below.

Example 1-1. Construct a model of a facultative lagoon with a dimension of 250m x 40m x 1.5m if the volume of the model is 10^{-6} of the prototype. The facultative lagoon will be used for treating domestic wastewater.

Solution:

From Equation 1 – 4 :

$$\frac{V_m}{V_p} = 10^{-6} = K^3 \;;$$

$$K = 10^{-2}$$

Hence from Equation 1 – 1 ,

$$X_m = X_pK = 250 \times 10^{-2} = 2.5\text{m}$$

$$Y_m = Y_pK = 40 \times 10^{-2} = 0.4\text{m}$$

$$Z_m = Z_pK = 1.5 \times 10^{-2} = 0.015\text{ m}$$

It should be noted that as the magnitude becomes smaller, any error in construction is magnified. Buckling and other environmental effects on the model could easily cause distortion exceeding 20% of the depth. Furthermore, oxygen diffusion takes place on the surface of the lagoon. The depth of oxygen penetration is approximately 0.5m under quiescent condition with this particular waste strength. Hence, the model lagoon system will be completely aerobic. The experimental results obtained from the model will not be representative of the performance of the prototype.

Distorted Models In the above example, a geometrically distorted model may be used. For a geometrically distorted model, Eq. 1-1 is modified as:

$$\frac{X_m}{X_p} = K_1; \quad \frac{Y_m}{Y_p} = K_2; \quad \frac{Z_m}{Z_p} = K_3 \qquad (1-5)$$

where K_1, K_2, K_3, are constants and the other terms are as defined before. Similarly, the other geometric properties could be expressed as:

$$\frac{A_{c,m}}{A_{c,p}} = \frac{X_m Z_m}{X_p Z_p} = K_1K_3; \qquad (1-6)$$

$$\frac{A_{r,m}}{A_{r,p}} = \frac{X_m Y_m}{X_p Y_p} = K_1K_2 \qquad (1-7)$$

and

$$\frac{V_m}{V_p} = \frac{X_m Y_m Z_m}{X_p Y_p Z_p} = K_1K_2K_3 \qquad (1-8)$$

The constants K_1, K_2, and K_3 may depend on physical phenomena such as mass transfer, heat transfer, mixing, etc. or the chemical reaction kinetics. In the case cited in Example 1-1, dissolved oxygen concentration on the lagoon is dependent on sunlight intensity. Hence, if the turbidity of the wastewater used in the model and prototype are equal, to maintain the same proportion of aerobic and anaerobic degradation, the following should hold true:

$$\frac{Z_m}{Z_p} = 1 \qquad (1-9)$$

The longitudinal and horizontal length may be adjusted as shown in Example 1-1.

Example 1-2. Construct a model which considers the effect of aerobic reaction in the lagoon system given in Example 1-1.

Soln: $K_3 = 1$ from Eq. 1 – 9

$K_1 = K_2 = K$

Hence,

$$\frac{V_m}{V_p} = K_1K_2K_3 = K^2 = 10^{-6}$$

or

$$K = 10^{-3}$$

$X_m = 0.001X_p = 250 \times 0.001 = 0.250\text{m}$
$Y_m = 0.001Y_p = 40 \times 0.001 = 0.04\text{m}$
$Z_m = Z_p = 1.5\text{m}$

The ratio of the surface area to the wastewater volume is very small. Since the quantity of oxygen supplied to the wastewater is dependent on the surface area of the lagoon, there is a tendency for the model to become anaerobic. This illustrates the problem of constructing bench scale wastewater treatment systems. The dimensions should not be arbitrary as commonly practised today.

Pilot plant construction will be discussed in detail in Chapter IX after the reader has become conversant with the different physical and chemical phenomena affecting the efficiency of the different waste treatment processes as discussed in Chapters III to VII . Simulation of the prototype behavior could be obtained by manipulating the detention time and wastewater strength, or the addition of dividers, mechanical aerators, liners and other process components aside from varying the geometric dimensions.

DIMENSIONAL ANALYSIS

Formal mathematical solution to a problem is not always possible. Instead of resorting to purely empirical study, dimensional analysis may be utilized. Dimensional analysis is based on the fact that if a theoretical relation exists between the variables affecting the performance of a process, then the dimensions of length, time, mass and temperature must be homogeneous. Force, heat (cal), work, gravitational acceleration and power may be used as dimensions to simplify the analysis, although they can be expressed further in terms of the basic dimensions of length, time, mass and temperature.

Dimensional analysis does not yield a definite equation. Experimentation has to be carried out to determine the values of the exponents and constants. However, dimensional analysis has one major advantage of identifying the trade-offs among the different variables and utilizing dimensionless groups, thus reducing the number of experimental variations drastically. To illustrate the principles of dimensional analysis, consider the following example:

Example 1-3. Determine the equation which defines the friction factor, f, for a liquid with velocity, $\overline{V}$, density, ρ, and viscosity, μ. The pipe has a diameter D and a roughness ϵ.

Solution: Step 1. The friction factor could be expressed as:

$$f = g\,(\overline{V}, \rho, \mu, D, \epsilon) \qquad (1-10)$$

The sum or difference of values with different dimensions has no meaning, e.g., lm + 1 sec. However, two values with different dimensions may be divided or multiplied to yield a new variable. For example, distance divided by the time required to transverse it, yields velocity which has the dimension of m/sec.

Eq. 1-10 could be rewritten as:

$$f = k\,(\overline{V})^a\ (\rho)^b\ (\mu)^c\ (D)^d\ (\epsilon)^e \qquad (1-11)$$

where, k, a, b, c, d, and e are constants and the other terms are as defined before. The dimensional homogeneity of Eq. 1-11 depends on the constants a, b, c, d, e. For dimensional homogeneity to exist, the sum of the exponents for a particular dimension on the left hand side of Eq. 1-11 should be equal to that on the right side.

Step 1. The different variables in Eq. 1-11 could be expressed in terms of the basic dimensions of length, L, mass, M and time, T, as follows:

$$\begin{aligned} \bar{V} &= \text{m/sec} = L/T \\ \rho &= \text{kg/m}^3 = M/L^3 \end{aligned} \qquad (1-12)$$

$$\mu = \frac{\text{kg}}{\text{m-sec}} = \frac{M}{LT}$$

$$\begin{aligned} D &= \text{m} = L \\ \epsilon &= \text{m} = L \\ f &= \text{dimensionless} = 1 = L^o T^o M^o \end{aligned}$$

Putting Eq. 1-12 into Eq. 1-11 yields:

$$M^o T^o L^o = K \left(\frac{L}{T}\right)^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{LT}\right)^c (L)^d \quad (L)^e$$

where the terms are as defined before. If the dimensions are homogeneous, then the sum of the exponents on the left hand side should equal that on the right hand side. Hence, for length L,

$$0 = a\text{-}3b\text{-}c\text{+}d\text{+}e \qquad (1-14)$$

for time T,

$$0 = -a\text{-}c \text{ or } a = -c \qquad (1-15)$$

and for mass M,

$$0 = b\text{+}c \text{ or } b = -c \qquad (1-16)$$

Substituting Eq. 1 – 15 and 1 – 16 for a and b, respectively, into Eq. 1 – 14 yields:

$$0 = -c\text{+}3c\text{-}c\text{+}d\text{+}e \qquad (1-17)$$

or d = -c -e. It should be noted that we have five unknowns a, b, c, d, and e but only three linear equations can be set up from the equations of length, mass and time. The remaining two unknowns will have to be determined experimentally. Substituting the values of a, b, and d as defined from Eq. (1-15) through (1-17) into Eq. (1-11) yields:

$$f = k \left(\frac{\mu}{\rho \bar{V} D}\right)^c \left(\frac{\epsilon}{D}\right)^e \qquad (1-18)$$

If c is a constant, its negative is still a constant, hence Eq. (1-18) could be rewritten as:

$$f = k \left(\frac{\rho \bar{V} D}{\mu}\right)^{-c} \left(\frac{\epsilon}{D}\right)^{e} \qquad (1-19)$$

The terms $(\rho \bar{V} D/\mu)$ is nothing more than the Reynolds number and the term ϵ/D is the roughness factor. The graph showing the relationship among the friction factor f, the Reynolds number and roughness factor is very familiar to all engineers as it can be found in all standard hydraulic textbooks, as shown in Fig. 1-1.

It should be noted from Eq. 1-19 that the Reynolds number behaves as a pseudo-variable defining the trade-off and interaction between the velocity, density, and viscosity of the liquid and the pipe diameter. Although it would be correct to say that the friction factor is dependent on the Reynolds number, it would be incorrect to conclude that the friction factor is independent of the viscosity, velocity and density of the liquid and the pipe diameter.

The major advantage of dimensionless groups is in the reduction of experimental permutations required to establish the graph and the compactness of result presentation as the following example would illustrate.

Example 1-4. (a) If it takes ten values to define the relationship between the friction factor f and any of the variables, ρ, μ, D, $\bar{V}$, and ϵ, determine the total number of experimental combinations required to establish such relationship. (b) If Eq. 1 – 19 is utilized, determine the number of experimental combinations required to give the same results as those in (a).

Solution – (a) in order to facilitate the determination of the causation and effect relationship, the experiment is carried out by varying one variable at a time while holding the other constant. Hence, f is measured while varying ρ ten times, and the other variables are held constant. After that, ρ is maintained constant while, μ is varied ten times and so on down to the last variable. The number of experimental combinations is:

$$n = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000 \qquad (1-20)$$

If a page will contain ten lines, then 1,000 pages will be required for result presentation. (b) From Eq. (1-19) by varying the Reynolds number ten times for each ϵ/D, the total number of experimental combinations is equal to:

$$n = 10 \times 10 = 100$$

The results could be drawn in one page similar to Fig. 1-1 which gives the same amount of information as the 1,000 pages of graphs discussed in the preceeding example.

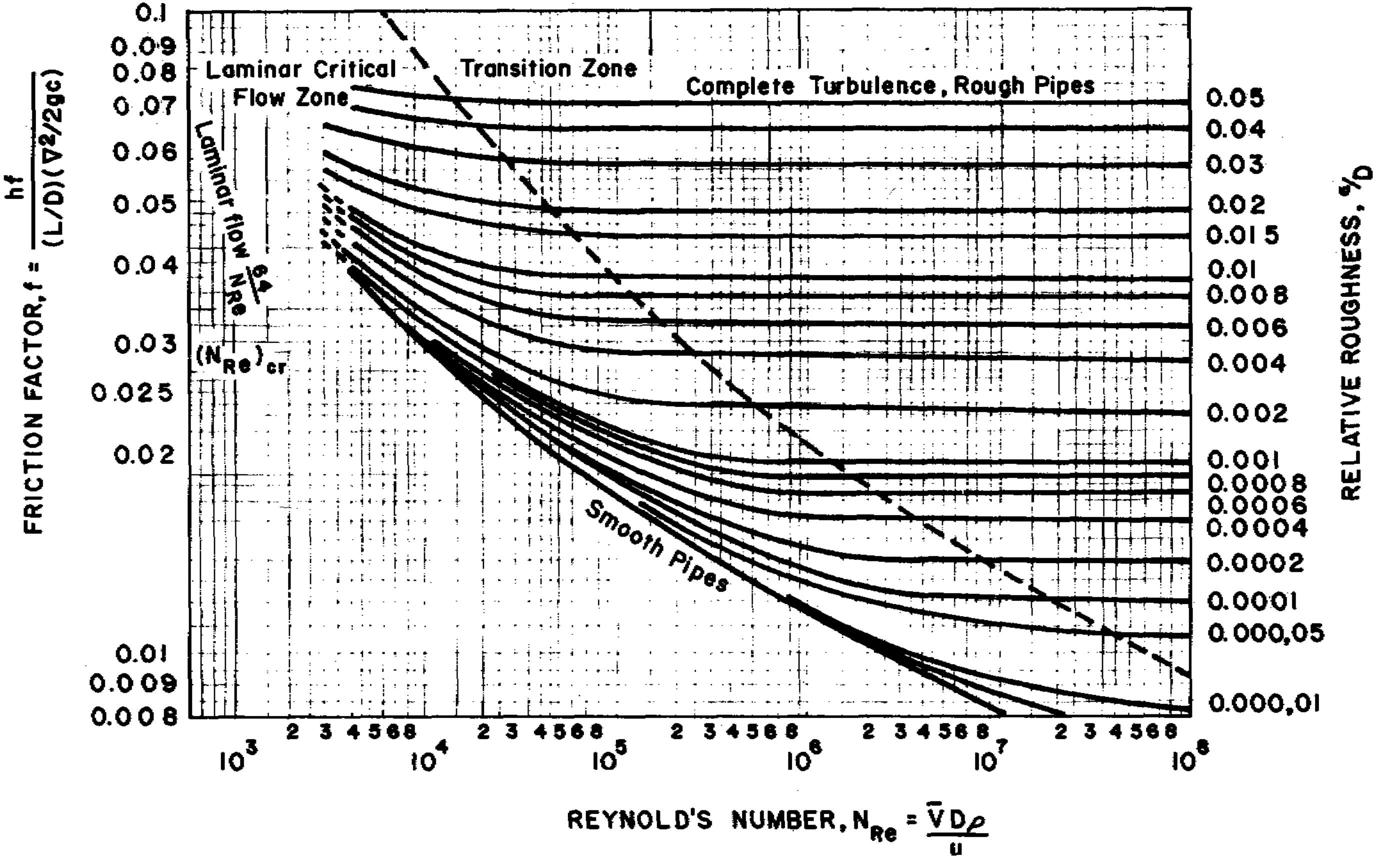


FIG. 1-1 Moody diagram for friction in pipes.

Scale Up – The dimensionless groups are also useful for constructing models of unit processes or scaling up of results from models to prototypes. Table 1-1 shows the common dimensionless groups for scaling up various physical and chemical phenomena for biological waste treatment processes. In addition to dimensionless groups, lumped parametric systems could be derived from dimensional analysis. Those lumped parameters are very useful in design specially in evaluating tradeoffs and interaction of the different operating variables and their effects on biological wastewater treatment process performance. The lumped parametric systems commonly encountered in biological waste treatment process are also listed in Table 1-1. Their derivation and use are described in the succeeding chapters.

For scale up and scale down, the following should hold true:

$$(N)_m = (N)_p \qquad (1-21)$$

where N is the dimensionless variable with m and p as subscripts denoting model and prototype, respectively.

> Example 1-5. Construct a model to simulate the friction factor in a 4m diameter concrete pipe with water flowing at 3m/sec. The temperature is 30°C. The model is to have a diamater of 60mm only.
>
> Solution: For the friction factor in the model to be the same as in the prototype, the following should hold true:
>
> $$\left(\frac{\epsilon}{D}\right)_m = \left(\frac{\epsilon}{D}\right)_p \qquad (1-22)$$
>
> and
>
> $$\left(\frac{\rho D\bar{V}}{\mu}\right)_m = \left(\frac{\rho D\bar{V}}{\mu}\right)_p \qquad (1-23)$$
>
> such that Eq. 1-21 will be satisfied. Concrete has an ϵ value of 0.003 to 0.01. For large pipes, the higher value is more reasonable. Hence,
>
> $$\left(\frac{0.01}{4000}\right) = \left(\frac{\epsilon}{60}\right)$$
>
> $$\epsilon = 0.00015$$

From Unit Operation by Brown (1950), wrought iron or commercial steel pipes have an ϵ value of 0.00015. So, either of those materials could be used. The Reynolds number in the prototype must be equal to that of the model. Hence,

$$4000 \times 0.3 \times 10^3 \text{mm/sec} \times \left(\frac{\rho}{\mu}\right)_p = 60 \times \left(\frac{\bar{V}\rho}{\mu}\right)_m$$

or

$$\bar{V}_m = \frac{4000 \times 0.3 \times 10^3}{60} = 20 \times 10^3 \text{ mm/sec}$$

or 20.0 m/sec

It will be difficult to pump water at such a high velocity of 20.0 m/sec. The designer may fix the velocity at 10 m/sec and use a liquid which has a ρ/μ value of approximately twice that of water at 30°C. This could be derived from:

$$\left(\frac{\rho D\bar{V}}{\mu}\right)_p = \left(\frac{\rho D\bar{V}}{\mu}\right)_m \qquad (1-24)$$

$$4000 \times 0.3 \times 10^3 \left(\frac{\rho}{\mu}\right)_{water} = 60 \times 10 \times 10^3 \left(\frac{\rho}{\mu}\right)_m$$

or

$$\left(\frac{\rho}{\mu}\right)_m = 2 \left(\frac{\rho}{\mu}\right)_{water} \qquad (1-25)$$

From chemical handbooks, benzene at 30°C has properties which satisfy Eq. (1-25). The velocity may be slightly adjusted to maintain the equality defined by Eq. (1-24).

It should be noted that there is in fact a wide range of alternatives in designing the model. The velocity, diameter of the pipe and even the liquid may be altered to simulate the conditions of dynamic similitude.

The succeeding chapter will discuss how to adjust the strength of wastewater, residence time, microbial mass concentration, as well as geometric similarity in the design of pilot plants. However, in some instances wherein difficulty is encountered in satisfying all the scale up parameters, the need arises for the use of best judgement in identifying the parameter which has the dominant effect on the performance of the process. For this reason, there should be a limit on the ratio of the model and prototype sizes. The effect of the neglected parameters in the model will tend to be magnified thus introducing a large error, if the scale up ratio is very large.

DIMENSIONLESS GROUPS AND DIFFERENTIAL EQUATIONS

Even if the chemical and physical phenomena could be defined by differential equations with numerical solutions, engineers often prefer to use nomographs to facilitate calculations. In other instances, the differential equations may be non-linear which could require expensive utilization of computers, in which case nomographs are preferred.

If every parameter of the differential equation is treated independently, the nomographs will be very voluminous. The utility of the nomograph will be limited since the tradeoffs among different design variables could not be easily visualized. As an alternative, the differential equation may be reduced to dimensionless form. The effect is similar to that of dimensional analysis discussed in the previous section. The following example illustrates the procedure and advantage of reducing differential equations to dimensionless form.

Example 1-6. At steady state the concentration of a pollutant in a biological reactor could be expressed by the following differential equation:

$$D_L \frac{d^2C}{dx^2} - \overline{V} \frac{dC}{dx} - kC = 0 \qquad (1-26)$$

where D_L is the diffusivity of the pollutant, m^2/day; C is the concentration of pollutant, mg/1; x is the distance of the particular biological reactor section of interest from the influent end, m; $\overline{V}$ is the cross sectional velocity of the wastewater stream, m/day; and k is the kinetic constant of degradation, day^{-1}. The boundary conditions for Eq. (1-26) are:

$$x = 0, \quad C = C_o \qquad (1-27)$$

and

$$x = L \quad C = C_e$$

where L is the length of the biological reactor, m; C_o is the initial concentration of pollutant, mg/1 and C_e is the effluent concentration of pollutant, mg/1. Construct a nomograph for Eq. 1-26 which when subjected to the boundary conditions of Eq. 1-27 has the following solu-

$$C(x) = C_o \exp\left\{ \left(\frac{\overline{V} - \sqrt{\overline{V}^2 + 4kD_L}}{2\,D_L} \right) x \right\} \qquad (1-28)$$

$$C_e = C_o \exp\left\{ \left(\frac{\overline{V} - \sqrt{\overline{V}^2 + 4kD_L}}{2\,D_L} \right) L \right\} \qquad (1-29)$$

Solution: C_e from Eq. 1-29, is a function of $C_o, \overline{V}, k, D_L$ and L. Hence, if a nomograph is constructed allowing for 10 variations affecting C_e, Eq. 1-29 has to be solved 100,000 times and 10,000 graphs have to be constructed. Eq. 1-26 could be reduced to dimensionless form as follows:

$$\begin{aligned} \text{Let } C^* &= C/C_o \\ x^* &= x/L \\ \Theta &= L/\overline{V} = \frac{LA}{\overline{V}A} = \frac{\text{Vol}}{\text{flow rate}} = \frac{V}{Q} \end{aligned} \qquad (1-30)$$

then:

$$\begin{aligned} \frac{dC}{dx} &= \frac{C_o dC^*}{L\,dx^*} \\ \frac{d^2C}{dx^2} &= \frac{C_o}{L^2}\frac{d^2C^*}{dx^{*2}} \end{aligned} \qquad (1-31)$$

Substituting Eq. 1-30 and 1-31 into Eq. 1-26 yields:

$$D_L \frac{C_o}{L^2}\frac{d^2C^*}{dx^{*2}} - \overline{V}\frac{C_o}{L}\frac{dC^*}{dx^*} - k\,C_oC = 0 \qquad (1-32)$$

Eq. 1-32 could be rearranged as:

$$\frac{D_L}{\overline{V}L}\frac{d^2C^*}{dx^{*2}} - \frac{dC^*}{dx^*} - k\Theta C^* = 0 \qquad (1-33)$$

The boundary conditions defined by Eq. 1-27 could be modified using the relationship given in Eq. 1-30 as:

$$\begin{aligned} x^* &= 0 \qquad C^* = 1 \\ x^* &= 1 \qquad C^* = C_e/C_o \end{aligned} \tag{1-34}$$

The solution to Eq. 1-33 and 1-34 is:

$$\frac{C_e}{C_o} = \exp\left\{ \frac{1 - \sqrt{1 + 4k\Theta \dfrac{D_L}{\bar{V}L}}}{2\dfrac{D_L}{\bar{V}L}} \right\} \tag{1-35}$$

Hence, the term C_e/C_o could be defined as a function of the two dimensionless group $k\Theta$ and $D_L/\bar{V}L$. Fig. 1-2 shows the nomograph of Eq. 1-35 with some 10 lines in one page instead of the 10,000 lines in 1,000 pages required for a nomograph of Eq. 1-29.

Scale Up – The differential equation defines the characteristic change a system induces on the inputs or external stimuli. The inputs are defined by the boundary conditions at the influent end.

Hence the output of the system or biochemical waste treatment process will be the same with that of the bench scale model if all the variables are the same. In the above mentioned example, the effluent pollutant concentrations in the prototype and bench scale model will be equal if $\bar{V}$, D_L, k, C_o and L of the prototype are also equal to those of the bench scale model.

As described in the previous section, it would be very difficult to construct a model which scales down all those variables. For example, for the length and the cross sectional velocity of the prototype and the model to be equal then the width and depth should be equal too. Hence, the model must be of the same size as the prototype.

On the other hand, from the dimensionless form of the differential equation, the scale up parameters could be established without even solving the differential equations. From Eq. 1-33 or Eq. 1-35, for the waste treatment process to attain equal waste treatment efficiencies, then the following should hold true:

$$(k\Theta)_m = (k\Theta)_p \tag{1-36}$$

and

TABLE – 1-1

SUMMARY OF COMMON DIMENSIONLESS GROUPS

NAME OF GROUP	LINEAR MOTION	ROTATIONAL MOTION
Dispersion Number (d)	$\frac{D_L}{\bar{V}L}$	
Reynolds Number (N_{Re})	$\frac{D\bar{V}\rho}{\mu}$	$\frac{D^2\omega\rho}{\mu}$
Froude Number (N_{Fr})	$\frac{\bar{V}^2}{gl}$	$\frac{\omega^2 D}{g}$
Nusselt Number (N_{nu})	hD/r	
Power Number (Np)		$\frac{Pg}{\rho\omega^3 D^5}$
Prandt Number (N_{pr})	$\frac{C_{p}\mu}{r}$	
Shearing	$\sqrt{\frac{P}{\mu V}}\ \theta$	

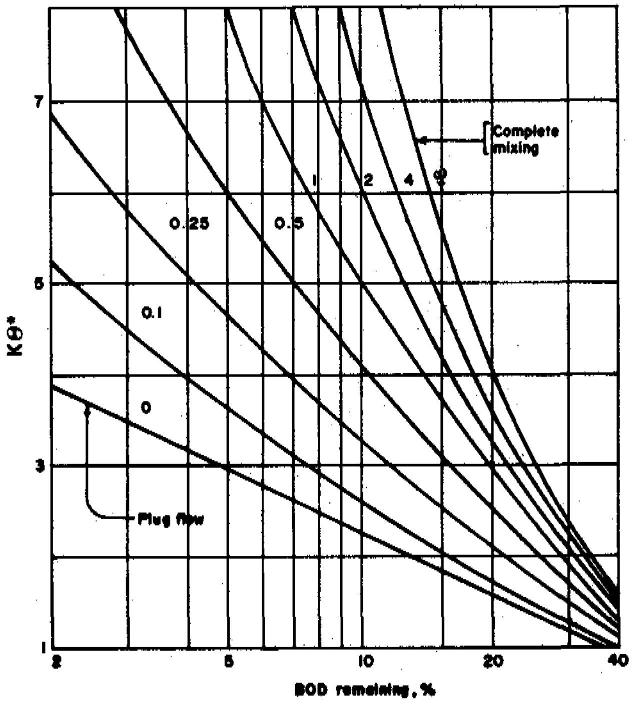


FIG. 1-2 Thirumurthi Chart for Wehner – Wilhelm equation. The numbers adjacent to each curve are corresponding dispersion numbers.

$$\left(\frac{D_L}{\bar{V}L}\right)_m = \left(\frac{D_L}{\bar{V}L}\right)_p \qquad (1-37)$$

in which case, the interactions and tradeoffs of the different variables affecting the waste treatment system performance could be analyzed.

> Example 1-7. Scale up the results of a model treating $10m^3$/ day of wastewater to a prototype to treat $100m^3$/day of the same waste under similar environmental conditions.
>
> The model test shows that to attain 95% reduction of the pollutant, the wastewater has to be retained for 6 hours in the biological reactor. The model is 0.5m wide, 2m long and 0.5m deep.
>
> Solution: a) Since the wastewater is to be treated under similar environmental condition, then $k_m = k_p$ or Eq. (1-36) is simplified to $\Theta_m = \Theta_p$. If Θ_m = 6 hours, then Θ_p is also 6 hours. b) The volume of the prototype should be:
>
> $$V_p = \frac{6 \text{ hrs}}{24 \text{ hrs/day}} \times \frac{100m^3}{\text{day}} = 25 \text{ m}^3$$
>
> For geometric similarity to hold true, then:
>
> $$K^3 = 25/.5 = 50 = \frac{V_p}{V_m}$$
>
> $$K = 3.68$$
>
> The other dimensions could be calculated as follows:
>
> $X_p = 3.68 \times X_m = 7.36$ m say 7.5 m
> $Z_p = 3.68 \times Z_m = 1.8$ m say 1.9 m
> $Y_p = 3.68 \times Y_m = 1.8$ m say 1.9 m
>
> c) The cross sectional velocity of the model
>
> $V_m = 10 \text{ m}^3 / (0.5\text{m} \times 0.5\text{m}) = 40\text{m/day}$
>
> For the prototype
> $V_p = 100 \text{ m}^3 / (1.9 \times 1.9) = 27.7$ m/day

$$\left(\frac{D_L}{\bar{V}L}\right)_m = \left(\frac{D_L}{\bar{V}L}\right)_p$$

$$\left(\frac{D_L}{40\ \text{m/day} \times 25}\right)_m \doteq \left(\frac{D_L}{27.7 \times 7.36}\right)_p$$

$$(D_L)_p = \frac{27.7 \times 7.36}{40 \times 2}\ (D_L)_m$$

$$(D_L)_p = 2.54\ (D_L)_m$$

The mixing and its effect on the dispersion of pollutants in the prototype should be approximately 2.54 times when compared to the model.

COMMON DIMENSIONLESS GROUPS

Table 1-1 summarizes the different dimensionless groups required for scaling up the results from bench scale units to prototypes of wastewater treatment processes. The detailed discussions on the utilization of those dimensionless groups in the different wastewater treatment processes are made in later chapters. Derivations are made in this section of some commonly encountered dimensionless groups.

Dispersion – Dispersion is the movement of the pollutant, the microbial mass, and other reactants in the biochemical reactor. For reaction to take place, the pollutant, microbial mass, oxygen and nutrients should come in contact with each other. Hence the higher the dispersion or mixing, the higher the chance is of the reactant to come in contact with each other and the more effective the waste treatment becomes.

However, the higher the dispersion or mixing, the faster the reactants move from the influent to effluent end. Hence, the reactants are prematurely released from the biochemical reaction tank. This phenomenon is known as short circuiting which lowers the efficiency of the wastewater treatment process. It is therefore important to control the dispersion in such a way that the wastewater treatment process performance is optimized. The dispersion phenomenon and the corresponding equation are derived as described.

Consider a rectangular aeration tank shown in Fig. 1-3 with wastewater flowing along the X-axis. Since the width and depth of the tank is very small compared to the length, then instantaneous mixing in the Y and Z direction could be assumed. For the section along the X-axis of thickness ΔX, the accumulation of incoming and outgoing pollutant concentration could be expressed as:

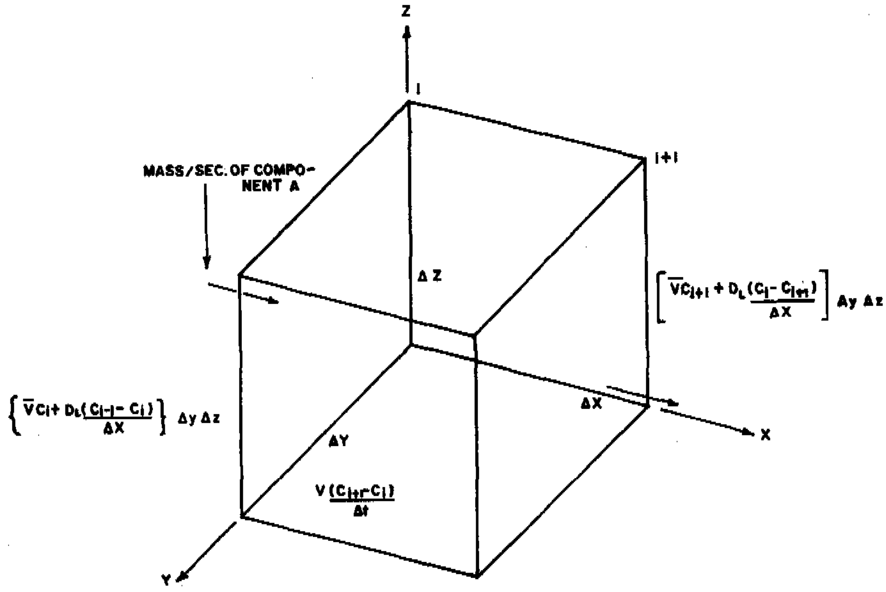


FIG. 1-3 Mass flux of component A in X direction.

Mass in = inflow + inward dispersion

$$= \bar{V}AC_{i-1} + D_LA\left(\frac{C_{i-1} - C_i}{\Delta X}\right) \qquad (1-38)$$

Mass out = outflow + outward dispersion

$$= \bar{V}AC_i + D_L A \left(\frac{C_i - C_{i+1}}{\Delta X} \right) \qquad (1-39)$$

$$\text{Accumulations} = V \left\{ \frac{C_{i+1} - C_{i-1}}{\Delta t} \right\} \qquad (1-40)$$

where the terms are given in Fig. 1-3.

From the law of conservation of mass,

Mass in = mass out + accumulation (1 – 41)
or putting in Eq. 1-38 to 1-40 into Eq 1-41 yields:

$$\bar{V}AC_i + D_L A \left(\frac{C_{i-1} - C_i}{\Delta X} \right) = \bar{V}AC_{i+1} + D_L A \left(\frac{C_i - C_{i+1}}{\Delta X} \right) + V \left(\frac{C_i - C_{i-1}}{\Delta T} \right) \qquad (1-42)$$

but since V = AΔX, Eq. 1-42 could be simplified further when $\Delta T \rightarrow 0$ and $\Delta X \rightarrow 0$ as:

$$\frac{\partial C}{\partial t} = -\bar{V} \frac{\partial C}{\partial X} + D_L \frac{\partial^2 C}{\partial X^2} \qquad (1-43)$$

If the pollutant undergoes first order degradation in the section ΔX, the mass out expression in Eq. 1-39 could be rewritten as:

Mass out = outflow + outward dispersion + degradation

$$= \bar{V}AC_i + D_L A \frac{\left(C_i - C_{i+1} \right)}{\Delta X} + k\, V\, C_i \qquad (1-44)$$

where k is the degradation rate constant and the other terms are as defined before. Substituting Eq. 1-44 into Eq. 1-41 in lieu of Eq. 1-39 yields:

$$\bar{V}AC_i + D_L A \frac{(C_{i-1} - C_i)}{\Delta X} = \bar{V}AC_{i+1}$$

$$+ D_L A \frac{(C_i - C_{i+1})}{\Delta X} + k\ VC_i + V \left[\frac{C_{i+1} - C_{i-1}}{\Delta\ t} \right] \qquad (1-45)$$

which attains the limit

$$\frac{\partial C}{\partial t} = -\bar{V} \frac{\partial C}{\partial X} + D_L \frac{\partial^2 C}{\partial X^2} - kC \qquad (1-46)$$

when $\Delta X \rightarrow 0$ and $\Delta T \rightarrow 0$. At steady states $\partial c/\partial t$ is equal to zero. The partial differential equation simplifies to:

$$D_L \frac{\partial^2 C}{\partial x^2} - \bar{V} \frac{\partial C}{\partial x} - k\ C = 0 \qquad (1-47)$$

For Example 1-6, the dimensionless groups required for scaling Eq. (1-47) are:

$$(k\Theta)_m = (k\Theta)_p \qquad (1-48)$$

and

$$\left(\frac{D_L}{\bar{V}L}\right)_m = \left(\frac{D_L}{\bar{V}L}\right)_p \qquad (1-49)$$

Eq. (1-49) defines the conditions required for scaling the reaction rate and detention time. The term $D_L/\bar{V}L$ is known as the dispersion number. It is this dimensionless group which is used for scaling the dispersion in a biochemical reactor.

The reaction rate constant, k, depends on the chemical composition of the wastewater, the temperature and presence of nutrients or poisons which could increase or reduce its value. The detention time could be controlled by increasing or reducing the flow rate or the volume of the wastewater treatment tank.

The dispersion number is controlled by adding baffles perpendicular to the wastewater flow, adding mixers to increase the turbulence, or increasing the length of the tank. Increasing the tank length and adding baffles tend to decrease the dispersion number.

Tracer study – The dispersion number is measured by injecting a conservative chemical such as sodium chloride or flourescent dye at the inlet of the tank either as an impulse load or as a continuous feed. The concentration of sodium chloride in the outlet or effluent end of the tank is measured at different time intervals. The impulse load is more commonly used as it does not require large quantities of tracer materials. Eq. 1-46 could be reduced to a dimensionless form as

$$\Theta \frac{\partial C}{\partial t^*} = \frac{D_L}{\bar{V}L} \frac{\partial^2 C^*}{\partial X^{*2}} - \frac{\partial C^*}{\partial X} - C^*\Theta k \qquad (1-50)$$

where $t^* = t/\Theta$ and the other terms are as defined before. If the tracer is a conservative substance which does not degrade or undergo any chemical reaction, k is equal to zero. If the tracer is added as an impulse then the boundary conditions for Eq. (1-50) are:

$$\begin{aligned} &C^*(X^*, 0) = 0.0 \quad X^* > 0 \\ &C^*(0, t^*) = C_o\,\delta(t) \qquad (1-51) \\ &C^*(X^*, \infty) = 0.0 \end{aligned}$$

where $\delta(t)$ is the Dirac delta function and the other terms are defined before

The solution to Eq. (1-50) and (1-51) are shown in Fig. 1-4. It should be noted that when the dispersion number is equal to zero, the impulse tracer injected at the influent end of the tank is unaltered in shape as it comes out in the effluent section. In fact, the tracer or input function is merely shifted by one unit in the t^* axis. The flow under this condition is known as plug flow.

As the dispersion number increases, the tracer in the effluent is spread out over a longer period as shown in Fig. 1-4. If the spreading of the tracer in the effluent around $t^* = 1$ is calculated as the standard deviation of the output function, then a correlation between this variable and the dispersion number should exist. Thirmuthi (1969) determined the correlation between the dispersion number and the standard deviation of the output function as:

$$\sigma^2 = 2\left(\frac{D_L}{\bar{V}L}\right) + 8\left(\frac{D_L}{\bar{V}L}\right)^2 \qquad (1-52)$$

where σ is the standard deviation of the output function and the other terms are as defined before.

From Fig. 1-4, when $D_L/\bar{V}L = 0$, all the tracer comes out at the same time at $t^* = 1$. Since the tracer represents the movement of the wastewater bulk in the tank, then at $D_L/\bar{V}L = 0$, all the wastewater particles have a residence time equal to the detention time, since by definition $t^* = t/\Theta$. Under this condition we could visualize the wastewater particles to move uniformly under rigid discipline with velocity, $\bar{V}$ in the tank. This condition takes place when mixing is negligible. This type of flow pattern is known as plug flow.

As $D_L/\bar{V}L$ increases, some tracer comes out of the tank at $t^* < 1$, as shown in Fig. 1-4. Hence, those particles stay inside the tank for a period less than Θ. Those particles which stay less than the average detention time are said to undergo short circuiting. Short circuiting is often detrimental to the efficiency of waste treatment unit processes. Briefly, it means that watewater is rejected before its expected time for degradation or stabilization. However, in some instances such as in autocatalytic reactions discussed in Chapter III, short circuiting to some extent is desirable.

Again as $D_L/\bar{V}L$ increases, a larger quantity of tracers stay longer than at $t^* = 1$, as shown in Fig. 1-4. Under this condition, a larger quantity of wastewater overstays after having been degraded or stabilized. The stabilized wastewater occupy portions of the tank volume which has become useless. Again, the effectivity of the unit process is lowered. This phenomenon is known as back mixing. For a long time, aeration tanks were designed with large length to width ratios to minimize short circuiting, and back mixing.

N – tank in Series – In a completely mixed tank, the concentration of tracer or any substance is uniformly distributed throughout the tank. This phenomenon takes place when dispersion is very high. In some instances, the biological waste treatment reactor may be subdivided into a number of chambers by baffles. Each chamber could be approximated as a completely mixed tank if the contents are stirred vigorously.

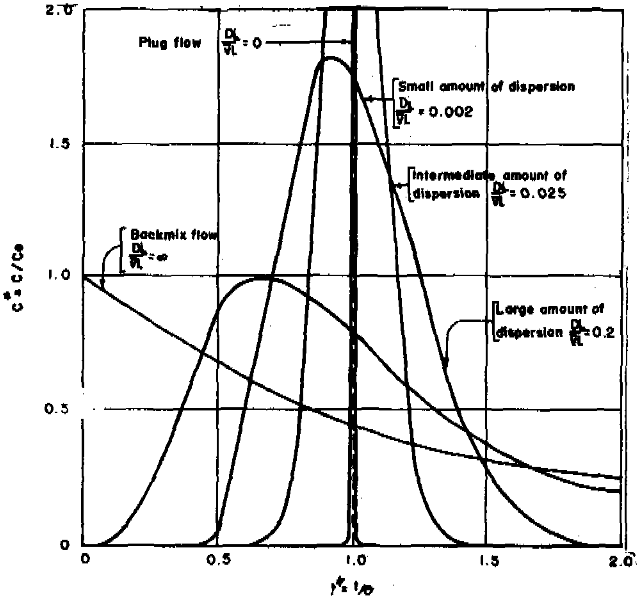


FIG. 1-4 Curves for various extents of backmixing as predicted by the dispersion model.

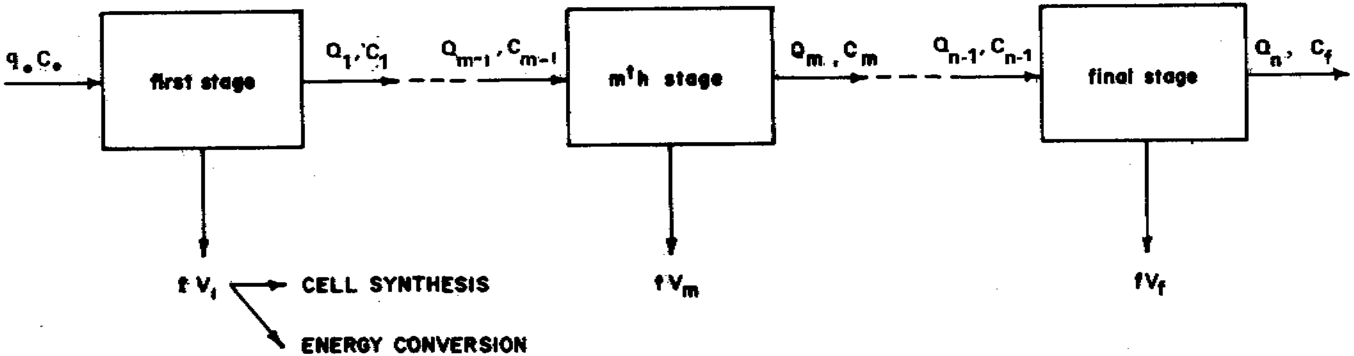


FIG. 1-5 Substrate mass balance chart.

Fig. 1-5 shows the schematic diagram of an n-tank in series reactor and the corresponding mass balance. If an impulse concentration of a tracer $\delta(t)$ is injected before the first tank, the mass balance yields:

$$\text{mass in} = q\,C_o\,\delta\,(t) \qquad (1-53)$$

$$\text{mass out} = q\,C_1 \qquad (1-54)$$

$$\text{accumulation} = \frac{V}{n}\,\frac{dC_1}{dt} \qquad (1-55)$$

From the law of conservation of mass, Eq. 1-53 should equal to the sum of Eq. 1-54 and 1-55 or

$$qC_o\ \delta\ (t) - q\,C_1 = \frac{V}{n}\,\frac{dC_1}{dt} \qquad (1-56)$$

where the terms are as defined before. Eq. 1-56 is best solved by method using Laplace transform, which yields:

$$C_1\,(t) = C_o\ \exp\,(-nq\ t/V) \qquad (1-57)$$

The mass balance in the second tank yields:

$$qC_1 = q\,C_2 + \frac{V}{n}\,\frac{dC_2}{dt} \qquad (1-58)$$

where C_1 is defined by Eq. 1-57. Since, $C_2 = 0$ at $t = 0$, solving Eq. 1-58 with Eq. 1-57 for C_1 results to:

$$C_2 = C_o\ t\exp\,(-nq\,t/V) \qquad (1-59)$$

The process could be repeated to the nth tank. This results to an effluent concentration,

$$C_n = \frac{C_o\ t^{n-1}}{(n-1)!}\exp\,(-nq\ t/V) \qquad (1-60)$$

As n becomes very large, C_n approaches the value given when $D_L/\bar{V}L \rightarrow 0$ as shown in Fig. 1-4. Hence, if $n > 5$, the dispersion number, $D_L/\bar{V}L$ could be approximated by the following formula

$$\frac{D_L}{\bar{V}L} \approx 0.5/n \qquad (1-61)$$

To prevent short circuiting, baffles are often added to the unit process tank decreasing the dispersion number defined by Eq. 1-61. When baffles are added especially in up and down configuration, the flow is often constricted as it changes direction from vertical to horizontal and back to the vertical direction as shown in Fig. 1-6. Even if the channel's cross sectional area is increased in the section of changing flow direction in relation to the area between the baffles, turbulence is created at the top of the baffle shown by point A in Fig. 1-6, since the wastewater tends to follow the shortest route or path of least resistance. Mixing takes place at this section. For this reason, the baffle arrangement is used in flocculators for mixing.

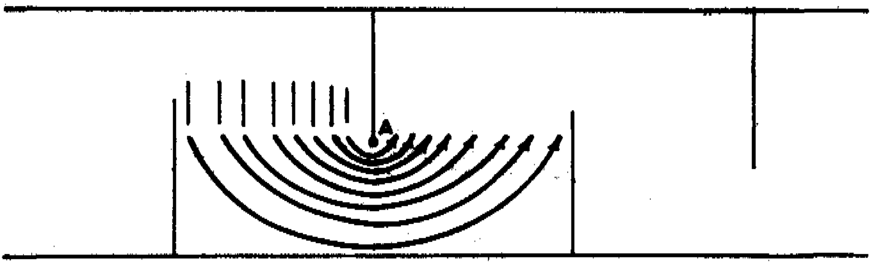


FIG. 1-6 Flow pattern around baffles.

However, the mixing process is lateral and vertical in nature with very minimal effect on longitudinal mixing. As the dispersion number defined in Eq. 1-47 is for longitudinal mixing, the effect of baffles on the dispersion number is to decrease it. Tracer studies have confirmed that as the number of baffles becomes very high, the dispersion number approaches zero even if the lateral and vertical mixing increased proportionally with the number of baffles. This effect is very important in the design of equalization basins where complete mixing without shortcircuiting is envisaged as the ideal case.

It should be noted carefully that the baffles may not be physically present. For example, a long narrow reactor with very low cross sectional velocity will give tracer results as if it were divided into ten or more complely mixed tank in series. A tracer study has to be made before the equivalent number of completely mixed tank in series is determined. Merely counting the physical number of baffles or tanks may underestimate the equivalent number of completely mixed tank in series especially if the mixing in each tank is very small.

Since the completely mixed tank is an idealized case which could only take place when the mixing power per unit volume of reactor approaches infinity, the wastewater treatment processes usually have equivalent number of completely mixed tank in series exceeding the physical number of completely mixed tanks in series. In general analysis, a completely mixed tank is easier to evaluate as it simplifies the partial differential equation for dispersion – reaction to an ordinary differential equation. The ordinary differential equation could be solved manually while the partial differential equation usually requires a computer.

Power number – The energy required to operate a wastewater treatment process is very important in the evaluation and selection of the appropriate process equipment since the cost of energy has increased dramatically since 1972. In fact for wastewater treatment processes, the concentration of research and development before 1972 was on aerobic process which utilize huge quantities of energy with fairly rapid stabilization rates, odorless by-products and clear effluent. Thus before 1972, energy was not a decisive factor in process selection.

The increase in power consumption is exponential with respect to the increase in the process dimensions and operating parameter such as speed of revolution. In the early 1960's a number of high speed aerator manufacturers have difficulties in living up to their equipment performance guaranty in terms of the weight of oxygen transferred per unit of input power. The difference between the laboratory test and the performance in the field could have been corrected had the proper scale up parameters been evaluated carefully.

Most of the equipment used in wastewater treatment processes involve rotation of impellers, flocculator blades, or disks. The power consumed depends on the properties of the liquid like viscosity and density. Hence, the power P consumed by an equipment could be expressed as:

$$P = f\,(\mu, \rho, D, \omega, g) \qquad (1-62)$$

where g is the acceleration due to gravity and the other terms are as defined before. For dimensional homogeneity Eg. 1-62 could be expressed as:

$$P = K\,\mu^a \rho^b D^c \omega^d \; g^e \qquad (1-63)$$

Where K, a, b, c, d, and e are constants and the other terms are as defined before. In terms of the basic dimensions of time, T, length, L, and mass, M, the variables in Eq. 1-63 could be expressed as:

$$P = \frac{L^2 M}{T^3}$$

$$\mu = \frac{M}{LT}$$

$$\rho = \frac{M}{L^3}$$

$$D = L$$

$$\omega = 1/T$$

$$g = L/T^2 \qquad (1-64)$$

Substituting Eq. 1-64 into 1-63 yields:

$$\frac{L^2 M}{T^3} = K \left(\frac{M}{LT}\right)^a \left(\frac{M}{L^2}\right)^b (L)^c \left(\frac{1}{T}\right)^d \left(\frac{L}{T^3}\right)^e \qquad (1-65)$$

For dimensional homogeniety the exponents for L,M, and T are equated. This yields:

$$\begin{aligned} L &: \quad 2 = -a - 3b + c + e \\ M &: \quad 1 = a + b \\ T &: \quad -3 = -a - d - 2e \end{aligned} \qquad (1-66)$$

Hence,

$$b = 1 - a$$

$$d = 3 - a - 2e$$

$$c = 2 + a + 3b - e$$

or $$c = 2 + a + 3(1-a) - e = 5 - 2a - e \qquad (1-67)$$

Therefore:

$$P = K \mu^a \rho^{1-a} D^{5-2a-e} \omega^{3-a-2e} g^e$$

or

$$\frac{P}{\rho D^5 \omega^3} = K \left(\frac{\mu}{D^2 \omega \rho}\right)^a \left(\frac{g}{D\omega^2}\right)^e \qquad (1-68)$$

The dimensionless term $P/(D^5 \omega^3 \rho)$ is known as the Power number, while $\rho D^2 \omega/\mu$ is the Reynolds number and $\omega^2 D/g$ is the Froude number for a revolving liquid. Hence, for scale up, the power number, the Reynolds number and the Froude number must be conserved. At low turbulence or low Reynolds number, viscous forces are more prevalent than gravitational force such that the Froude number could be ignored. At very high speeds such as in a rapid mixer, the Reynolds number could be neglected while the Froude number becomes significant.

Oldshue (1956) correlated the power number in an aerator with the Reynolds number as shown in Fig. 1-7 and Eq. (1-69) as:

$$P = \omega^3 D^5 f \left\{\frac{D^2 \omega \rho}{\mu}\right\} \qquad (1-69)$$

where all the terms are as defined before,

At very high speed of revolution, Schmitche (1976) established the relationship between Froude number and the power number as shown in Fig. 1-8. For the rotating disk filter which normally operates at speeds of 1-10 rpm, Ouano and Siddiqui (1980) correlated the power number with the Reynolds number as shown in Fig. 1-9.

Mass transfer – Mass transfer is an important phenomenon in wastewater treatment process primarily in aeration, movement of substrate into bacterial sludges, scrubbing of ammonia gas, hydrogen sulfide, and carbon dioxide. Dispersion and mixing is a special type of mass transfer operation in which the pollutant molecule moves in a homogeneous medium. Of significance in wastewater treatment is the transport of matter from gaseous phase into liquid as in aeration, or the transfer of matter from liquid to solid state as in the absorption of pollutants by bacterial cells and activated carbon particles. Adsorption may be followed by absorption as the pollutants penetrate the bacterial cell for metabolic conversion. Oxygen mass transfer to wastewater is the most common mass transfer phenomenon in biological wastewater treatment. The rate of oxygen mass transfer into the wastewater is defined by the equation:

$$\frac{VdO}{dt} = K_L \; A \; \alpha \; (O_s - O) \qquad (1-70)$$

Where V is the volume of the wastewater, m^3; O is the dissolved oxygen concentration, mg/l; t is time, sec; A is the surface area for mass transfer, m^2; O_s is

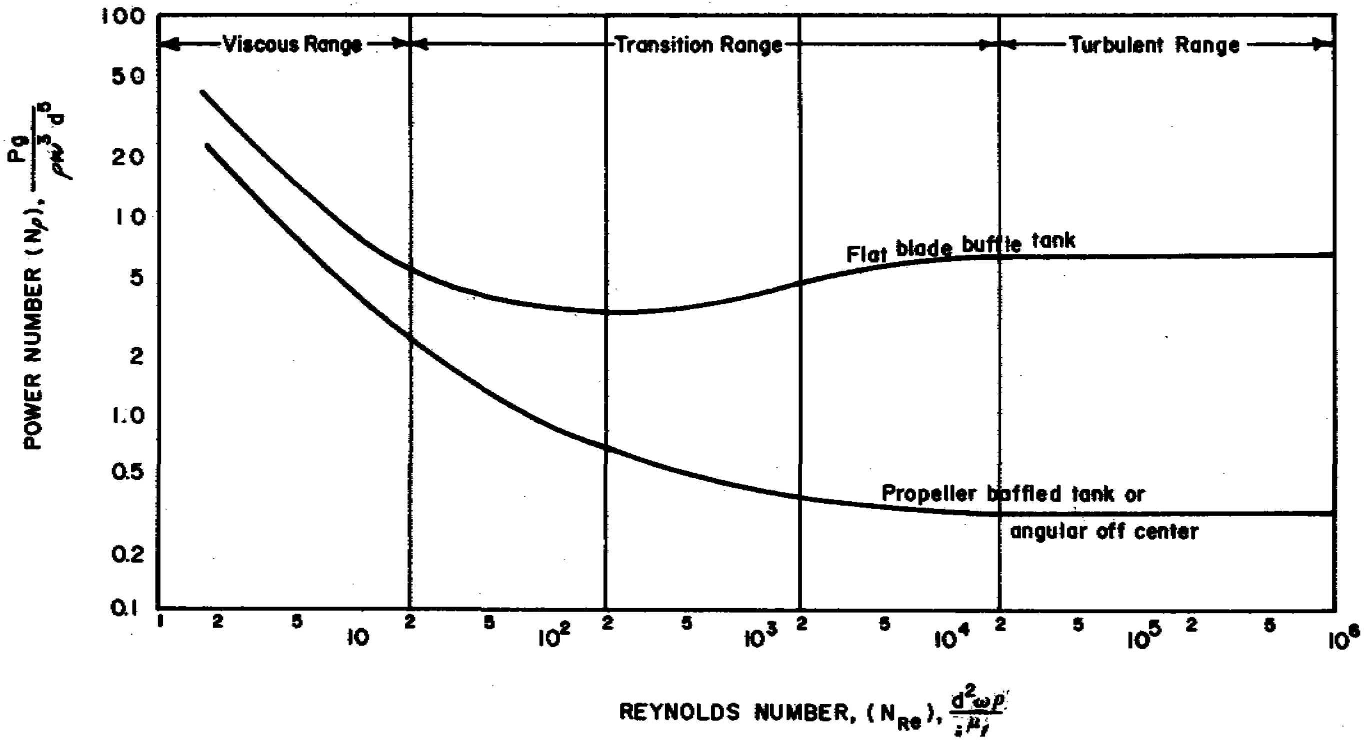


FIG. 1-7 Impeller power consumption.

the dissolved oxygen concentration at saturation, mg/1, K_L is the mass transfer coefficient, m/sec and α is the oxygen mass transfer efficiency.

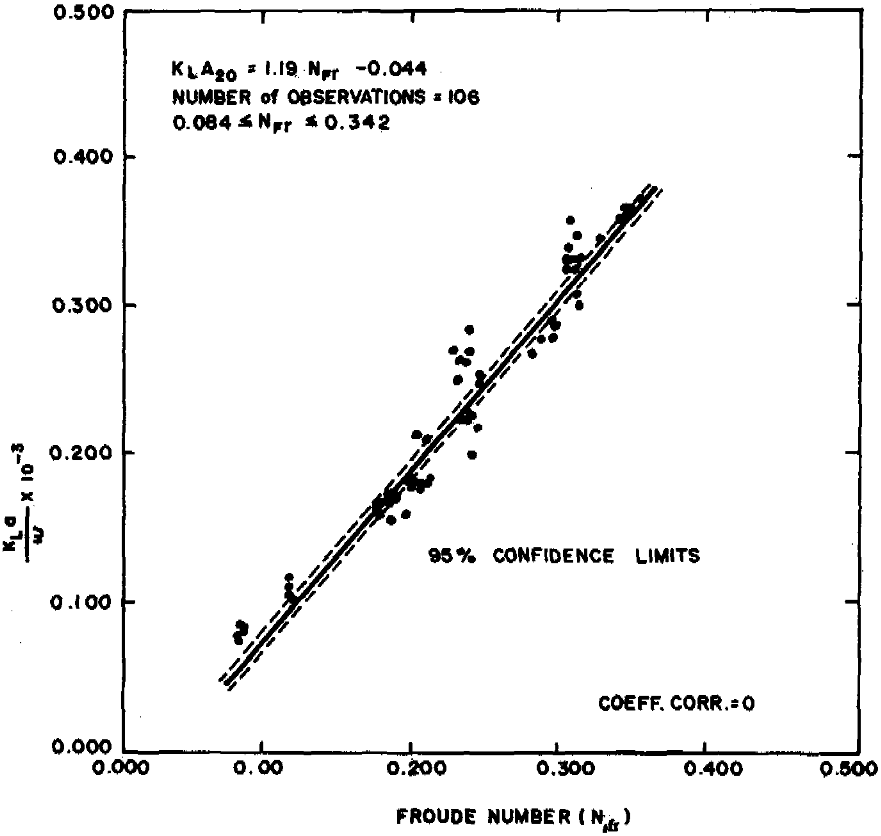


FIG. 1-8 Standard overall oxygen transfer coefficient (K_LA 20) as a function of FROUDE number (N_{fr}) for M.I

The dissolved oxygen concentration at saturation is dependent on the temperature and salt concentration. The mass transfer coefficient K_L depends on the presence, of surface active agents, the turbulence created by the aerator, the diffusivity of oxygen into the wastewater and temperature. As temperature and salt concentration are variable over which designers have little control, aerators are chosen to maximize the oxygen transfer efficiency by increasing K_L.

Schmitche (1976) was able to establish the dependence of the oxygen mass transfer coefficient on the Froude number (N_{fr}) and the physical properties of the impeller in addition to the geometric similarity of the aeration basins. This is expressed as:

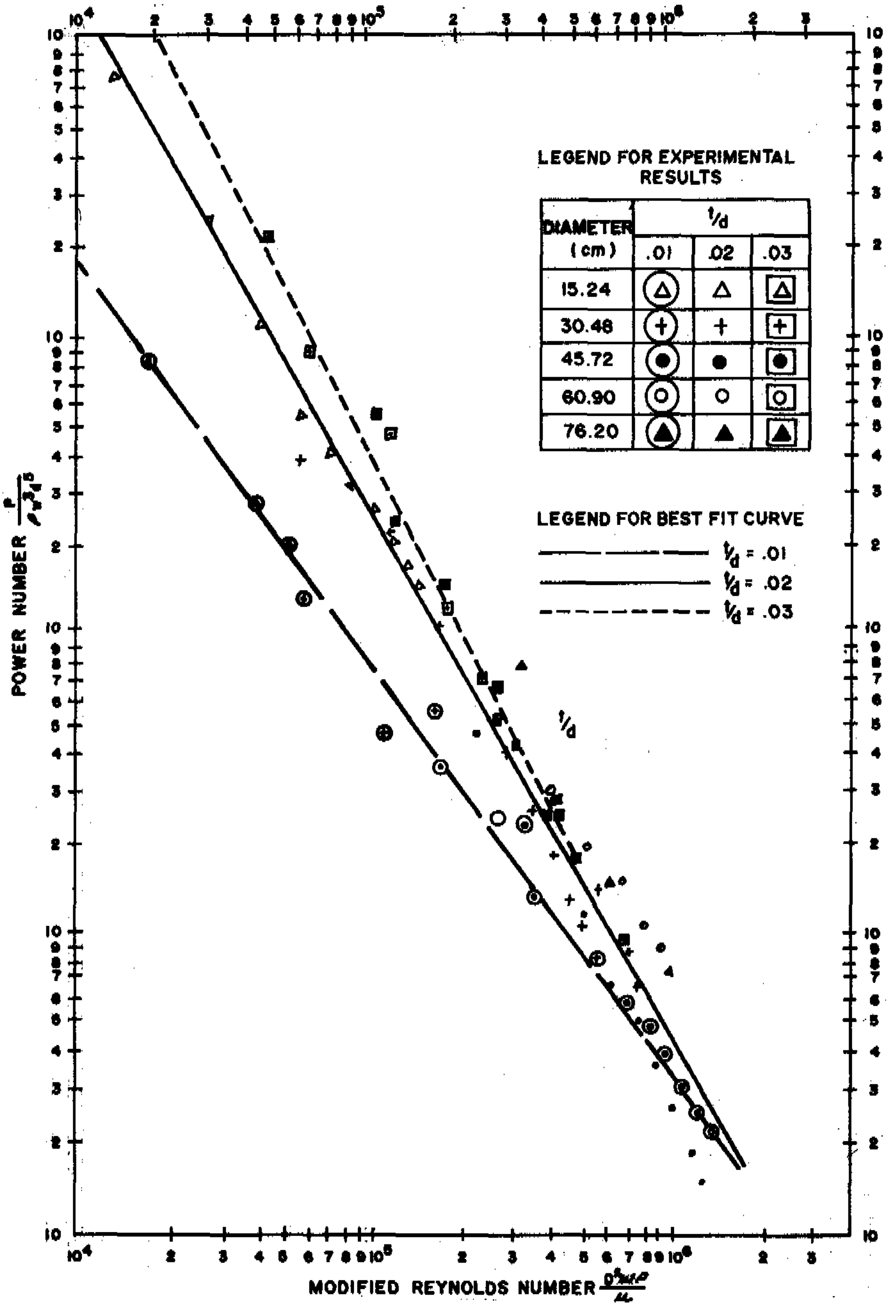


FIG. 1-9 Correlation of Power number and Reynolds number in a rotating disk filter.

$$\frac{K_L a}{\omega} = f\left(N_{fT}, \frac{h}{D}\right) \qquad (1-71)$$

where h is the height of the impeller, m; D is the diameter of the impeller, m; and the other terms are as defined before. On the other hand, Oldshue (1956) correlated the mass transfer efficiency with the Reynolds number power as:

$$K_L a = F^{0.8} \rho^{2.4} D^4 f(N_{re}) \qquad (1-72)$$

where F is the flowrate, m^3/min. Horvath (1966) correlated the oxygen mass transfer coefficient with the Reynolds number as:

$$\frac{K_L a}{\omega} = f\left(N_{re}, N_{sc}, \frac{h}{D}\right)$$

N_{Sc} is the Schmidt number defined as:

$$N_{sc} = \frac{\mu}{D_m M} \qquad (1-73)$$

where D_m is the molecular diffusivity in the liquid, gm/moles/m-hr; M is the molecular weight of the gas; and μ is the viscosity of the liquid, gm/m-hr. At constant temperature, N_{sc} remains constant as μ, D_m and M are independent of the other operational variables except for temperature.

For rotating disk filters, Ouano (1978) correlated the oxygen mass transfer coefficient with the Reynolds number as:

$$\frac{K_L a V/A}{D_m} = k N_{re}^{0.56} \qquad (1-74)$$

Shearing – Shearing is an important phenomenon in the formation of bacterial flocs and slimes. In the previous section, the increase of oxygen mass transfer coefficient with increase in turbulence was discussed. Although the increase in oxygen mass transfer coefficient is desirable, the turbulence created could shear the microbial films in rotating disk filters and trickling filters prematurely. The problem of floc destruction due to shearing has not been pinpointed in activated sludge process to create problems in the secondary settlers. However, fine flocs which do not settle very well have been observed in extended aeration process where the energy input is very high.

Camp (1942) formulated the dimensionless group Gt_d as a parameter determining the destruction of flocs due to shearing. The analysis is based on the shearing force created between two parrallel surfaces. In mathematical form,

$$Gt_d = \sqrt{\frac{P}{V\mu}}\ \Theta \qquad (1-75)$$

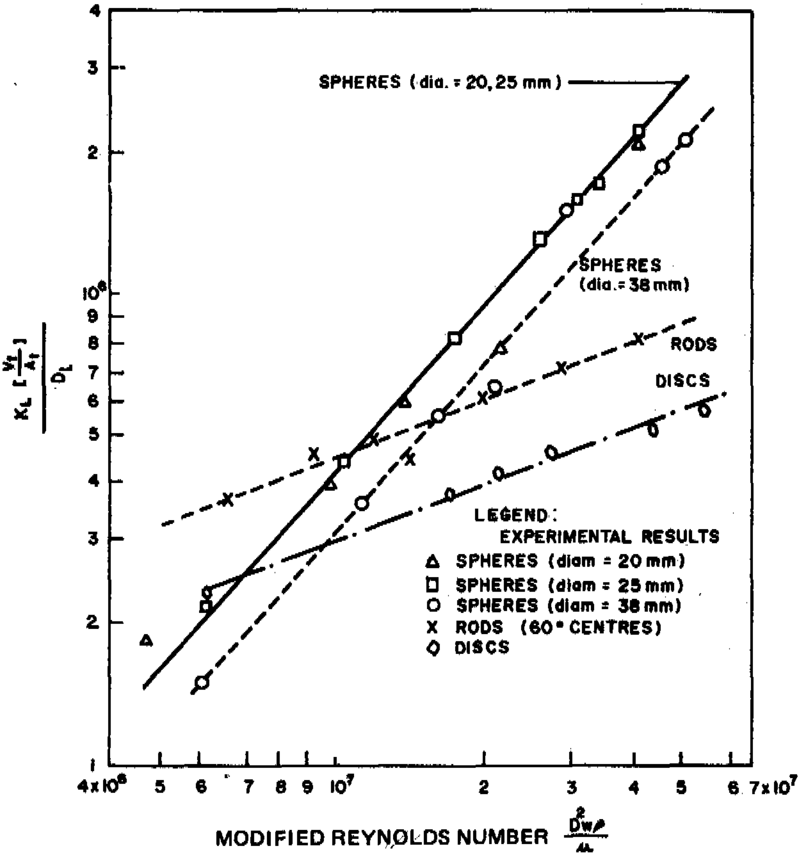


FIG. 1-10 **Effect of Reynold's number on the mass transfer coefficient.**

where the terms are as defined before. In the rotating disk filter the volume of film layer could be expressed as:

$$V = \frac{\pi D^2 \delta}{4} \qquad (1-76)$$

Schlichting (1962) summarized the formulas for the film layer thickness, δ, for laminar and turbulent flow. Ouano (1981) redefined Schlichting's function as:

$$\delta = D N_{re}^{\ a} \qquad (1-77)$$

If the time of shearing, $\Theta \approx 1/\omega$, then Gt_d for the rotating disk filter could be expressed as:

$$Gt_d = f(N_{re}) \qquad (1-78)$$

Details of the derivation is given by Ouano (1981). It should be noted that in literatures on rotating disk filter, the peripheral velocity has been intuitively described as the parameter defining the shearing of the film layer. The use of peripheral velocity (ωD) has no scientific basis and from Eq. 1-78 the continued utization of ωD is not advisable.

Heat transfer – Heat transfer phenomenon is applied in anaerobic digestion of sludges specially in cold climate. Current trends in anaerobic digestion of hot wastes such as alcohol slops and oil palm wastes have been to utilize thermophilic anaerobic digestion. Heat transfer could be more important in the future specially if energy cost continues to increase.

Heat exchangers are the most familiar equipment used in biological wastewater treatment process today. For heat transfer in tube, the dimensionless numbers required for scale up are the Nusselt, Reynolds and Prandt. For turbulent flow inside a tube the interaction of the diameter, D (cm); viscosity, μ, (gm/cm-sec); conductivity, r, (cal/sec-cm – °C); velocity, $\bar{V}$, (cm/sec), density, ρ (gm/cm^3); and specific heat C_p (cal/gm–°C) are defined by the following relationship commonly known as the Dithes Boelter equation

$$\frac{hD}{r} = .023 \left(\frac{D\bar{V}\rho}{\mu}\right)^{8.0} \left(\frac{C_p\mu}{r}\right)^{0.40} \qquad (1-79)$$

The term hD/r is known as Nusselt number, while $D\bar{V}\rho/\mu$ and $C_p\mu/r$ are the Reynolds number and Prandt number respectively. Eq. 1-79 is valid when heat transfer is towards the fluid flowing in the tube.

If heat is transferred from the fluid flowing inside the tube, the following relationship holds true for Reynolds number greater than 2000.

$$\frac{h}{C_p \bar{V}} = j_H \left(\frac{C_p\mu}{r}\right)^{-2/3} \qquad (1-80)$$

The term $h/C_p\bar{V}$ is known as Stanton number and j_H is the Colburn number. The Colburn number is a dimensionless number equivalent to the friction factor f, for calculating headloss. In fact, j_H is equivalent to 0.125 the friction factor f.

CHAPTER II

DATA HANDLING AND ANALYSIS

In the previous chapter, the derivation of the dimensionless groups and their application to the design of pilot units, scale up, experimentation and reporting of results were discussed. Although in the design of wastewater treatment plants the wastewater quality and quantity are assumed constant, in reality the quality and quantity fluctuate drastically depending on the industrial production cycle, the water use habits of the people, the availability of water, and the season. Perhaps the most difficult part of the design process is establishing the wastewater quality and quantity values to be used.

In this chapter, the different causes of wastewater quality and quantity fluctuation are discussed. The chemical and physical parameters affecting wastewater treatment plant design and operation are discussed in Chapter III together with biochemical kinetics. In the later part, a brief review of common statistical parameters, probability curves and their uses are presented as an introduction to curve fitting and parameter estimation. The chapter closes with the handling of stochastic data through spectral analysis.

QUALITY AND QUANTITY FLUCTUATION

Wastewater quality and quantity fluctuation could be classified as diurnal, weekly, monthly, seasonal or annual depending on its return cycle. For domestic sewage, the diurnal fluctuations are dependent on the water use habits. Generally, the water usage is very low before 5:00 AM, after which its starts to peak up until 10:00 AM. Between 5:00 AM and 9:00 AM people cook their breakfast, take a bath, go to the toilet, wash the dishes and do other early morning routines. The water consumption drops slightly between 9 and 11 AM. During this period most of the water consumption are for laundry and house cleaning.

A slight peak develops between 1100 and 1300 hours as housewives prepare lunch, wash the dishes and the users prepare themselves for lunch time. From 1300 to 1700 hours the water generation rate drops off considerably. After 1700 hours the water uses start to peak up until 2000 hours when people start retiring.

The wastewater generation rate follows the same pattern as the water consumption described above. In fact for domestic uses, most of the water used are discharged as wastewater after less than 5 minutes of use. The wastewater in flow cycle to the wastewater treatment plant is affected by the time required for the wastewater to flow from the user to the treatment plant. For example, if the wastewater treatment plant is 10.8 km from the city, and the wastewater has an average flow rate of 1 m/sec, then the wastewater generated at 5 AM reaches the wastewater treatment plant 3 hours later. Hence, the wastewater flow rate into the treatment plants starts peaking at 8:00 A.M.

The bigger the city, the higher the variation is on the wastewater transit time in the sewers. The wastewater in some section close to the wastewater treatment plant may have a transit time of only 10 minutes while the farthest section of the city may have a transit time exceeding six hours. The peak flow from the far section is superimposed with the low flow from the nearby section and vice versa. Hence, the bigger the city the more uniform the wastewater flow rate becomes. The ratio of the maximum peak flow to the average flow becomes smaller as the city becomes bigger. Instead of land area, the size of the city may also be expressed in terms of population. Fig. 2-1 shows the effect of population size on the ratio of the peak and average flow.

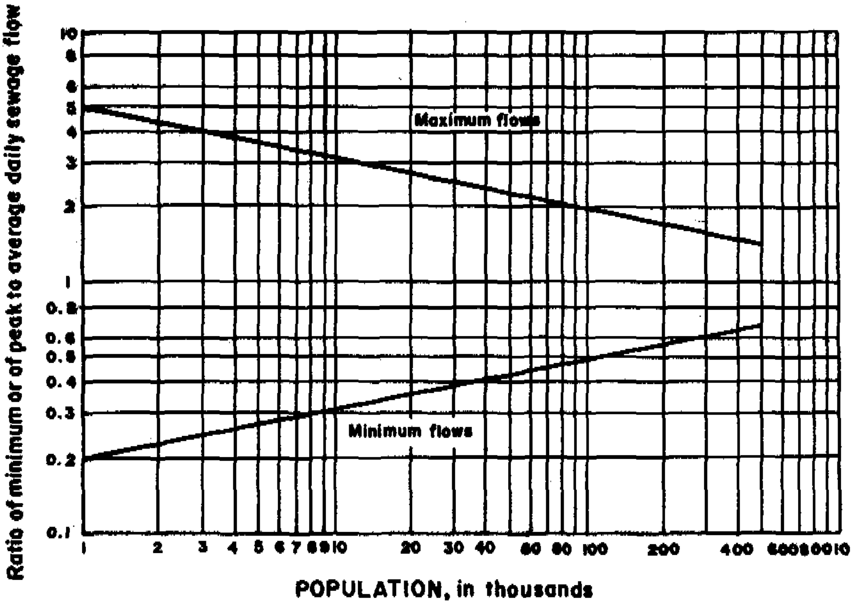


FIG. 2-1 Ratio of extreme flows to average daily flow.

Saturdays and Sundays are usually non-working days in most countries. In others, Fridays and Saturdays are religious holidays. The wastewater generation from commercial and industrial areas are expected to go down during those periods. On the other hand, the wastewater generation from residential areas may increase but the peak hourly demand may be lower since the residents need not follow the regimented schedule during working days. Those routines contribute to the weekly cycle or fluctuation of wastewater generation rates. During hot weather people are inclined to take a bath more frequently than during cold weather. This is one of the factors which lead to seasonal fluctuations.

On top of the diurnal, weekly, monthly and seasonal cycles there are long term effects of the standard of living, the business cycle, the general improvement in public health knowledge, communication and epidemics which also change the water use habits and the wastewater generation rates. The rainfall could affect the infiltration, volume and quantity of the wastewater. Those effects may have a cycle of 7 years to hundreds of years. In fact, the effect of improved knowledge on public health is an exponential increase on water consumption and wastewater generation for the last three hundred years without any possible decline in the future.

Commercial and industrial wastewater – The wastewater quality and quantity from commercial and industrial areas also fluctuate in stochastic manner. The diurnal fluctuations are affected by the office routine, the production shift, and production cycle. General maintenance schedule, cleaning and product demand also affect the weekly, monthly and long term fluctuations of wastewater quality and quantity. Although the effects of the different causes of wastewater generation rates could be generalized by impulses and sinusoidal functions,the cumulative result appears randomnized as shown in Fig. 2-2.

MONITORING PROGRAM

Taking a spot measurement of the wastewater quality and quantity as a basis for design is very risky. It could lead to serious under or over design. Moreover, the sampling, analysis and evaluation of samples are very expensive. The laboratory analysis of a single sample for five major chemical and physical parameters could cost almost $100 in 1980 excluding the cost of sampling.

To define the weekly and diurnal fluctuations the monitoring program could easily cost $50,000. To determine the fluctuations in a given period, the normal rule of thumb is to conduct a monitoring program whose length of observation is three times the time period to be studied. For example, to determine the diurnal variation, three days of the week are chosen at random and observed. For weekly variation, three weeks will be chosen in a month and observed continuously. The program to determine the long term fluctuations of wastewater quality and quantity may take years to evaluate. It should be noted that when the diurnal fluctuations are studied, only the wastewater generation patterns are observed. The seasonal effects on the diurnal variation are being discounted.

For this reason, a comprehensive monitoring program is very expensive. Engineers have to rely on common sense in selecting the appropriate factor of safety. The factor of safety applied is inversely proportional to the data available.

It is preferable to conduct the wastewater monitoring program under the worst expected condition. For example, the diurnal variation study conducted during summer tends to be higher than the same study conducted during cold season. For an industry, the best time to conduct the study is when it is operating at full capacity. In simple industries, the management may be agreeable to operate the factory at peak capacity within the duration of the study which may take a week or month.

STATISTICS

As discussed in the previous section, the wastewater quality and quantity vary constantly with time. Although the change may conform to some cycles such as the diurnal, weekly, monthly, seasonal, annual and hourly variations, the

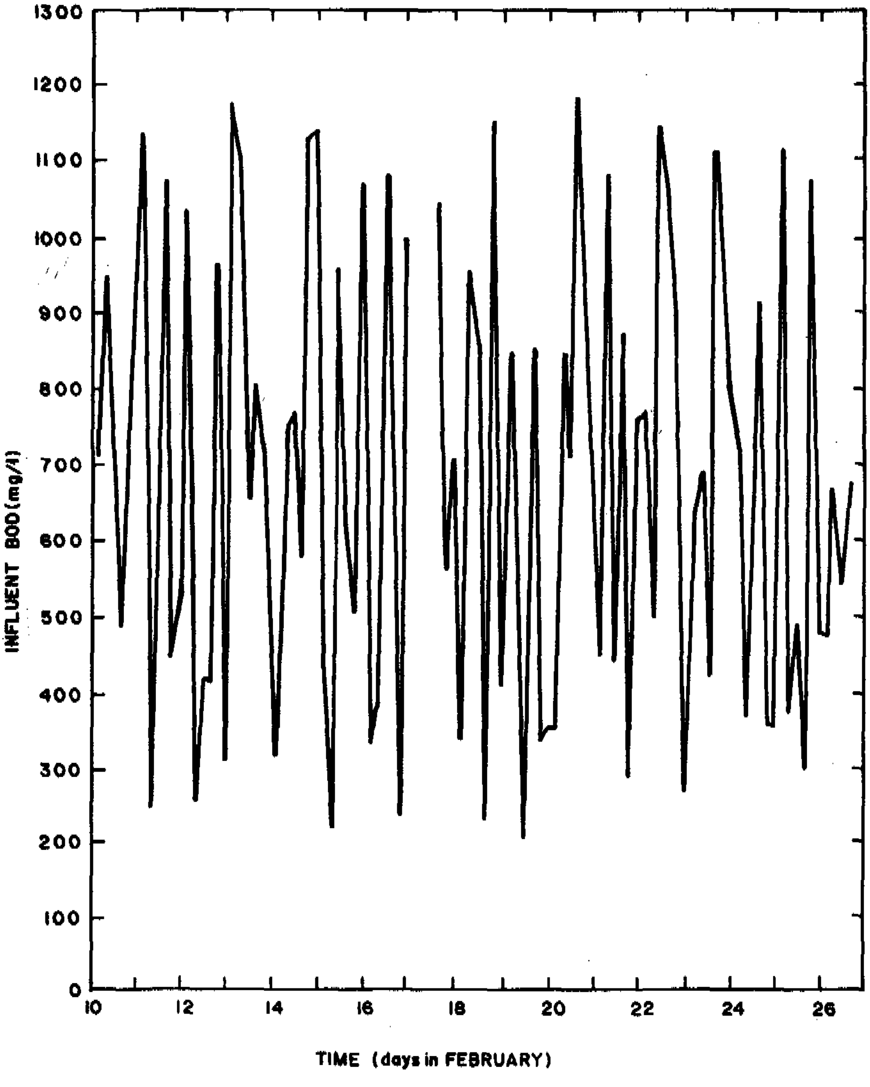


FIG. 2-2 Chronological variation in influent concentration.

superimposition of the different cycles introduces a semblance of randomness or unpredictability of the wastewater quality and quantity. For example, it is certain that the wastewater generation rate reaches a peak between 7-9 AM. If the peak is 10,000 cu. m/hr. today it may go down tomorrow although the peak flow will occur at the same time. Nobody could predict the exact wastewater flow rate even if he controls the available water supply during that period. The peak may not exceed the available water supply.

The occurrence of a particular wastewater quantity is known as stochastic event. As described above a stochastic event is an event which follows a cyclic pattern but whose value occurs at random. Random variables are defined by probability of occurrence or its chance of occurring. For example, if the average peak flow is 10,000 m^3/hr, there is also a 50% probability that tomorrow's peak of wastewater flow rate will exceed 10,000 m^3/hr.

Engineers would like to design processes with sufficient factor of safety for the process to function under the worst possible condition. The process could become very expensive and during its useful lifetime, the worst condition may not take place at all. For example, if a peak flow rate of 40,000 m^3/hr has only .000001% chance of occurring in the next ten (10) years, it makes no sense for the design of the wastewater treatment process to consider this value. On the other hand if a peak flowrate of 30,000 m^3/hr has a 5% chance of occurrence even if the average is only 10,000 m^3/hr, then the 30,000 m^3/hr may be adopted as peak flow for design purposes.

It is very important to be able to describe the probability of occurrence of a particular wastewater quality and quantity on the design value adopted or calculated from experimental measurements. The average may be a convenient parameter to apply if the process is expected to function satisfactorily 50% of the time only. The succeeding section will describe the procedure of calculating the design values from experimental results.

Sampling – Sampling of wastewater may be taken continuously or intermittently. For flow rate measurement, continuous sampling is common using an automatic recording system. The system involves a float placed before a weir in an open channel. To the float is attached a pen which moves up and down simultaneously with the float movement as the water level fluctuates. A lever arm is usually attached between the pen and the float to magnify the float fluctuations and facilitate the reading of results. The charts in which the pen writes the fluctuations of the water level moves at a particular speed. Normally, the time is printed in the chart and the speed is adjusted such that for a particular time, say 1 hour, the chart moves under the pen segment corresponding to the printed length of one hour. Although this device is useful, it may be too expensive and the designer may not need all the information.

Spot or intermittent sampling is more common. The sample or event is observed at fixed time interval. For example the water depth may be measured every hour. This method minimizes the number of samples to be analyzed. However, the fluctuations in wastewater quality in between the sampling interval is not recorded.

To minimize the effects of unrecorded fluctuations between the sampling interval, the sampling may be carried out continuously or more frequently. The samples are mixed afterwards to reduce the cost of analysis. For example, 250 ml wastewater sample may be taken every 15 minutes and mixed to form one sample representing the sample taken at one hour interval. Instead of analyzing four samples, only one sample is analyzed. Hence, the cost of laboratory analysis is cut down by 75%. The mixed sample is known as composite sample while the individual sample is known as grab sample.

The sampling interval of a composite sample is also known as averaging time since the composite sample represents the average wastewater quality in the particular sampling interval. The composite sample may be taken continuously by pump feeding and stored in suitable sampling bottles. For example if it is desired to collect a 60 liter composite sample every hour, the wastewater could be collected by a pump at the rate of 1 liter/minute.

The composite sample described above places equal significance on the wastewater quality regardless of the quantity or flow rate. The higher flow rate should have more significance to the wastewater quality in the sampling interval than the low wastewater flow rate. The error introduced by neglecting the wastewater flow rate fluctuations is shown in Example 2-1. The quantity of grab samples which are mixed to make up the composite should be proportional to the wastewater flow rate. This proportioning of grab samples may be done manually or collected by means of proportional sampler. A proportional sampler is a modification of the continuous sampler by attaching a variable valve controlled by a float. As the float moves up and down corresponding to the fluctuations of the water level in a channel, the valve opening changes correspondingly.

> Example 2-1. During the sampling interval the wastewater flow rate and BOD values are shown in Table 2-1. Calculate the BOD concentration if a) 6 liter composite sample is a mixture of equal volume of grab samples b) the volume of grab sample added is in proportion to the flow rate c) Compare the results of the two samples.

TABLE 2-1 Wastewater Quality and Quantity Fluctuation

TIME (mins.)	Flow Rate m3/hr	BOD mg/1
0- 5	5	250
5-10	7	300
10-15	9	400
15-20	12	600
20-25	15	500
25-30	12	400
30-35	9	300
35-40	8	350
40-45	6	250
45-50	4	250
50-55	2	100
55-60	1	80

Solution:

a) The volume of each grab sample should be equal to

$V = 6/12$ sample $= 0.5$ liter/sample

Weight of BOD in composite sample $= V_1 C_1 + V_2 C_2 + \ldots V_{12} C_{12}$

$= .5 \times 250 + 300 \times .5 + 400 \times .05 + 600 \times .5 \times .5 + 600$
$\times .5 + 400 \times .5 + 300 \times .5 + 350 \times .5 + 250 \times .5$
$+ 250 \times .5 + 100 \times .5 + 80 \times .5 = 1940$ mg

Concentration of BOD in composite sample = 1940 mg/6 1 = 323 mg/1

b) Total wastewater flow $= Q_1 t_1 + Q_3 t_3 + \ldots Q_{12} t_{12}$

$= 5 \times 10/60 + 7 \times 10/60 + 1.q \times 10/60 + 12 \times 10/60$
$+ 15 \times 10/60 + 12 \times 10/60 \times 9 \times 10/60 + 8 \times 10/60$
$+ 6 \times 10/60 + 4 \times 10/60 + 2 \times 10/60 + 1 \times 10/60 =$
$900/60 = 15$ m^3

Proportional volume to be collected per grab sample

$$V_n = \frac{6 \text{ liter}}{15 \text{ m}^3} \times Q_n t_n$$

hence

$V_1 = (6 \times 5 \times 10/60)/15 = .3343$ liters
$V_2 = (6 \times 7 \times 10/60)/15 = .467$ liters
$V_3 = (6 \times 9 \times 10/60)\ 15 = .600$ liters
$V_4 = (6 \times 12 \times 10/60)\ 15 = .800$ liters
$V_5 = (6 \times 15 \times 10/60)/15 = 1.000$ liters
$V_6 = (6 \times 12 \times 10/60)/15 = .800$ liters
$V_7 = (6 \times 9 \times 10/60)/15 = .600$ liters
$V_8 = (6 \times 8 \times 10/60)/15 = .5343$ liters
$V_9 = (6 \times 6 \times 10/60)/15 = .400$ liters
$V_{10} = (6 \times 4 \times 10/60)/15 = .267$ liters
$V_{11} = (6 \times 2 \times 10/60)/15 = .1343$ liters
$V_{12} = (6 \times 1 \times 10/60)/15 = .067$ liters

TOTAL 6 liters

Weight of BOD $= V_1 C_1 + V_2 C_2 + V_3 C_3 + \ldots V_{12} C_{12}$

$= .333 \times 250 + .467 \times 300 + .1600 \times 400 + 800$
$\times 600 + 1 \times 600 + .8 \times 400 + .650 \times 300 + .533$

$x\ 350 + .4 \times 250 + .267 \times 250 + .133 \times 150 +$
$.067 \times 8 = 2410$ mg

BOD concentration = 2410 mg/61 = 402 mg/1

c) The error between the composite sample which neglects the fluctuation in wastewater flow rate is:

$$\frac{(402-323) \times 100}{402} = 19.7\%$$

Continuous monitoring of wastewater quality and quantity specially for domestic sewage shows that the BOD and suspended solids concentration reach their peaks during the peak flow rate in a similar trend shown in Table 2-2 during a 24 hour period. Hence, a 24 hour composite sample which neglects the flow rate fluctuations may differ by more than 10% from a composite sample which is a mixture of grab samples in proportion to the flow rate.

Probability Distribution – Since the occurrence of a particular wastewater quality and quantity is stochastic, its occurrence in the future is expressed in terms of probability or chance of taking place. Stochastic function differs from deterministic function in that stochastic function only defines the chances of the event taking place whereas deterministic function defines the particular value of the event at a specific time. For example the deterministic function $s = \frac{1}{2}gt^2$ defines the distance s of a falling object at anytime t, i.e. at t = 1 sec and g = 9.8 m/sec^2, s is equal to 4.9 m. Whereas average s defined as $s = \frac{1}{2}gt^2$, at t = 1 states that there is 50% chance the object is 4.9 m below the point of release after 1 sec. A deterministic model defines the behavior of the falling object under ideal condition or in a vacuum. The probability function defines the uncertainties involved such as as a very strong wind, a bird getting hit or other events that could affect the motion of the falling object.

Probability – The probability of an event taking place is measured experimentally and is defined as:

$$P(x) = \lim_{n \to \infty} \frac{N_x}{n} \qquad (2-1)$$

where P(x) is the probability of x taking place, N_x is the number of times the event x takes place and n is the number of experimental trials. If x could assume any value between $-\infty$ to ∞, x is known as a continuous function. Since x could assume an infinite number of values between 0 and 1, the probability of a particular x value taking place is zero. The occurrence N_x is distributed over an infinite variation and hence would remain finite when n approaches infinity. The limit of Eq. 2-1 is equal to zero. For continuous function, the probability is defined for an interval of interest. Eq. 2-1 is modified as:

$$P(b < x < a) = \lim_{n \to \infty} \frac{N_{(b < x < a)}}{n} \qquad (2-2)$$

where $N(b < x < a)$ is the number of times the event occurred with the event x taking the value between a and b.

Since the number of occurrence N_x is always less than or equal to the number of experimental trial n, the maximum value of P(x) is equal to one. An event with P(x) = 1, is an event which is certain to take place. Eq. 2-1 defines the probability function of a discrete event. Discrete events are rarely encountered in wastewater treatment processes and further discussion will deal with the continuous probability function.

Histogram – There are a number of probability functions which are often encountered in data handling and analysis. The most common probability density functions (pdf) are the normal distribution, log normal, chi-square, student t and the exponential distribution. To select the pdf which fits the experimental results, a histogram is first constructed. If the event X during the experiment has a maximum value of X^* and a minimum value of X' then X^*-X' is known as the range of X. The range could be subdivided into a number of smaller intervals. A useful rule of thumb is to divide the range X into intervals equal to 0.1 times the range. If X is uniformly distributed throughout its range of values, approximately 10 results should fall within each interval.

The experimental results are tabulated into each interval and graphed as shown in Fig. 2-3.Hence, if the number of occurrence at each interval is divided by the total number of experimental trials, n, the histogram is reduced to a unit histogram. The sum of the strip area in a unit histogram is equal to 1. The unit histogram is compared with the common probability density function shown in Fig. 2-4. To illustrate the procedure of constructing a histogram consider the following example.

> Example 2-2. Table 2-2 shows the BOD of the raw wastewater from a tomato processing factory. Construct a histogram of the occurrence of the different BOD values and determine the probability density function.
>
> Solution: By inspection, the minimum BOD value is 221 mg/1 and the maximum is 1,174 mg/1. The range is then equal to 1,174-221 or 953 mg/1.
>
> The total number of experimental results is 100. Divide the range into 10 intervals. The increment per intervals is 95.3 or 100 mg/1 per interval. The results which fall within a particular interval are tabulated as shown below.

TABLE 2-2
SUMMARY OF BOD CHARACTERIZATION DATA
(4-hour composites)

DATE	BOD (mg/1)	DATE	(BOD (mg/1)	DATE	BOD(mg/1)	DATE	BOD(mg/1)	
2/10 4 am	717		758		940		1054	
8 am	946		769		233		888	
12 noon	623		574		1158		266	
4 pm	490		1135		407	2/23	619	
8 pm	666		1142	2/19	853		696	
12 pm	828	2/15	505		751		416	
2/11	1135		221		207		1111	
	241		957		852		973	
	396		654		318		807	
	1070		510		358	2/24	722	
	440		1067	2/10	356		368	2/10
2/12	534	2/16	329		847		686	
	1035		371		711		915	
	265		1001		1185		361	
	419	6	621		825		364	346
	413		235		618	2/25	1110	
	961		993	2/21	454		374	
	308	2/17	1019		1080		494	
2/23	1174		1023		440		268	
	1105		1167		872		1078	
	659		1056		294		481	
	801		560		763	2/26	472	
	720		708	2/22	776		671	
	454	2/18	340		502		556	
2/14	316		949		1146		672	

Sum of all values = 68,700 mg/l

BOD interval mg/1	Tabulation of results	Total	P(a<x<b)
200- 299	𝍸 ||||	9	.09
300- 399	𝍸 𝍸 |||	13	.13
400- 499	𝍸 𝍸 ||	12	.12
500- 599	𝍸 ||	7	.07
600- 699	𝍸 𝍸 |	11	.11
700- 799	𝍸 𝍸	10	.10
800- 899	𝍸 ||||	9	.09
900- 999	𝍸 |||	8	.08
1000-1099	𝍸 𝍸 |	11	.11
1100-1199	𝍸 𝍸	10	.10

TOTAL – – –100

The histogram is shown in Fig. 2-3. From Fig. 2-4, a rectangular distribution seems to give the best fit to the unit histogram.

Hypothesis Testing – The type of probability density function (pdf) which closely resembles the histogram could be selected from Fig. 2-4. However, two or more probability density functions may resemble the histogram. To determine whether the selected probability density function is acceptable, the chi-square test is used. The chi-square error is defined as:

$$\chi^2 = \frac{(X_o - X_e)^2}{X_e} \qquad (2-3)$$

where X_o is the observed value of X, X_e is the estimated value of X and χ^2 is the Chi-Square value.

The degrees of freedom (df) is equal to:

$$df = I - 1 - p \qquad (2-4)$$

where I is the number of intervals and p is the number of parameters estimated from the probability density function. Table A-2 gives acceptable sum of the chi-square errors defined by Eq. 2-3 depending on the degree of freedom and confidence interval. If the value given in Table A-2 exceeds the sum of the chi-square error, the hypothesis is acceptable. To the reader who is interested in more details, any standard textbook on probability and statistics such as Hieler and Libermann (1965) could be consulted. Normally the confidence level used is 90%. To illustrate the abovementioned principle, consider the following example.

Example 2-3. Determine the probability density function which conforms to the histogram given in Example 2-2. Check the acceptability of the hypothesis.

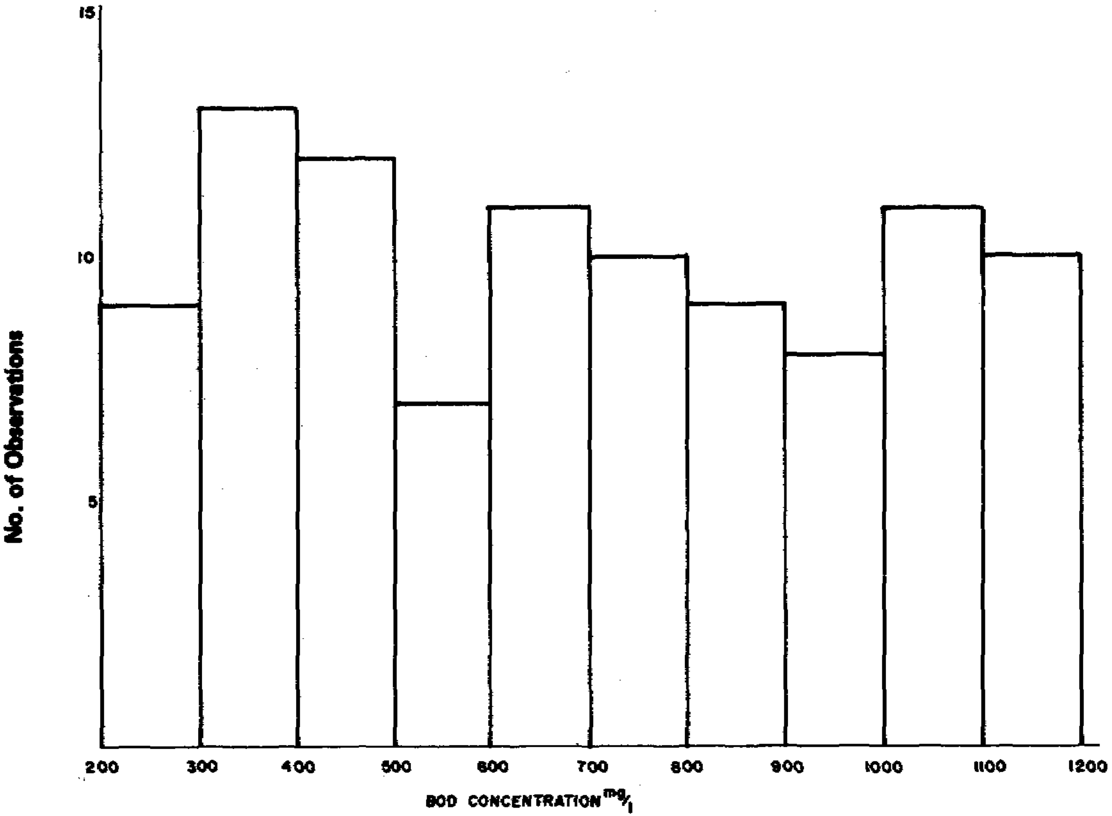


FIG. 2-3 Histogram on BOD concentration given in FIG. 2-2.

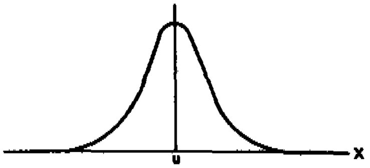
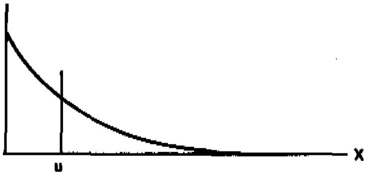
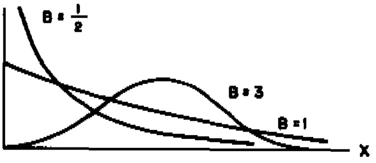
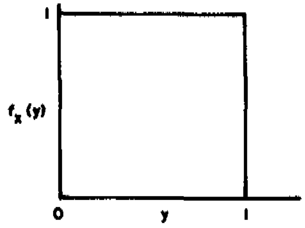
DISTRIBUTION	FORM	PROBABILITY FUNCTION
a. Normal		$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(X-u)^2}{2\sigma^2}}$ u = population average σ = population standard deviation
b. Exponential		$y = \frac{1}{u} e^{-u}$ u = population average
c. Weibull		$y = aB(X-\gamma)^{B-1} e^{-a(X-\gamma)^B}$ a = scale parameter B = shape parameter γ = location parameter
d. Rectangular		$f_X(y) = \begin{cases} \frac{1}{d-c} \\ 0 \end{cases}$

FIG. 2-4 Common probability density function.

Solution: From Fig. 2-4 a rectangular probability density function looks very similar to the histogram.

The parameter a-b or range is calculated as 1200 – 200 or 1000 mg/1 The probability of occurrence is calculated as:

$$P\ (200 < x < 300) = \int_{200}^{300} \frac{dx}{1000} = 0.1$$

$$P\ (300 < x < 400) = \int_{300}^{450} \frac{dx}{1000} = 0.1$$

The other probabilities are calculated in a similar manner. Hence,

$P\ (400 < x < 500) = P\ (500 < x < 600) = P\ (600 < x < 700)$
$P\ (700 < x < 800) = P\ (800 < x < 900) = P\ (900 < x < 1000)$
$P\ (1000 < x < 1100) = P\ (1100 < x < 1200) = 0.1$

The observed values are taken from the column shown in Example 2-2. The calculation of the X^2 chi-square error is summarized in Table 2-3. The X_e values are calculated as the product of the number of observations and the probability of an event X occurring in the particular interval. For example for the interval 200-299 mg/1, the expected value is equal to .1 x 100 = 10.

TABLE 2-3 Summary of Chi-Square error

BOD Interval	Observed Values	Expected Values	Error Square
200- 299	9	10	.1
300- 399	13	10	.9
400- 499	12	10	.4
500- 599	7	10	.9
600- 699	11	10	.1
700- 799	10	10	.0
800- 899	9	10	.1
900- 999	8	10	.4
1000-1099	11	10	.1
1100-1199	10	10	0
TOTAL	100	150	3.0

One parameter is required to define the rectangular probability density function which is the range (a-b). Hence, the degree of freedom is equal to 10-1-1 or 8. At 90% confidence level and 8 degrees of freedom the acceptable sum of chi-square error is equal to 15. Since 8 is lower than 15, then the hypothesis that the BOD fluctuations in the tomato processing plant conform to a rectangular distribution is acceptable.

As the confidence level increases, the acceptable sum of the chi-square error increases. The confidence interval of 95% means that there is only 5% chance of error or rejecting a correct hypothesis. If the confidence level reaches 100%, then the probability of rejecting a correct hypothesis is reduced to zero, since all probability distributions will be acceptable.

Example 2-5. Consider the same problem in Example 2-4 except that a normal distribution is fitted.

Solution: From Fig. 2-4, the parameters of the normal distribution to be calculated in order to define the function are the average $\bar{X}$ and standard deviation σ.

Using the data from Table 2-2,

$$\bar{X} = \frac{\Sigma X_i}{100} = 717 + 946 + 623 + \ldots 556 + 672$$

$$= \frac{169238}{100} = 692 \text{ mg/l}$$

$$\sigma^2 = \frac{\sum_{i=1}^{100} (X_i - \bar{X})^2}{n-1} = \frac{\sum_{i=1}^{100} X_i^2}{n} - (\bar{X})^2$$

$$\sigma^2 = \frac{(717)^2 + (946)^2 + (623)^2 + \ldots (556)^2 + (672)^2}{100} - (692)^2$$

$$= \frac{55{,}944{,}507}{100} - (692)^2 = 80581$$

$$\sigma = \sqrt{80581} = 283 \text{ mg/l}$$

The different values of $(X_L - \bar{X})/\sigma$ are calculated and shown in Table 2-4. The values under the normal curve from $-\infty$ to $(X_L - \bar{X})/\sigma$ are taken from the normal distribution table. Hence, the area of the normal distribution between $(X_e - \bar{X})/\sigma$ and $(X_u - \bar{X})/\sigma$ are calculated by subtracting the area covered by two adjacent values. The expected value is obtained by multiplying the number of observation with the area in the above mentioned interval.

Table 2-4 chi-square Error Calculations for Normal Distribution

X_L	X_u	X_o	$\frac{X_L - \bar{X}}{\sigma}$	$\frac{X_u - \bar{X}}{\sigma}$	$P[-\infty < x < (X_l - X)/\sigma]$	$P[-\infty < x < (X_u - \bar{X})/\sigma]$	X_e	$\frac{(X_o - X_e)^2}{X_e}$
200	299	9	-1.74	-1.39		.0823	8.0	.125
300	399	13	-1.39	-1.04	.0823	.1492	6.7	5.92
400	499	12	-1.04	- .68	.1492	.2483	9.9	.45
500	599	17	- .68	- .33	.2483	.3707	12.2	1.89
600	699	11	- .33	.025	.3707	.5050	13.4	.43
700	799	10	- .025	.378	.5050	.6480	14.3	1.29
800	899	9	.378	.731	.6480	.7673	11.9	.71
900	999	8	.731	1.08	.7673	.8599	9.2	.16
1000	1099	10	1.08	1.44	.8599	.9251	6.5	1.89
1100		11	1.44	1.79	.9251	1.000	7.5	1.63
						TOTAL		14.5

The degrees of freedom = 10-1-2 = 7

For 90% confidence $X^2 = 12.01$ which is less than 14.5. Hence, there is a 90% possibility that the function does not fit the normal distribution.

The construction of a histogram is an important step in determining the probability density function which fits the experimental data. The histogram allows visualization of the distribution of the experimental results which could not be made by simply reviewing the tabulated data.

The X^2 test is a very lenient test. A number of pdf could pass the chi-square test for a given set of experimental results. The chi-square test allows for adoption of the hypothesis even if there is only 10% probability that the hypothesis is correct.

As shown by Example 2-5, the BOD fluctuation shown in Fig 2-2 does not conform to the normal distribution. However, USEPA (1962) suggested fitting the same set of experimental data on a normal distribution without any confirmation whether the normal distribution is applicable. The absence of a histogram was the major cause of the errors.

Properties of the pdf – If the pdf is integrated over the entire range of possible values of X, the result is equal to one. An event $b < X < a$ where $a = X_{max}$ and $b = X_{min}$, is 100% sure to occur.

Mathematically this is expressed as:

$$\int_{X_{min}}^{X_{max}} \text{pfd}(X)\, dX = 1 \qquad (2-6)$$

The area in the normal distribution table is the area covered between $-\infty$ to $(X-\overline{X})/\sigma$ or cdf of a standardized normal distribution. The standardized normal distribution is obtained by converting the variable X to a dimensionless form $X^* = (X-\overline{X})/\sigma$. As discussed in the previous chapter the conversion of equation to dimensionless form facilitates the presentation and tabulation of results.

The mean or average value is obtained by summing all the experimental results of X and dividing it by the number of experimental results as shown in the previous examples.

$$\overline{X} = \frac{\sum_{i=1}^{n} \overline{X}_i}{n} \qquad (2-7)$$

The standard deviation is the measurement of the spread of the experimental outcomes around the mean. The higher the standard deviation the wider the spread of the experimental results is around the mean. The effect was briefly described in the previous chapter on the discussion of dispersion numbers. The standard deviation is calculated as:

$$\sigma = \sqrt{\sum_{i=1}^{n} \frac{(X_i-\overline{X})^2}{n-1}} \qquad (2-9)$$

or

$$\sigma = \sqrt{\sum_{i=1}^{n} \frac{(X_i)^2}{n} - (\overline{X})^2} \qquad (2-8)$$

The square of the standard deviation is known as variance. If the event X is a random event whose occurrence is independent of the previous events, then as the number of trials increases, the probability distribution of X tends to follow or conform to the normal distribution. For this reason, the normal distribution is commonly used in practice.

Example 2-6. Determine the BOD values which have less than 5% chance of occurring if the experimental results were those shown in Fig. 2-2, a) Use a rectangular distribution b) Use the normal distribution

Solution: From Example 2-4, the cdf (cumulative density function)

$$\text{cdf} = \frac{1}{1000}\int_{200}^{X} dX =$$

$$\text{cdf} = \frac{X - 200}{1000}$$

if X has less than 5% chance of occurring then cdf = 1−0.05 = .95. Solving for X from the above equation yields:

$$\text{cdf} = \frac{X - 200}{1500} = 0.95$$

$$X = 1150 \text{ mg/l}$$

b) Instead of integrating the normal distribution function, the cdf table for standardized normal curve is consulted. For a cdf of 0.95 from the table of normal function, $(X-\bar{X})/\sigma = 1.65$. From example 2-5, $\bar{X} = 693$ and $\sigma = 283$. Substituing those values to $(X-\bar{X})/\sigma$ yields:

$$\frac{X - 693}{283} = 1.65$$

Solving for X, results to

$$X = 1{,}160 \text{ mg/l}.$$

Therefore if the designer is to select the BOD design value such that 95% of the time he would expect the actual influent to be lower than his selected value, then he chooses a BOD value of 1150 or 1160 mg/l.

It should be noted that if $\bar{X}$ is used for design purposes, at $(X-\bar{X})/\sigma$ the cdf is equal to 0.5. Hence, values of $X > \bar{X}$ is expected to occur 50% of the time.

The wastewater treatment plant design at $\bar{X}$ is expected to function satisfactorily 50% of the time when $X \leq \bar{X}$.

Since, the normal and log normal distributions are often encountered in practice, special graphing papers are commercially produced to facilitate the presentation of experimental results. The graph of cdf (X) vs. X is transformed to a linear graph in the normal probability paper. In a log normal probability paper, the graph of cdf (X) vs. log X is transformed to a linear graph.

When $(X\text{-}X)/\sigma = 1$, the cdf is equal to 0.8413. So, at the point $(X\text{-}\bar{X})/\sigma, = 1$, the corresponding ordinate on the normal probability paper is 84.13%. A straight line between (X, 50%) and ($X + \sigma$, 84.13%) defines the best fit normal curve.

The cumulative probability could be calculated from the data obtained from experimentation. This is shown in the following example.

Example 2-7. Assuming that experimental values shown in Table 2-2 conform to a normal distribution, graph the best fit line and the experimental results on a normal probability paper.

Solution:

X_l	X_u	$\frac{\Sigma N_x}{n}$	**Graphed Points**
250	299	.09	(299,89%)
300	399	.22	(399,22%)
400	499	.34	(499,43%)
500	599	.41	(599,41%)
600	699	.52	(699,52%)
700	799	.62	(799,62%)
800	899	.71	(899,71%)
900	999	.79	(999,79%)
1000	1099	.90	(1099,90%)
1100	1199	1.00	(1199,100%)

n is 100, the number of experimental measurements. The best fit curve is the straight line connecting (693,50%) for (X,50%) and (976,84.1%) for ($\bar{X} + \sigma$, 84%) with the results shown in Fig. 2-5. The designer could easily select and change the design BOD value depending on the degree of reliability.

Spetral analysis – The statistical analysis described in the previous section provides a method of determining and estimating the probability of occurrence of a particular wastewater quality and quantity. However, the analysis assumes that the occurrence of particular range of values of the wastewater quality and quantity is purely randomized. This means the occurrence of one event is inde-

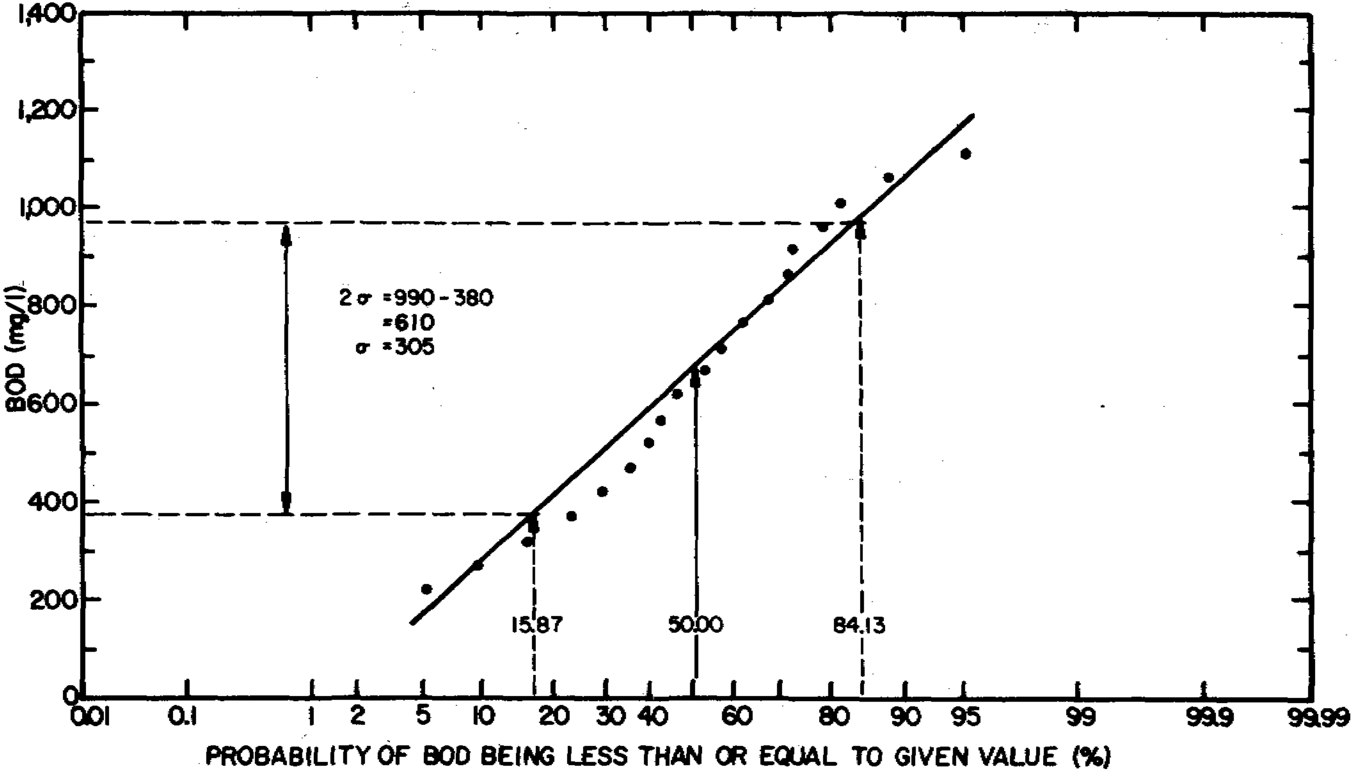


FIG. 2-5 Probability Plot of Graphical Method II.

pendent of the past history or it does not exhibit any cyclic pattern. As previously discussed, the occurrence of particular wastewater quality follows a cyclic pattern depending on the water use habit of the people, the production cycle and other factors.

The performance of biological wastewater treatment processes is highly dependent on the sequence of occurrence or cyclic behaviour of the wastewater quality and quantity fluctuations. For example, the effect of a single shock load on the performance of an activated sludge could be accomodated by the capability of the sludge to absorb the excess food as cellular material. A number of shock loads occurring at close interval could result in failure of the system. The dynamic performance of auxiliary processes, such as an equalization system, grit tanks, and sedimentation tanks is dependent on the cyclic pattern of wastewater quality and quantity fluctuations.

The values of the trigonometric functions such as sine and cosine are repeated at increments of 360^0 or 2π. For example $\sin(t/2) = \sin(2\pi + t/2) = \sin(4\pi + t/2)$ etc. For a function such as $f(t) = A \sin(2\pi\, t/T)$, A is known as the amplitude of the function and $T/(2\pi)$ is the period of oscillation or fluctuation. The amplitude is the maximum value of the function above the average. By increasing $T/(2\pi)$, the time interval for the values to repeat is increased.
The converse holds true when T is reduced.

For example the graph of $f(t) = 10 \sin t$ has a maximum value of 10 which takes place when $t = \pi/2$. The function f(t) is again equal 10 when $t = (2\pi + \pi/2)$, $6\pi + \pi/2$, etc. The function has a period of 2π since its values are repeated at increment of 2π. Consider the case when $f(t) = 10 \sin(2t)$. The maximum value of the function is still equal to 10 at $t = \pi/4$. The value is repeated when $t = \pi + \pi/4$; $2\pi + \pi/4, 3\pi + \pi/4$. The time it takes for the function to repeat is reduced from 2π to π, since T is reduced to π.

Instead of graphing 10 sin t with respect to time, the function could be also expressed in the phase plane by graphing the absolute value of the amplitude and the frequency of oscillation. The phase representation of the function simplifies the graph from a long series of waves to a single line in the phase plane. The phase plane presentation of a function is significant in graphing complicated series of periodic functions such as the Fourier series and Fourier transform. As will be shown in Chapter IV and the succeeding chapters, the dynamic analysis of equalization systems and biological wastewater treatment processes is easier to evaluate using the phase plane mapping than in the real time domain analysis.

Trigonometric series is defined by:

$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{in\omega t} \qquad (2-11)$$

where

$$a_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-in\omega t}\, dt \qquad (2-12)$$

where ω is equal to $2\pi f$ or $2\pi/T$ referred to as the angular velocity. The amplitude A, of the Fourier series is equal to

$$A = 2|a_n| \qquad (2\text{-}13)$$

If the Fourier series is convergent then as $n \rightarrow \infty, |a_n| \rightarrow 0$. The contribution of the high frequencies or large ω, to the series becomes very small and could be neglected. To illustrate the fourier series consider the following example.

Example 2-8. Determine the Fourier series of a function shown in Fig. 2-6(a) and draw its frequency spectrum.

Solution: From Eq. 2-12

$$a_n = \frac{1}{6}\int_0^3 2\exp\left\{\frac{(-in2\pi)t}{6}\right\}dt + \frac{1}{6}\int_3^6 0\exp\left(\frac{-in2\pi t}{6}\right)dt$$

$$= \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)\exp\left(\frac{-in\pi}{2}\right)$$

Hence

$$f(t) = \frac{2}{\pi}\sum_{n=-\infty}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)\exp\left\{\frac{in2\pi}{3}(t-3/2)\right\}$$

It should be noted that it would be time consuming and difficult to plot f(t) from the series function.

b) The frequency spectrum of f(t) could be determined as:

$$A(\omega) = 2/a_n/ = 2\frac{2}{\pi n}\sin\left(\frac{n\pi}{2}\right)$$

$$= \frac{4}{\pi n}\sin\left(\frac{n\pi}{2}\right)$$

The graph of A vs. n ω is shown in Fig. 2-6 (b) which reduces f(t) to discrete lines instead of a continuous polot of f(t) shown in Fig. 2-6 (a)

The BOD of a tomato processing factory shown in Fig. 2-2 could be theoretically expressed by a Fourier series. However, current numerical techniques for estimating the Fourier series constant from experimental results have not been developed satisfactorily. The problem is further complicated by the discrete nature of the graph in phase plane of the amplitude and the angular velocity as shown in Fig. 2-6 (b).

$T_p = 3 \qquad A = 2$

$T = 6$

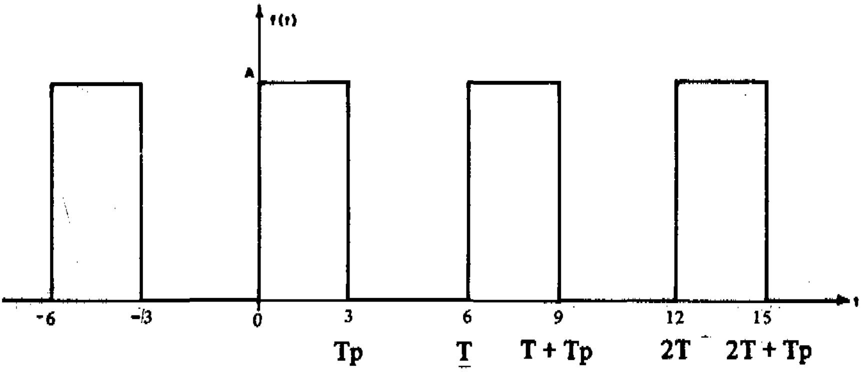


FIG. 2-6(a) A periodic rectangular pulse train.

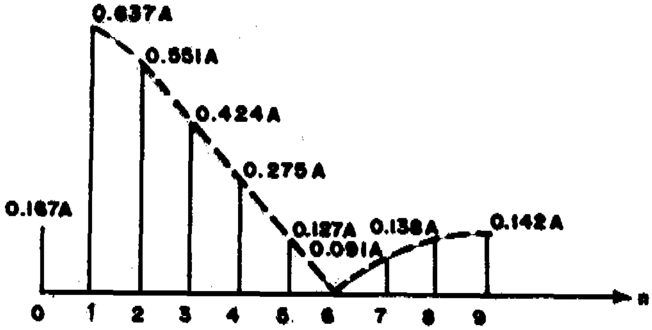


FIG. 2-6(b) Frequency spectrum of $f(t)$ in FIG. 2-6(a) for $T_p/T = 3/6$

Fourier Integral – If experimental data are obtained during an interval T^*, the experimentalist has completely no idea on the value of $f(t < 0)$ and $f(t > T^*)$. Instead of extrapolating the results of $f(0 \leq t \leq T^*)$ for $f(t < 0)$ and $f(t > T^*)$, the values for those ranges are assumed as zero. Then, the Fourier series approximation of the $f(t)$ within the range $0 \leq t \leq T^*$ could be expressed as:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left\{ \int_{-i\infty}^{i\infty} f(t)\, e^{-i\omega t} \right\} e^{i\omega t} \quad d\omega \quad dt \qquad (2-14)$$

The term inside the bracket of Eq. 2-14 is known as the Fourier transform $F(\omega)$ of the $f(t)$.

The Fourier transform $F(\omega)$ is expressed as:

$$F(\omega) = \int_{-\infty}^{i\infty} f(t)\, e^{-i\omega t}\, d\omega \qquad (2-15)$$

The plot of $/F(\omega)/$ and ω shows the relative frequency distribution of f(t). $F(\omega)$ is a continuous function of ω unlike A (ω) which exists only at discrete values of $n\omega$.

The function $F(\omega)$ could be estimated from experimental results by means of the auto-correlation analysis. The procedure is presented in this section without any mathematical proof. The reader is advised to consult publications by Tukey (1949), Kinsman (1965), and Walter (1953).

For a given set of data subdivided or taken at interval Δt, the frequency range that could be evaluated from the experimental data is given as:

$$f_r = 1/(2 \cdot \Delta t) \qquad (2-16)$$

The frequency range could be subdivided further into m sections. A useful rule of thumb is to use $m \leq 0.1n$ where n is the number of experimental data. In case of continuous records, n is equal to the length of the record divided by Δt or $n = T/\Delta t$.

The values of $|F(\omega)|^2$ are calculated from the autocorrelation function, P_m. The autocorrelation function is expressed as:

$$P_m = \frac{1}{n-m} \sum_{j=1}^{n-m} \left[f_i(t_j) - \bar{f}(t)\right]\left[f(t_{j+m}) - \bar{f}(t)\right] \qquad (2-17)$$

where $\overline{f(t)}$ is the average value of the data. The Fourier cosine transform, (C_r) of P_m is calculated from the following equation.

$$C_r = \frac{k}{m} \left\{ p_o + p_m \cos(rN) + 2 \sum_{j-1}^{m-1} p_i \cos\left(\frac{jr\pi}{m}\right) \right| \qquad (2-18)$$

where k = 1/2 when r = 0 and r = m, otherwise k = 1. Like m, r has the value ranging from 0, 1, 2 . . . to m.

The Fourier cosine transform obtained from Eq. 2-18 may be very unreliable due to errors and fluctuations inherent in the experimental procedure and equipment. The Fourier cosine transforms are defined using the following equations:

$$C_o^* = 0.54\, C_o + 0.46\, C_1$$

$$C^*_r = .23\ C_{r-1} + .54\ C_r + .23\ C_{r+1} \quad \text{where } r = 1,2\ldots m\text{-}1 \qquad (2-19)$$

$$C^*_m = .46\ C_{m-1} + .54\ C_m$$

where C^*_o; C^*_r and C^*_m are the smooth estimates of the power spectrum or the Fourier cosine transform.

It should be noted that when $n \gg m$, Eq. 2-17 could be approximated as:

$$P_m = \frac{1}{n\text{-}m} \sum_{j=1}^{n-m} f(t_j)\ f(t_{j+m}) - \left[\overline{f(t)}\right]^2 \qquad (2-20$$

The use of the power spectrum estimates for analyzing the cyclic nature of the wastewater quality and quantity is shown in the following example.

Example 2-9. Determine the power spectrum of the BOD fluctuations from a tomato processing factory given in Fig. 2-2 and Table 2-2.

Solution: i) The first step is to add all the BOD values and calculate f(t) which is equal to 687 mg/1. ii) Since the total number of experimental points is equal to 100, then select m=0.1n = 100 x 0.1 = 10. iii) The sampling interval t = 6 hours or 0.25 day.
Hence, the frequency range

$f_r = 1/(2 \times 0.25) = 2$ cycles/day.

iv) Since m=10, then each m corresponds to an increment of 0.2 cycle/day. v) the autocorrelation function is calculated using Eq. 2-17. Typical case is shown below:

(717-687) (717-687) =	(717-687) (946-687) =
(946-687) (946-687) =	(946-687) (623-687) =
(623-687) (623-687) =	(623-687) (490-687) =
.	.
.	.
;	;
.	.
$\frac{\qquad}{100} = 87{,}476$	$\frac{\qquad}{99} = -9289$

The values of P_m calculated are as follows:

$P_0 = 87{,}476$, $P_1 = -9289$, $P_2 = -8304$,
$P_3 = 12{,}348$, $P_4 = -8078$, $P_5 = 14{,}340$,
$P_6 = 1{,}196$, $P_7 = 11{,}849$, $P_8 = -4{,}380$,
$P_9 = 4520$ and $P_{10} = 9438$.

v) The values of C_r are calculated using Eq. 2-18.

Typical calculation for C_o is:

$$C_o = \frac{1}{2 \times 10} \{87476 + 9438\,(1) + 2\,(-9289 - 8304 + 12348 - 8078 - 14{,}740 + 1{,}196 + 11{,}489 - 4380 + 4520)\} = 3322.$$

The calculated values are $C_1 = 4285$, $C_2 = 10{,}724$, $C_3 = 7300$, $C_4 = 4219$, $C_5 = 6734$, $C_6 = 18417$, $C_7 = 10{,}763$, $C_8 = 9317$, $C_9 = 9121$, $C_{10} = 2456$.

vi) The smooth values C_r^* are calculated using Eq. 2-19. Hence,

$$C_0^* = .54 \times 3322 + .46 \times 4285 = 3765$$

$$C_1^* = .23 \times 3322 + .54 \times 4255 + .23 \times 10724 = 5544$$

$$C_2^* = .23 \times 4285 + .54 \times 10724 + 23 \times 7300 = 8455$$

$$C_3^* = .23 \times 4285 + .54 \times 7300 + .23 \times 4219 = 7379$$

$$C_4^* = .23 \times 7300 + .54 \times 4219 + .23 \times 6734 = 5506$$

$$C_5^* = .23 \times 4219 + .54 \times 6734 + .23 \times 18417 = 8842$$

$$C_6^* = .23 \times 6734 + .54 \times 18417 + .23 \times 10763 = 13970$$

$$C_7^* = .23 \times 18417 + .54 \times 10763 + .23 \times 9317 = 12190$$

$$C_8^* = .23 \times 10763 + .54 \times 9317 + .23 \times 9121 = 9604$$

$$C_9^* = .23 \times 9317 + .54 \times 9121 + .23 \times 2456 = 7633$$

$$C_{10}^* = .46 \times 9121 + .54 \times 2456 = 5521$$

viii) The graphs of C_T^* and f are shown in Fig. 2-7 . It is important to note that the major components of the BOD have a frequency of 4 to 1.2 cycle/day which corresponds closely to the diurnal fluctuation. Although no mention was given on the production cycle from Fig. 2- 7 , the production cycle is estimated at 1.2 cycles/day.

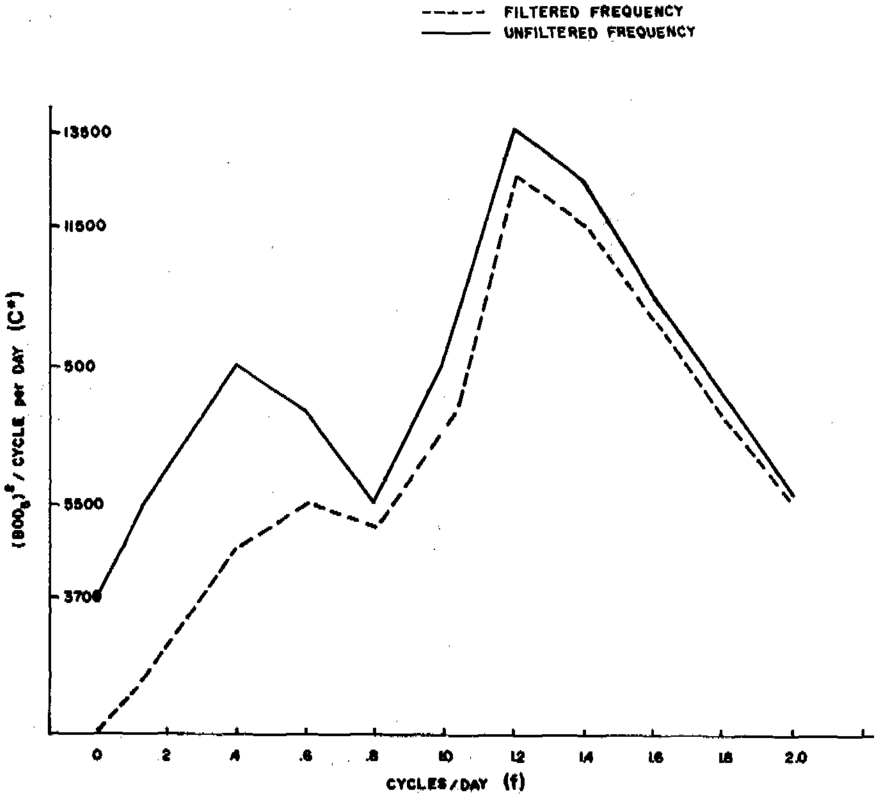


FIG. 2-7 Power spectrum of BOD fluctuations given in FIG. 2-2

At intervals of 2.0 to 3.5 days, the machine and storage space seems to undergo general cleaning and maintenance as could be determined from the secondary peak at 0.4 and 0.6 cycle/day.

Spectral analysis could also be utilized to determine the appropriate sampling interval. In the above example, the highest frequency determined is 2 cycles/day. Since the normal working hour is only one third of a day or 8 hours/ shift, the effect of the shift change, the sampling interval or averaging time between samples should be reduced from 6 hours to 3 hours. The maximum frequency range at 3 hours or 0.125 day is equal to $1/(2 \times .125)$ or 4 cycles/day. The reader should note that the number of samples to be analyzed is doubled correspondingly to account for a wider frequency range to be studied. The monitoring or duration of experimental observation may be shortened from 25 days to 12 days. By shortening the duration of the monitoring program, the reliability of low frequency events on the wastewater quality and quantity fluctuations is reduced. In the above example, the effects of weekend and weekly cleaning are lumped in the residuals between $f = 0$ and $f = 0.2$. Had the sampling been carried out at shorter time intervals a decrease in the value of $|C(\omega)|$ at $f=0$ would have taken place to correspond with the weekend operation.

Filtering – If the monitoring program in carried out for 25 days only, then the values of $|C(\omega)|$ at frequencies lower than 0.04 cycle/day are questionable since the experimental data do not even cover one cycle of those events. The reliability of the values for $|C(\omega)|$ also declines as f decreases.

$|C(\omega)|$ at low frequencies may be adjusted to account for its low reliability. The process of adjusting the $|C(\omega)|$ at low frequencies is known as filtering. A common filtering equation is:

$$Y(\omega) = \frac{((\omega)\Delta t)^2}{a^2 + ((\omega)\Delta t)^2} \qquad (2\text{-}21)$$

where a is a suitable constant, Y is the transmission efficiency and ω is the radian frequency equal to $2\pi f$.

> Example 2-10. Suppose the reliability of the power spectra at frequencies lower than 0.4 cycle/day is only 50% reliable, determine the filtered spectrum of the data used in Example 2-9.
>
> Solution: $\Delta t = 0.25$ day
>
> $\omega = 0.4 \times 2\pi = 2.51$ radians/day
>
> at $\omega = 2.51$ radians, $Y(\omega) = 0.5$ or 50%.
>
> Solving for a from Eq. 2-21 yields:
>
> $$a^2 = \left[\frac{1}{.5} - 1\right](2.51 \times .25)^2$$
>
> $a = .394$

For other frequencies, $Y(\omega)$ could be expressed as:

$$Y(\omega) = \frac{\omega^2}{6.3 + \omega^2}$$

The value of $C^*(\omega)$ are adjusted as follows:

$$C^*(0)=3765 \times \frac{0^2}{6.3+0^2} = 0.0$$

$$C^*(.2) = 5544 \times (.2 \times 2)^2 / (6.3 + (.2 \times 2)^2)$$
$$= 1111$$

$$C^*(.4) = 8455 \times (.4 \times 2)^2 / (6.3 + (.4 \times 2)^2)$$
$$= 4227$$

The other values are calculated as shown before. This yields

$C^*(.6) = 5112$, $C^*(.8) = 4407$, $C^*(1.0) = 7625$, $C^*(1.2) = 12576$, $C^*(1.4) = 11272$, $C^*(1.6) = 9040$, $C^*(1.8) = 7274$, $C^*(2.0) = 5309$. The graph of the filtered spectra is shown in Fig. 2-7.

Moving average – The calculation involved in the spectral analysis is very tedious as illustrated above so that computers are often required for calculations. A moving average will also provide a quantification of the cyclic nature of the wastewater quality and quantity as shown by Berthouex and Hunter (1979). The major advantage of the moving average is the simplicity of calculations which could be done manually. The charts could be updated easily as data are made available. The moving average and CUSUM charts are also used to determine if the performance of the wastewater treatment plant is satisfactory.

The moving average $\overline{Y}(t)$ is calculated using the following equations:

$$\overline{Y}(t) = Y(t - \Delta t) + \frac{1}{K}\left[Y'(t) - Y(t\text{-}K\,\Delta t) \right] \qquad (2-22)$$

where $\overline{Y}(t)$ is the moving average value for time t, $\overline{Y}(t\text{-}\Delta t)$ is the moving average one sampling interval behind, $Y'(t)$ is the experimental value obtained at time t, $Y(t\text{-}K\,\Delta t)$ is the experimental value K interval before and K is a constant.

The constant K would depend on the receiving stream, the type of pollutant discharged, or stream standard. For example an impulse discharge of highly concentrated BOD may be tolerated by aquatic life whereas the results from three day continuous discharge could be disastrous. Hence, K is set for 3 days in this particular case. For carcinogens the effect may take 30 years before any effect is noticed, hence K may be adjusted for 10-year or 30-year moving average. For example the USEPA standard for BOD discharges per unit of product produced is stated in terms of 30-day moving average.

In the moving average, the present values and distant past values are weighted equally. The moving average tends to dampen the effect of random variation such as error in analysis and it tends to highlight the long term trend change of those components with low frequencies. One of the major shortcomings of the moving average is the inclusion of the long past effects which may have little significance to the present performance of the biological wastewater treatment processes. To overcome this problem, the most recent event could be given a higher weight in averaging than the past event. Eq. 2-22 could be modified as:

$$\overline{Y}'(t) = \varphi\,\overline{Y}(t-\Delta t) + (1-\varphi)\,\overline{Y}(t) \qquad (2\text{-}23)$$

where φ is a constant which is less than one but greater than zero. Eq. 2-23 is known as exponentially weighted moving average. As φ becomes smaller the effect of the current value $\overline{Y}(t)$ becomes more significant.

> Example 2-11. Construct a moving average chart from the data shown in Table 2-2 using a K value of 3 days, and 7 days.
>
> Solution: a) Since the data are taken every six hours, the number of samples per day is equal to 4. For a 3 day moving average K = 4 x 3 = 12
>
> $$\begin{aligned}\overline{Y}_1 = (&717 + 946 + 623 + 490 + 666 + 828 + 1135 + 241 + 396\\ &+ 1070 + 440 + 534)/12 = 673.3\end{aligned}$$
>
> $$\overline{Y}_2 = 673.3 + (1035 - 717)/12 = 699.8$$
>
> $$\overline{Y}_3 = 699.8 + (265 - 946) = 643.$$
>
> The other values are calculated in a similar manner and shown in Fig. 2-8.
>
> b) For a 7 day average, K = 4 x 7 = 28
>
> $$\begin{aligned}\overline{Y}_1 = (&717 + 946 + 623 + 490 + 666 + 828 + 1135 + 241 + 396\\ &+ 1070 + 440 + 534 + 1035 + 265 + 419 + 413 + 961 + 308\\ &+ 1174 + 1105 + 659 + 801 + 720 + 454 + 316 + 758 + 769\\ &+ 574)/28 = 672\end{aligned}$$
>
> $$\overline{Y}_2 = 672 + (1135 - 717)/28 = 686.$$

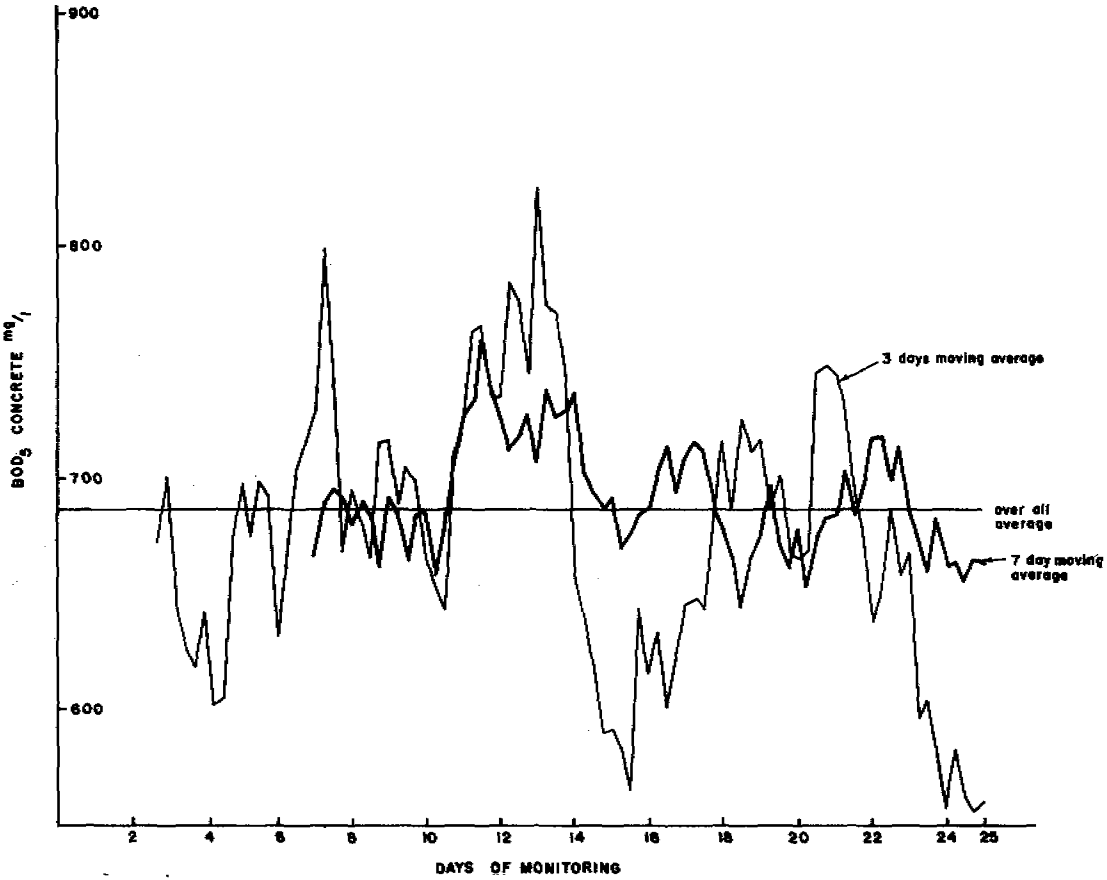


FIG. 2-8 Moving average graph of BOD_5 fluctuations given in TABLE 2-2

The other values are calculated in a similar manner and plotted in Fig. 2-8. It should be noted from Fig. 2-8 that as K increases, the fluctuations of the BOD values are dampened. For a process operating at a steady state the moving average tends to fluctuate around the long term average defined by Eq. 2-7 If the process is deteriorating then the moving average tends to move away from the long term average. By using the moving average chart the process operator could make the necessary adjustment before complete breakdown takes place. The operator could differentiate the random effects from the deterioration of the process equipment.

CURVE FITTING

As discussed in Chapter I, experiments have to be carried to determine the different exponents of the dimensionless equations. The kinetic constants, design equations and other process parameters are obtained from experimental results. In this section, the principles of parameter estimation and curve fitting will be discussed.

Let Y $(Y_1, Y_2, Y_3, \ldots Y_n)$ be a set of experimental results corresponding to the change in the independent variable defined by the set $X = (X_1, X_2, X_3, \ldots X_n)$.

If the relationship between Y and X is hypothesized as:

$$Y(X) = aX + b \qquad (2-24)$$

where a and b are constants, then the sum of the squares of the error between the hypothesis and the experimental values could be expressed as:

$$E^2 = \sum_{i=1}^{n} (Y_i - aX_i - b)^2 \qquad (2-25)$$

The best estimate for a and b should minimize E^2 By differentiating Eq. 2-25 with respect to a and b, and setting the differential to zero, a and b could be calculated which gives the minimum value of E^2 as shown below:

$$\frac{dE^2}{da} = 2\sum_{i=1}^{n} (Y_i - aX_i - b)(-X_i) = 0$$

$$= 2\sum_{i=1}^{n} (X_iY_i - aX_i^2 - b)(-1) = 0 \qquad (2-26)$$

$$\frac{dE^2}{db} = 2\sum_{i=1}^{n} (Y_i - aX_i - b)(-1) = 0 \qquad (2-27)$$

Since

$$\frac{d^2(E^2)}{d\,a^2} = 2\sum_{i=1}^{n} + (X_i^2) > 0 \qquad (2-28)$$

and

$$\frac{d^2E^2}{db^2} = 2\sum_{i=1}^{n}(1) > 0$$

then Eq. 2-26 and 2-27 defines the minimum $\overline{E^2}$ value. Solving Eq. 2-26 and 2-27 simultaneously for a and b yields:

$$a = \frac{\sum_{i=1}^{n} Y_i\ (X_i - \bar{X})}{\sum_{i=1}^{n} (X_i - \bar{X})^2} \qquad (2-29)$$

$$b = \bar{Y} - a\bar{X} \qquad (2-30)$$

Where

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \qquad (2-31)$$

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

The above analysis could be extended when Y is linearly dependent on two or more variables $X_{(1)}, X_{(2)}, X_{(3)}, \ldots$ The sum of the squares of the error could be expressed as.

$$E^2 = \sum_{i=1}^{n} (Y_i - a_0 - a_1X_{i,1} - a_2X_{i,2} - a_3X_{i,3} \ldots - a_nX_{i,n})^2 \qquad (2-32)$$

The derivatives $dE/da_1 \ldots dE/da_n$ are calculated and equated to zero. The values of $a_o, a_1 \ldots a_n$ are calculated from the matrix equation defined as:

$$\left|\begin{array}{cccc} n+1 & \sum_{i=1}^{n} X_{i,1}\cdots & \sum_{i=1}^{n} X_{i,2}\cdots & \sum_{i=1}^{n} X_{i,n} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^{n} X_{i,1} & \sum_{i=1}^{n} (X_{i,1})^2\cdots & \sum_{i=1}^{n} X_{i,1}X_{i,2}\cdots & \sum_{i=1}^{n} X_{i,1}\cdot X_{i,n} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^{n} X_{i,n} & \sum X_{i,n}X_{i,1} & \sum_{i=1}^{n} X_{i,n}X_{i,2}\cdots & \sum_{i=1}^{n} (X_{i,n})^2 \end{array}\right\| \left|\begin{array}{c} a_o^2 \\ \vdots \\ \vdots \\ \vdots \\ a_n \end{array}\right| = \left|\begin{array}{c} \sum_{i=1}^{n} Y_i \\ \vdots \\ \sum_{i=1}^{n} X_{i,1}Y_i \\ \vdots \\ \sum_{i=1}^{n} X_{i,n}Y_i \end{array}\right| \qquad (2-33)$$

A number of non-linear equations could be rearranged into a linear form so that Eq. 2-29, 2-30, and 2-33 could be used for estimating the values of the parameters. For example $Y = n/(b + Qx)$ could be expressed in linear form as $z = b + aX$ where all the values of $z_i = 1/Y_i$. The function $Y = b \exp(-ax)$ could be expressed as $\ln(Y) = \ln(a) - bx$ or $Z = a' - bx$ where $a' = \ln a$ and $Z = \ln(Y)$. After calculating the value of ln a the value of Y is calculated as $Y = \exp(a)$. Fig. 2-10 shows the linear transformation of some common functions.

Example 2-11. Determine the values of a and b which fit the experimental result shown in Table 2-5.

X_i	Y_i	$\ln(Y_i) = Z$
0.1	1.228	0.205
0.2	1.005	0.005
0.3	.823	-0.195
0.4	.674	-0.395
0.5	.552	-0.594

Table 2-5 Summary of experimental results on chlorine removal

Solution: The values of Y_i vs. X_i are graphed as shown in Fig. 2-9. From Fig. 2-10, the experimental results seem to conform to the equation $Y = a \exp(-bx)$

ii) The function could be linearized as $\ln Y_i = \ln(a) - bx$ or $Z = a - bx$

iii) $\underline{x} = (.1 + .2 + .3 + .4 + .5)/5 = .3$

$\overline{z} = (.205 + .005 - .195 - .395 - .594)/5 = -.195$

$$b = \frac{.205\,(-.2) + .005\,(-.01) - .195\,(0) - .395\,(.1) - .594\,(.2)}{(-.2)^2 + (-.1)^2 + 0^2 + (a.1)^2\ x\,(.2)^2}$$

b = -1.998

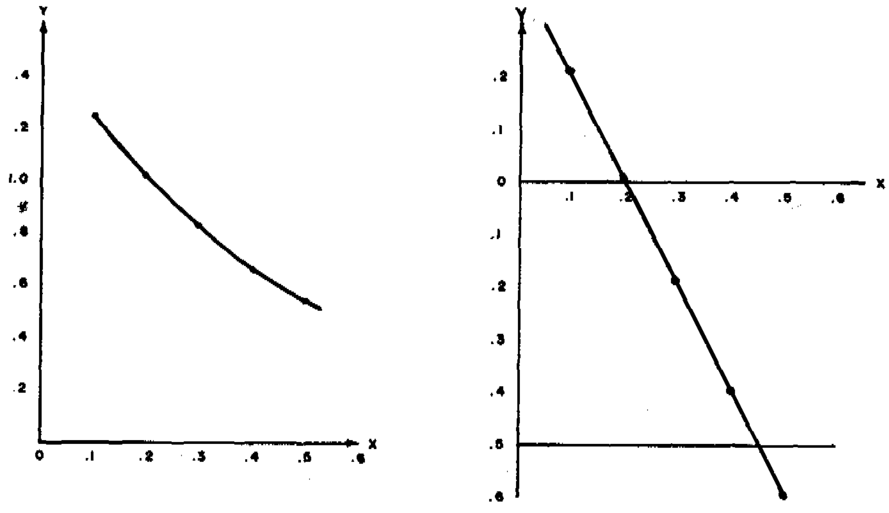
$$
\begin{aligned}
a' &= \overline{Y} - b'\overline{X} \\
&= -.195 - (-1.998)(.3) \\
&= .4044
\end{aligned}
$$

iv) since $a' = \ln(a)$, the $a = \exp(a')$
or $a = \exp(.4044) = 1.50$
Hence, the curve which fits the data is:
$Y = 1.50 \exp(-1.998X)$

Confidence Interval – Once the equation of the straight line which best fits the experimental data has been determined, the confidence limit around the line is calculated. The confidence limit around the best fit line provides an estimation of the acceptable deviation of the present and future data from the equation established. The higher the confidence interval the wider is the band of the upper and lower confidence interval around the best fit line.

The upper confidence limit is defined as:

$$
Y_u = \overline{Y}_i + t_{(v,a|2)} \sqrt{\frac{\sigma^2}{n} + \frac{(X_i - \overline{X})^2}{\sum_{i=1}^{n} (X_i - \overline{X})^2}} \qquad (2-33)
$$



RAW PLOT

LINEAR TRANSFORM

FIG. 2-9 Raw data and linearized plot of experimental data shown in Table 2-11.

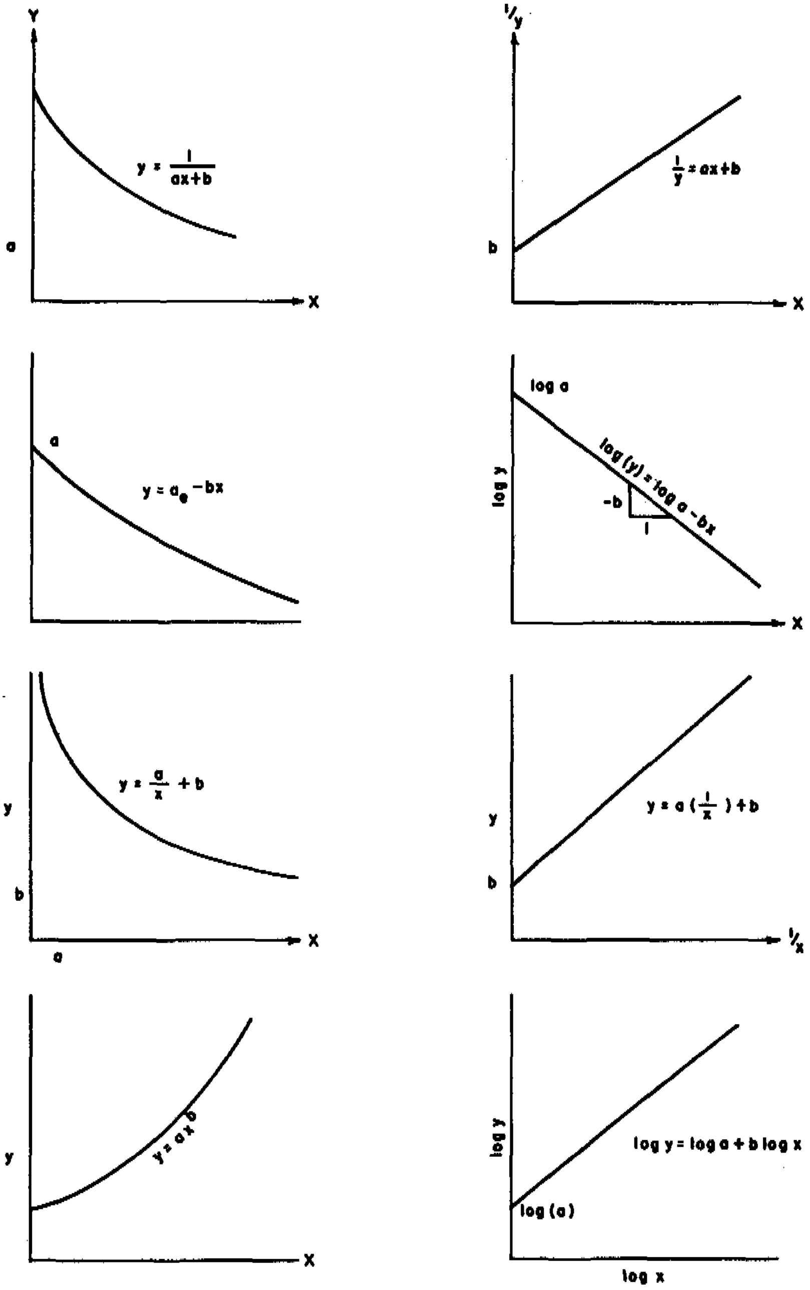


FIG. 2-10 Common curve and their linear transformation.

where t is the student -t value, v is the degree of freedom equal to n-2, a/2 is the confidence limit desired, σ^2 is defined by Eq. 2-34 and the other terms are as defined before. The estimates for σ^2 is the sum of the square around the line which is:

$$\sigma^2 = \sum_{i=1}^{n} \frac{(Y_i - aX_i - b)^2}{n-2} \qquad (2-34)$$

where the terms have been previously defined. Normally, 95% confidence limit is used for a/2. Similarly the confidence limit of the parameters a and b are respectively equal to:

$$a \pm t_{(v, a/2)} \; \sigma \sqrt{\frac{1}{n} + \frac{\bar{X}^2}{\sum_{i=1}^{n} (X_i - \bar{X})^2}} \qquad (2-35)$$

$$b \pm t_{(v, a/2)} \; \frac{\sigma}{\sqrt{\sum_{i=1}^{n} (X_i - \bar{X})^2}} \qquad (2-36)$$

where the terms are as defined before.

It should be noted from Eq. 2-35 and 2-36 that the range of values of a and b decreases as the sum of $(X_i - \bar{X})^2$ increases. Hence, it is important in the experiment to maintain a wide scatter of X_i as much as possible. The positive sign is used in Eq. 2-35 and 2-36 to determine the upper confidence limit.

Closure — An experiment is carried out to establish the inter-relationship of the different process variables and wastewater properties and the efficiency of the wastewater treatment processes. The experimentalist may develop hypothesis on the interaction of the different variables which he will have to confirm through experimentation. He may use dimensional analysis to reduce the number of experimental variation and facilitate the presentation of the results as discussed in Chapter I.

The development of the hypothesis is the most difficult part as the experimentalist will have to depend on his intuition, previous results published in journals and books, and principles from related fields of science which have not yet been utilized in the particular problem he is investigating. It is in the development of the hypothesis that the experimentalist introduces his original ideas and sometimes his own personality. There is a strong tendency for him to confirm his hypothesis which could lead to personal bias in the interpretation of the experimental results.

Lines and curves fitted by eye tend to shift closer to the hypothesis than the best fit curve obtained from mathematical calculations. For this reason it is important to confirm the significance of causation and effect of the variations of the different variables in the interpretation of the experimental data. It should be borne in mind that statistical tests are always lenient with regards to the acceptance of the hypothesis and should not be considered as an unnecessary hindrance to the establishment of sound engineering equations. The statistical tests often discard a hypothesis only when the probability of discarding a wrong hypothesis is greater than 95%. There is almost a 95% chance that a wrong hypothesis is accepted.

In this chapter we have scanned briefly the statistical techniques required in the analysis and presentation of experimental data.A detailed presentation of those techniques is outside the scope of this text. The reader is encouraged to consult standard reference materials on statistics.

Although statistical analysis is a useful tool in establishing the relationship between experimental variables it does not confirm the correctness of the hypothesis or the relationship of the experimental variables. Hence, the hypothesis and the theoretical development should be made rationally considering all the scientific facts available. A common mistake of experimentalist is to add as many variables as possible and conduct a regression analysis between the independent variables and dependent variables without considering if the independent variables have any significance at all on the experimentals result. Kawata (1978) found a large number of public health data are correlated to variables which have no scientific basis. For example, he cited that, the number of typhoid cases has dropped drastically since 1900 whereas the number of telephone poles have increased drastically. A simple regression analysis will show a very strong correlation between the decrease in typhoid incidence and the increase in the number of telephone poles. Common sense will dictate that telephone poles have minimal effect at all in the control of typhoid epidemics.

CHAPTER III

BIOCHEMICAL KINETICS AND REACTORS

Most of the researches on biochemical reactions are carried out under asceptic condition, with a homogeneous microbial population and stringently controlled environmental conditions. Commercial biochemical reactors are often batch type to eliminate unwanted biochemical by-products from microbial mutants. Typical of those reactors are fermentation tanks for the production of wine, alcoholic beverages, penicillin and other drugs. Biochemical reactors could be as simple as the primitive earthen jars used for vinegar production or as sophisticated as fermentation vats used for penicillin production.

In wastewater treatment, the major objective is to remove or stabilize the carbonaceous substances dissolved or suspended in water at a minimum cost and time. Recovery of the biochemical by-products is of minimal importance. The wastewater flow rate could be very large which would make stringent environmental control impractical. Likewise, the wastewater quality or chemical composition could vary from time to time which would make quality control for any by-product recovery difficult. The stability of a homogeneous microbial population under varying wastewater influent quality would be low. Hence, wastewater is treated in biochemical reactors with minimal environmental control, a heterogeneous microbial population and preferably in a continuous flow type reactor. Quantification of the physical and chemical phenomena of the wastewater treatment process is often approximation from studies conducted using pure culture under controlled environmental condition. Like in ancient wine making, tradition and superstition are prevalent in current wastewater treatment technology to cover up the difference between theory and practice. The major difference between the ancient and the current superstition is the presence of experimental data in the latter which might not have been analyzed thoroughly.

MICROBIAL POPULATION

The microbial population could be added with carbonaceous wastes as in domestic sewage, or introduced from the soil as the wastewater flows through the drains, or from the microbial spores carried by the wind. The microorganisms which are important in wastewater are yeast, fungi, bacteria, algae and protozoa. The microorganisms may be dispersed in the wastewater or they could aggregate to form flocs or slimes.

Bacteria – Bacteria are the predominant microbial group in wastewater. They multiply very rapidly consuming the carbonaceous substances dissolved in wastewater. The E. Coli group is most prevalent in sewage. Since bacteria are very important, the succeeding section deals mostly with this organism.

Protozoa – Protozoa are unicellular animals which prey on bacteria. For a long time the growth of protozoa in wastewater was considered detrimental to the efficiency of the process since they consume and reduce the bacterial population. Current studies however, have shown the importance of protozoa in the production of clear effluents. Protozoa have a higher preference to dispersed bacterial population which causes the turbidity or cloudiness in the treatment plant effluent.

Algae – Algae are unicellular plants with chlorophyll. Algae are important in facultative and aerobic lagoon systems as a source of oxygen. The chlorophyll produces oxygen from carbon dioxide and water in the presence of sunlight in a process known as photosynthesis. Algal cells are high in protein which could make them important by-products from wastewater treatment. Algae are also important food source for higher fauna such as fish. Although algae produce oxygen during daytime, a large portion of the excess dissolved oxygen is released to the atmosphere. At night time, the algae compete with the fish and bacteria for the remaining dissolved oxygen. In the absence of sunlight algae produce carbon dioxide from the stored cellular materials and oxygen by a process known as respiration. Fishkill could result due to oxygen depletion.

Yeast and fungi – Fungi form mostly filamentous colonies which could result to bulking sludge in activated processes. Due to its poor settling properties, bulking sludge could reduce the activated sludge efficiency.

In fixed film reactor, the filamentous growth increases drastically the surface area for contact as the filament waves in the wastewater. For this reason, filamentous growth in rotating disk filter is desirable. Yeast exists more in unicellular form specially in wastes where the pH has dropped below 6. In normal pH range of 6 to 9, yeast could not compete with bacteria and would normally remain dormant.

BACTERIAL CELL AND GROWTH

A typical bacterial cell is shown in Fig.3-1. The carbonaceous matters are first adsorbed in the bacterial cell. Enzymes are then excreted by the cells, after which the adhering substances are absorbed and converted to cellular tissues. The bacteria multiply by cellular fission after they have matured. The cellular reproduction is fairly rapid.

Under adverse condition, the bacterial cell may form spores and remain dormant until the environmental conditions are again suitable for its growth. When a bacterial culture is introduced into a solution containing carbonaceous substances, the cells adjust their enzymatic system to suit the food available. The growth rate at this stage is very slow. This phase of growth is known as lag phase or acclimatization stage.

After the bacterial cells have been acclimatized, they grow and multiply rapidly at a growth rate that is exponential in nature. This period of rapid growth is known as log phase or exponential growth rate. When almost all the food is used up, a balance is established between the bacterial population and the food supply and the growth rate remains close to zero. This period is known as stable growth phase.

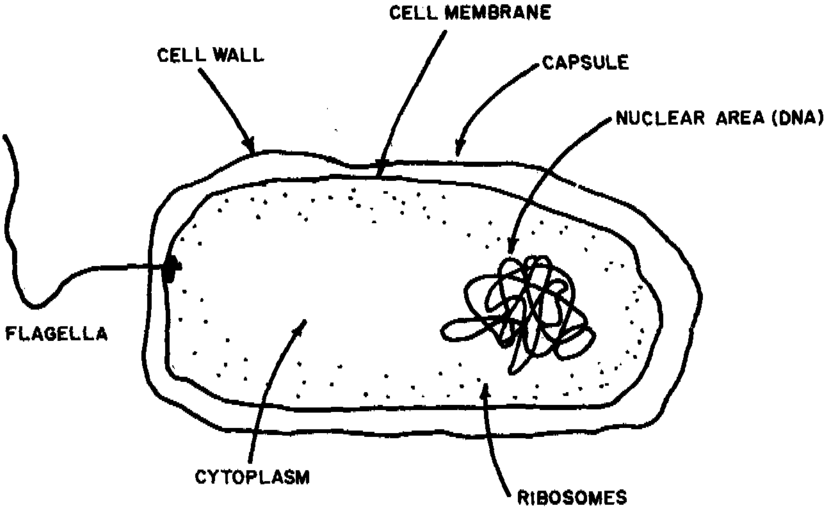


FIG. 3-1 Generalized schematic diagram of bacteria cell.

When the food is used up, the bacterial cells start to consume the stored cellular reserves. The bacterial cells stop reproducing and they lose weight. This phase of growth is known as endogenous phase. Fig. 3-2 shows the different stages of bacterial growth.

In biological wastewater treatment processes, the exponential growth phase is desirable since the removal of the organic or carbonaceous waste is fastest. However, to minimize the disposal of excess bacterial cells and provide better stability of the process to shock loads the bacterial cells are best slightly starved. The starved cells are more adapted to absorb any excess loads introduced into the wastewater treatment than fully fed cells.

High rate wastewater treatment processes utilize the exponential growth phase while low rate or extended aeration phase processes use the endogenous phase. In conventional processess, the growth phase could vary between the two extremes. The stationary growth phase is difficult to attain in practice due to the varying quality and quantity of wastewater influent to the wastewater treatment processes.

CONDITIONS AFFECTING GROWTH

In the previous section, the effect of food supply and acclimatization period on bacterial growth was briefly described. However, like all living substances bacteria need a balance diet, sufficient temperature, and other nutrients. Poisons could stop or slow down bacterial growth while vitamins and other metabolic

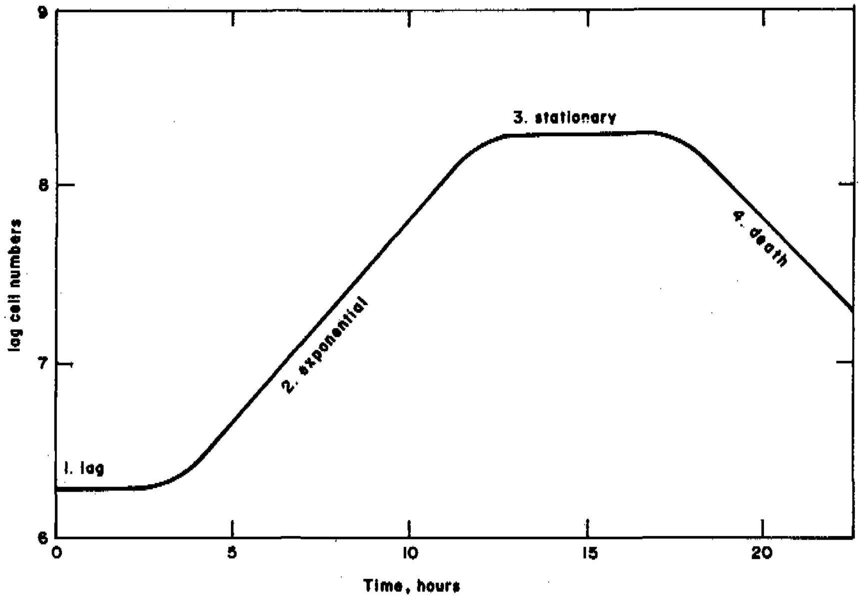


FIG. 3-2 The bacterial batch culture curve (axis numbers are illustrative only).

catalysts could accelerate growth.

Carbonaceous substances – In biochemistry, a substrate is a mixture of essential nutrients such as nitrogen, phosphorous, carbohydrates, fats, vitamins and minerals. In wastewater terminology substrate often refers to the carbonaceous substances measured as BOD (Biochemical Oxygen Demand) or COD (Chemical Oxygen Demand) or TOC (Total Organic Carbon).

BOD is a parameter originally developed in England to estimate the oxygen required to aerobically decompose the organic waste when discharged to the rivers. Since the British rivers are very short, after 5 days it reaches the sea where it is diluted by sea water. After dilution with sea water, the organic waste becomes very dilute that there is no chance of the receiving water to turn septic or anaerobic. The test is carried out at 20°C to simulate water temperature in England. Actually, BOD measures also the non-organic oxygen demand such as the oxygen required for oxidation of ammonia to nitrites and nitrates.

At 20°C, the oxygen consumed during the first six days are for oxidation of the carbonaceous matter. After almost all the carbonaceous matter is used up, the nitrogeneous compounds are oxidized. Fig. 3-3 shows the cumulative oxygen required to oxidize a typical sewage sample. The dotted line shows the cumulative oxygen required for oxidizing the carbonaceous matter if the oxidation of the nitrogeneous compounds have been suppressed. In tropical climates where the rate of biochemical reaction is much faster, the oxidation of nitrogenous substances could take place earlier than the third and fourth day. Hence, the oxygen requirements for biological wastewater treatment process in the tropics

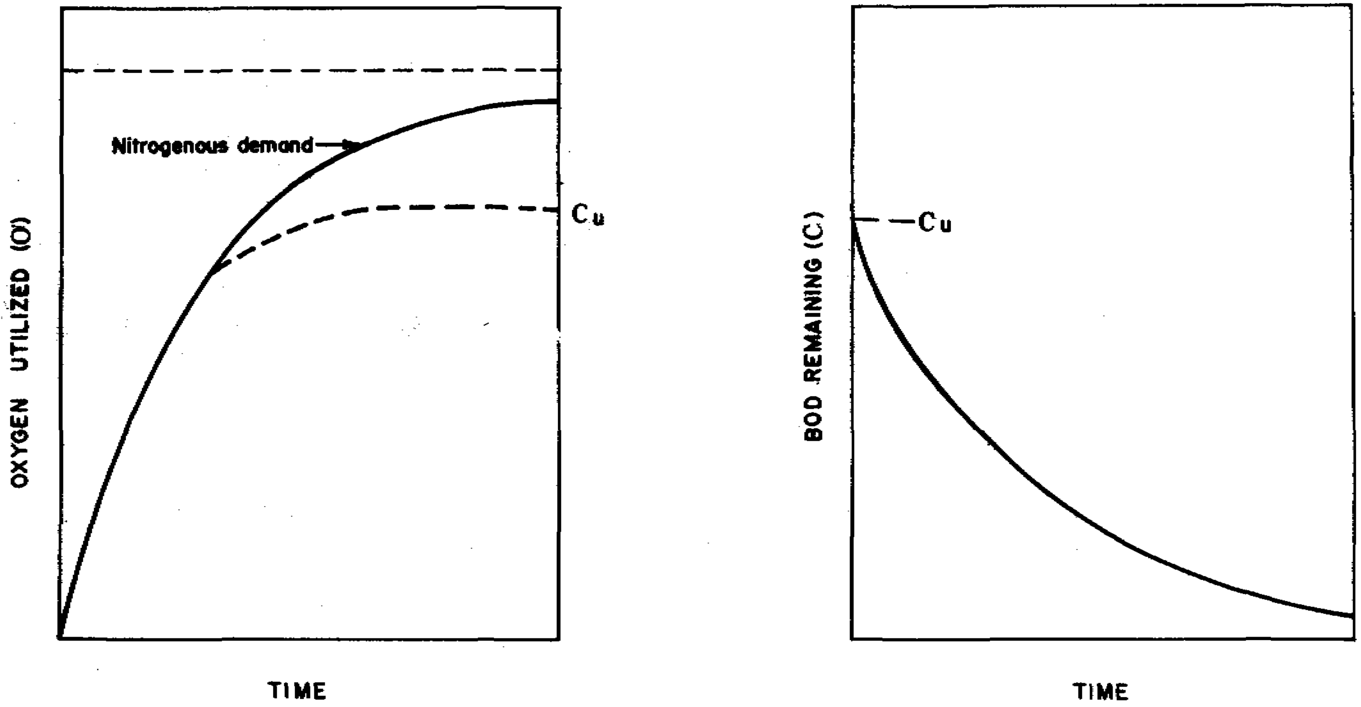


FIG. 3-3 Generalized BOD curve.

are 20% to 30% higher than in temperate climates.

If oxidation of nitrogenous compounds is suppressed, the total oxygen required to oxidize the carbonaceous substances is known as ultimate BOD. The relationship between the ultimate BOD and the BOD at any particular time, t is expressed as:

$$C = C_u \, (1 - \exp(-kt)) \qquad (3\text{-}1)$$

where C is BOD or cumulative oxygen concentration consumed after t days of incubation, mg/l; and C_u is the ultimate BOD. Hence, the relationship of the 5 day BOD or BOD_5 and the ultimate BOD could be expressed as:

$$BOD_5 = C_u \, (1 - \exp(-5k)) \qquad (3\text{-}2)$$

The BOD test is carried out in standard BOD bottles. The dilution water is saturated with oxygen and provided with sufficient nutrient and microbial seeds. The result of the BOD test is highly variable. Heavy metals in the dilution water could catalyze the oxidation of the carbonaceous substances when very dilute or they could poison the microbes thereby inhibiting the reaction.

The second major disadvantage of the BOD test is the time required for analysis. It takes more than five days to know the results after the sample is taken. By the time the results are known, the wastewater quality in the wastewater treatment plant or sampling point has changed. Hence, it is very difficult to use the BOD test results in the control of wastewater treatment plant performance. However, the BOD test is still popular today inspite of all its shortcomings. Most of the standards, design criteria and performance of wastewater treatment plants have been reported in terms of BOD. Current trends have been to shift to the COD or TOC test as those tests are more reliable and reproducible than the BOD test.

Wastewater such as domestic sewage is composed of thousands of organic compounds in very dilute concentration. To the environmental engineer, his major interest is the prevention of septic conditions in the receiving water of the wastewater treatment plant effluent. Hence, the BOD, COD or TOC is used as a parameter to represent the effects of all the organic compounds which are degradable.

COD – The chemical oxygen demand test uses a strong oxidizing agent such as an acidic solution of potassium dichromate with silver ion catalyst and high temperature. Since a strong reagent is used almost all the carbonaceous compounds are oxidized. The COD value approximates the ultimate BOD.

TOC – The total organic carbon test measures the quantity of carbon dioxide released when the organic wastes are burned in the presence of excess oxygen.

Nutrients – In early 1975, one of the sugar centrals in the Philippines had a wastewater treatment plant using physico-chemical processes which called for pH neutralization, addition of alum and flocculating aids. The process operation and maintenance cost was almost ten times that of a simple biological wastewater treatment process such as an aerated lagoon or even a facultative lagoon, which could have been used since a large tract of land was available around the factory.

The selection of the physico-chemical processes was supported by experimental results which showed that sugar wastes are non-biodegradable and such results were obtained by aerating sugar mill wastewater. With an initial COD removal of 10% on the first day, the COD removal efficiency was less than 25% even after one week of continuous aeration. Further, the pH declined to less than 4 and the wastewater remained very turbid.

Like man, bacteria do not live by bread or carbohydrates alone. Bacteria and the microorganisms in biological wastewater treatment processes require nitrogen, phosphorous, traces of iron, calcium, sodium, and other minerals. Microorganisms are more flexible in a sense that the nutrients may be supplied in salt or mineral form not necessarily in the form of proteins and organo-metallic compounds. Commercial fertilizers such as urea, ammonium phosphate, ammonium sulfate and sodium nitrate are common additives to augment any nitrogen deficiency in the wastewater. Ammonium phosphate and super phosphate fertilizer is a common chemical additive to make up for any phosphorous deficiency. Iron, calcium, sodium and traces of other elements are often found in sufficient quantities in natural and tap waters so that use of additives is unnecessary.

Bacterial cells contain approximately 10% nitrogen and 2% phosphorous. Since approximately 50% of the carbonaceous matter expressed as BOD is converted to bacterial cells, then the required ratios between BOD, nitrogen and phosphorous is:

$$\text{BOD} : \text{N} : \text{P} = 100 : 5 : 1 \qquad (3-3)$$

The effect of nutrient deficiency is more pronounced in high rate aerobic processes where the growth rate and metabolic rate are fairly rapid. The nutrient concentration could become rate limiting. Once the microbial cell dies off it releases the nutrients in the cell back to the aqueous media. This process is known as lysis. Under this condition, the requirements of Eq. 3-3 may be relaxed.

It should be borne in mind that the need for nutrient is transitory or temporary in nature. The microorganism needs the nutrient for growth and reproduction. Any excess nutrient in solution or nutrient released from cellular lysis will have to be removed if the treated effluent is discharged to a water body subject to eutrophication.

Nitrogen in solution may exist in its lower oxidation state as ammonia, protein, or nitrite. When discharged to water it exerts an oxygen demand on the receiving water. Hence, it is important to control the addition of nutrients to the wastewater specially for very strong waste such as alcohol slop, oil palm and dessicated coconut as the cost of nutrient addition and removal could be very expensive.

Temperature – Microorganisms, on the basis of their response to temperature, could be classified into three groups. The thermophiles or thermophilic bacteria grow well at temperature above 45°C reaching a peak between 60-70°C after which the growth abruptly falls off. Since the natural ambient temperature even in the tropics is below 45°C, the thermophiles are not very important in biological wastewater processes.

However, some wastewater are by-products of distillation or evaporation such as alcohol slop and oil palm wastes. The wastewater temperature at discharge is often close to boiling. Since thermophilic bacteria produce methane at a much faster rate than the anaerobic mesophiles, their potential is currently being studied.

The second group of microorganisms grows well within the range of 20°C to 40°C with peak growth rate at 35-37°C. These microorganisms are known as mesophiles or mesophilic bacteria. Since most of the ambient temperatures are within 15-37°C, the mesophilic bacteria form the most important group in biological wastewater treatment processes. Generally, the effect of temperature on the microbial growth rate below 37°C is exponential in nature. The effect of temperature on the microbial activity could be expressed as:

$$K_{(T)} = K_{20}\,(1.042)^{T-20} \qquad (3-4)$$

where K_T is the microbial activity at temperature, T, °C, and K_{20} is the microbial activity at 20°C. Fig. 3-4 shows the typical growth or reproduction rate of mesophilic bacteria at different temperatures.

It should be noted from Fig. 3-4 that the growth rate of E. Coli organism, a typical mesophile, increases with temperature. In fact in the presence of food, or carbonaceous matter and nutrients, E. Coli could multiply rapidly at 35°C. This is the basis for the E. Coli and fecal coliform test. However, Parker (1962) and Marais (1974) observed higher E. Coli and fecal coliform removal in facultative and anaerobic lagoon systems during summer than in winter time. Marais formulated the following equation to correlate the fecal coliform removal rate and temperature:

$$k_{b,T} = 0.30\,(1.05)^{T-20} \qquad (3-5)$$

where $k_{b,T}$ is the fecal coliform die off rate at temperature, T, in day^{-1}. In effect, Eq. 3-5 is in direct contradiction to the mesophile growth rate shown in Fig. 3-4. The survival of fecal coliform in tropical streams tends to confirm Fig. 3-4 rather than Eq. 3-5. Yet experimental data have shown the higher removal efficiency of the facultative lagoon system during *summer* than in *winter* time. Higher temperature is just one effect of summer. Increased solar radiation, or stimulated growth of fecal coliform predator compared to fecal coliform growth rate could account for higher fecal coliform removal efficiency in summer time. There is a serious need to reexamine the basis of Eq. 3-5. Hopefully it is not similar to the equation between typhoid incidence and the number of telephone poles described in Chapter II.

Studies by McGarry (1966) showed the effects of areal loading on pathogen kill off. Since the surface area is inversely proportional to the areal loading rates if the initial BOD concentration is constant, then the lower the areal loading the

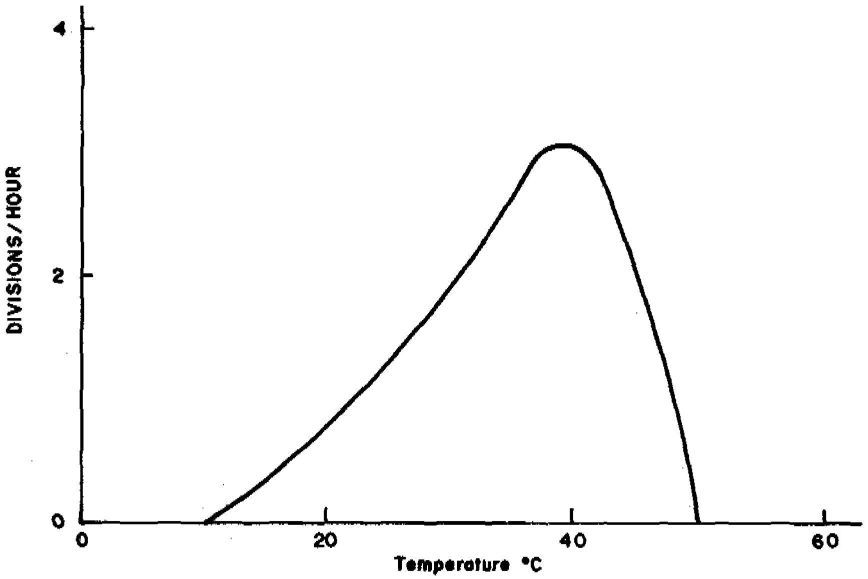


FIG. 3-4 The influence of temperature on the rate of growth of a typical mesophile.

higher is the available solar radiation. Marais (1974) reported a first order Fecal Coliform die off. The effect of solar radiation seems to increase the k_b value in a similar manner as chlorine concentration affects the rate of bacterial removal.

Bacteria whose optimum growth rate is below 20°C are known as psychrophiles. This group of bacteria is important only in biological waste treatment in very cold climates.

Dissolved oxygen – Microorganisms could also be classified depending on their capability to utilize free oxygen molecules in solution. Anaerobic bacteria can only utilize the oxygen in organic or inorganic compounds such as carbohydrates, nitrates and sulphates. In the presence of molecular dissolved oxygen, anaerobic bacterial activity is inhibited. Strict anaerobes are even exterminated by molecular oxygen.

Some anaerobic bacteria could shift their metabolism such that they could utilize molecular dissolved oxygen to oxidize carbonaceous matter. This group of organisms is known as facultative bacteria.

The third group of microorganisms could only utilize molecular dissolved oxygen to oxidize carbonaceous compounds and is called aerobic organism.

The by-products of anaerobic reaction are often dark and smelly. The smell is caused by hydrogen sulphide with the reduction of the sulphate radical. The nitrate in the wastes is often reduced to gaseous nitrogen. The hydrogen sulphide, methane, carbon dioxide and nitrogen gases reduce the specific gravity of organic solids to which they adhere. As a result the solids are bouyed up and are commonly called scum. The rate of carbonaceous degradation is much slower in anaerobic reaction than in aerobic reaction. For this reason, anaerobic reaction was limited to the treatment of strong wastes such as sludges. In fact in the early 1970's aerobic sludge stabilization started to gain acceptance due to smaller reactor sizes required.

One of the gaseous by-products of anaerobic reaction is methane gas. Methane gas is a useful fuel with a heating value of 1000 Btu/ft^3(STP). Normal gaseous by-products of anaerobic digestion could contain from 60-70% methane by volume with the remainder made up of carbon dioxide, hydrogen sulphide and hydrogen. When the energy cost started to escalate in 1972, anaerobic reaction became popular. Methane is commonly known as biogas to the layman.

The by-products of aerobic reaction are carbon dioxide, nitrate, sulphates and extra cellular matters. Since, the only gaseous by-product is carbon dioxide the fermentation is odorless. However, the solubility of oxygen in water is very low. Large quantity of energy is required to introduce the oxygen from the air into the wastewater. The major operating cost of aerobic wastewater treatment plants is the energy required for oxygen dissolution or aeration.

pH – pH is the negative logarithm of the molar concentration of hydrogen ion in solution. Wastewater with pH below 7 are acidic while those with pH above 7 are alkaline. Each microbial group has its own range of tolerance to hydrogen ion concentration. Acid forming bacteria such as those responsible for vinegar production could tolerate pH below 5 although most bacterial growth is inhibited at this condition. The ideal pH range for bacterial growth is from 6.5 to 9. At pH below 5, yeast and fungi start to predominate. Methane forming anaerobes are very sensitive to pH changes.

Some ions such as carbonates and bicarbonates form a buffer. In the presence of H^+, the carbonate undergoes the following reaction:

$$CO_3^{=} + 2\,H^+ \rightarrow HCO_3^- + H^+ \rightarrow H_2O + CO_2 \qquad (3-6)$$

In this condition, the added hydrogen ion is eliminated, thus maintaining the pH constant. The reverse reaction takes place with the addition of alkaline substance or hydroxyl ions

$$HCO_3^- + OH^- \rightarrow CO_3^{=} + H_2O \qquad (3-7)$$

which again maintains the pH constant. Since carbon dioxide is a common by-product of biological degradation, the carbonate and bicarbonate buffers are very important in maintaining the stability of the processes.

The presence of metallic ions such as calcium and magnesium retains the carbonate and bicarbonate ions in solution effectively. For this reason wastewater treatment plants with "hard" wastewater are easier to operate or are more stable. Liming or addition of calcium hydroxide is often practised in anaerobic digestion not only to increase the pH but also to improve the buffering capacity of the wastewater. However, care should be exercised in adding lime or sodium carbonate or ammonium hydroxide as excessive concentration of sodium, calcium or ammonium could be also toxic to the bacteria.

Table 3-1 summarizes the concentration of common ions which inhibit bacterial growth. It should be noted that some nutrients become poisonous when found in excess.

Table 3-1 Effects of Common Substances on Performance of Activated Sludge Process (mg/l)

		Concentration and Effect	
Chemical Substances	**No Effect**	**Inhibitory**	**Upset**
Ammonia	100		480
Arsenic	0.1	0.1	0.1
Borate	0.1	1.0	0.1
Cadium	–	1-100	50
Chromium (hexavalent)	–	1-10	–
Chromium (trivalent)	–	50	–
Copper	0.2	.1	1.0
Cyanide	0.1	13-100	200
Iron	–	1-1000	1000
Lead	0.1	.1	–
Manganese	10	20	60
Mercury	–	.1	200
Nickel	–	1.0	20
Silver	–	9	30
Sulphide	–	20	–
Zinc	10	.01	–

BIOCHEMICAL KINETICS

Aerobic degradation of organic wastes could be generalized by the following reaction:

bacterial cell + organic matter + oxygen + nutrient → CO_2 + H_2O + more bacterial cells (3 – 8)

Anaerobic degradation could be generalized by a two stage symbiotic reaction :

acid forming bacteria + organic matter + nutrient → organic acids + CO_2 + more acid forming bacteria (3 – 9)

and

methane forming bacteria + organic acids + nutrients → CH_4 + CO_2 + more methane forming bacteria (3 – 10)

For biochemical reaction to take place, there should be contact between the different reactants. The chances of contact increase as the reactant becomes more concentrated or numerous. For example, consider the possible number of collision between the black and white balls shown in Fig. 3-5 . Black ball (B—1) could collide with either W-1, W-2 or W-3. Similarly B-2 could collide with the three white balls. Hence, the total number of collision per unit time is proportional to the product of the number of black and white balls. Using the above analogy the rate of organic matter degradation could be expressed as:

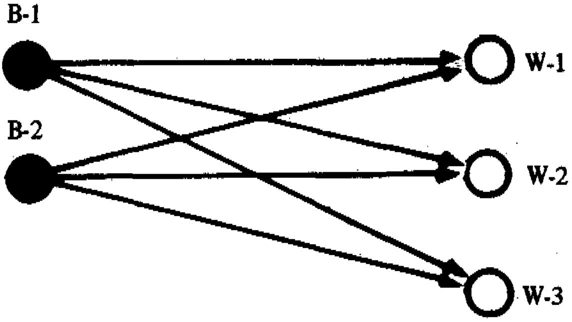


FIG. 3-5 Effect of particle concentration on collision rate.

$$\frac{d\,(C)}{dt} = -K\,C\,X_a\,O\,N \qquad (3-11)$$

when C is the concentration of carbonaceous matter, moles/liter; X_a is the number of bacterial cells per liter; 0 is the molar concentration of oxygen, and N is the product of concentration of the different essential nutrients.

Consider the case shown in Fig. 3-5 wherein the number of white balls is too large compared to the black balls. The number of collision per unit time could depend on the number of black balls present since the white balls are almost everywhere.

The same phenomenon has been observed in biological reaction. The reactants in excessive quantities could be treated as constant or non-rate limiting. Fig. 3-6 shows the effect of carbonaceous matter concentration on microbial growth rate. The microbial growth rate increases proportionally with increasing concentration of carbonaceous matter at low concentration. At very high concentration, the growth rate becomes almost independent of the carbonaceous matter concentration.

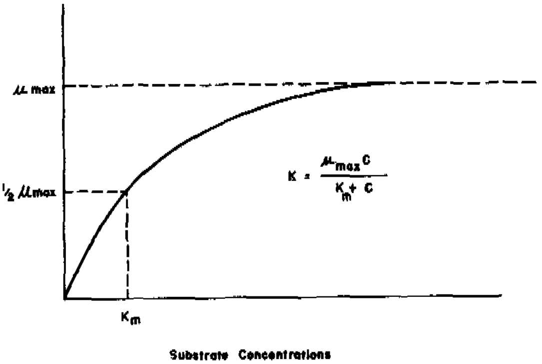


FIG. 3-6 Monods relationship between assimilation rate and substrate concentration.

A number of equations have been proposed to quantify this phenomenon. The most popular being the simplest is:

$$\frac{dC}{dt} = \frac{-K\ C}{K_m + C} \qquad (3-12)$$

where K_m and K are constants, the other terms are as defined before.

Hence, if the effect of carbonaceous matter concentration defined by Eq. 3-8 is generalized for the other reactants, Eq. 3-12 could be modified as:

$$\frac{dC}{dt} = -K\left(\frac{K_1 C}{K_m + C}\right)\left(\frac{K_2 X_a}{K'_m + X_a}\right)\left(\frac{K_3 0}{K''_m + 0}\right)\left(\frac{K_4 N}{K'''_m + N}\right) \qquad (3-13)$$

where K_1, K_2, K_3, K_4, K_m, K_m', K_m'' and K_m''' are constants. Wastewater treatment plants are often operated with excess nutrients and oxygen. When $0 >> K''_m$ and $N >> K'''_m$ then Eq (3-13) simplifies to:

$$\frac{dC}{dt} = -K\left(\frac{K_1 C}{K_m + C}\right)\left(\frac{K_2 X_a}{K'_m + X_a}\right) \qquad (3-14)$$

The number of microorganisms is very low compared to the concentratio carbonaceous matter. In one liter of wastewater, the microbe population the region of 10 to 10^{14}, whereas the carbonaceous matter moleculés coul in the range of 10^{23} to 10^{27}. In most instances $K_m' \gg X_a$ hence, Eq. 3-14 is simplified further as:

$$\frac{dC}{dt} = -k'\left(\frac{C}{K_m + C}\right)X_a = f(X_a, C) \qquad (3-15)$$

where k' is the product of K, K_3, K_4, K_2 and K_1.

In wastewater, the carbonaceous matter is a mixture of a wide variety of organic compounds, making it difficult to determine C in molar concentration. For this reason C is approximated by the ultimate carbonaceous BOD concentration in mg/l. Similarly X_a is approximated by the mixed liquor volatile suspended solids (MLVSS) concentration in mg|l. Eq. 3-15, using the above mentioned definition, is commonly used in practice today.

From Eq. 3-8, 3-9, and 3-10, more bacterial cells are produced from the reaction among bacteria, food and nutrients. Some of the food and nutrients consumed by the bacteria are used for energy production, repair of cellular tissues, or stored in the cells for future use. The remainder is utilized for the production of new cells. Hence, the rate of increase of bacterial mass X_a could be approximated as:

$$\frac{dX_a}{dt} = -\frac{dC}{dt}Y \qquad (3-16)$$

where Y is the yield coefficient, mg of bacterial cells produced|mg of BOD; and the other terms are as defined before.

It should be noted that Eq. 3-16 includes the production of new cells and increase in cellular weight due to storage. The derivation of the original Eq. 3-13 and 3-15 defined X_a in terms of cells per unit volume. The use of MLVSS for X_a is only an approximation which includes the increase in cellular weight and new cells produced. The yield coefficient could also vary depending on the age of the microbial colony, floc or slime. However, to simplify the calculations Y is assumed constant.

Combining Eq. (3-16) and (3-15) yields:

$$\frac{1}{X_a}\frac{dX_a}{dt} = Y\frac{kC}{K_m + C} = \hat{\mu} \qquad (3-17)$$

where $\hat{\mu}$ is known as the specific growth rate. The significance of specific growth rate will be discussed in succeeding sections on biochemical reactors.

As C >> K_m, then from Eq. 3-15, $k \rightarrow \hat{\mu}$ max. The original form of Eq. 3-15 was derived by Monod (1949) as:

$$\frac{1}{X_a}\frac{dX_a}{dt} = \frac{\hat{\mu}_{max}\, C}{K_m + C} \qquad (3-18)$$

Autocatalytic reactions – A catalyst is a chemical substance which hastens the rate of chemical or biochemical reaction by providing active reaction sites or linkages. A catalyst is unaltered or unchanged in the reaction although it may form intermediary products. Although the bacterial mass participates in the biochemical reaction the changes of its biological characteristics are not very significant after the reaction.

From Eq. 3-15, the rate of degradation of the organic wastes C, is dependent on the microbial mass concentration. Since, more microbial mass is formed from the organic waste as given in Eq. 3-8 to 3-10, the rate of organic waste degradation is hastened until C becomes very low. The reaction is known as autocatalytic since it is hastened by one of the reaction by-products which is the microbial mass. Eq. 3-16 has a general solution:

$$X_a = Y(C_o - C_e) + X_{a,o} \qquad (3-19)$$

where $X_{a,o}$ is the initial microbial concentration, and C_o is the initial organic waste concentration and C_e is the effluent organic waste concentration. Substituting Eq. 3-19 into Eq. 3-15, yields:

$$\frac{dC}{dt} = -k' \frac{C}{K_m + C}\left\{Y(C_o - C_e) + X_{a,o}\right\} = f(C) \qquad (3-20)$$

Fig. 3-7 is a graphical illustration of Eq. 3-20. At the start, the rate of reaction (dC/dt) is slow as shown by point A in Fig. 3-7. As more microbial mass is produced, the reaction increases until it reaches point B, after which it decreases as the organic waste concentration approaches zero or becomes rate limiting.

FUNDAMENTALS OF REACTOR DESIGN

A biochemical reactor is a vessel wherein the wastewater, oxygen, microbial cells and nutrients are added and mixed. The organic constituents are degraded as discussed in the previous section. The biochemical reactor may be an aerated lagoon, an activated- sludge, an aeration tank, a trickling filter, a rotating disk filter, a facultative or an anaerobic lagoon. Before discussing the principles of biochemical reactor, it is worthwhile discussing the classification and common terminology used in design.

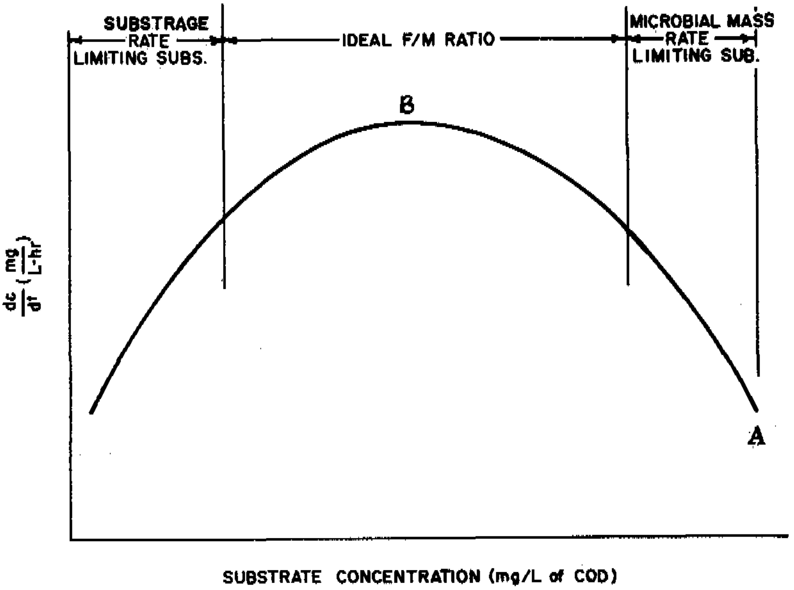


FIG. 3-7 Autocatalyctic substrate degradation curve.

Feeding Pattern – Biochemical reactors are classified either as batch or continuous, depending on the feeding and discharge pattern.In a batch reactor, all the reactants, i.e., wastewater, microbial solids, and nutrients are mixed until the aeration tank is filled up. The mixing may be carried by aerators or diffusers which supply the oxygen simultaneously. A common example of this type is a draw and fill activated sludge process which does away with the secondary sedimentation tank. This type is quite popular in very small communities or industries where the major wastewater flows may take place only at certain hours of the day with an interval of six to eight hours of very low wastewater flows.

The batch or draw and fill reactor is filled in the early morning after which it is continuously aerated for four to six hours. The aeration is stopped and the sludges allowed to settle for 1 to 2 hours. The treated wastewater is drained leaving the settled sludge in the bottom of the tank. The tank is again filled with wastewater and the settled sludge is resuspended. Fig. 3-8 is a typical operation sequence of a batch reactor.

The batch reactor functions as equalization, aeration, clarification and sludge stabilization tanks simultaneously. Although tremendous savings in construction cost is realized for a very small wastewater treatment system, the operational efficiency is lowered as the tank function is varied. If different tanks of specialized functions are constructed and the wastewater is continuously fed into the reactor, the reactor is known as a continuous reactor. Most of wastewater treatment processes are continuous reactors. Under this category falls the different activated sludge variations, trickling filter and lagoons.

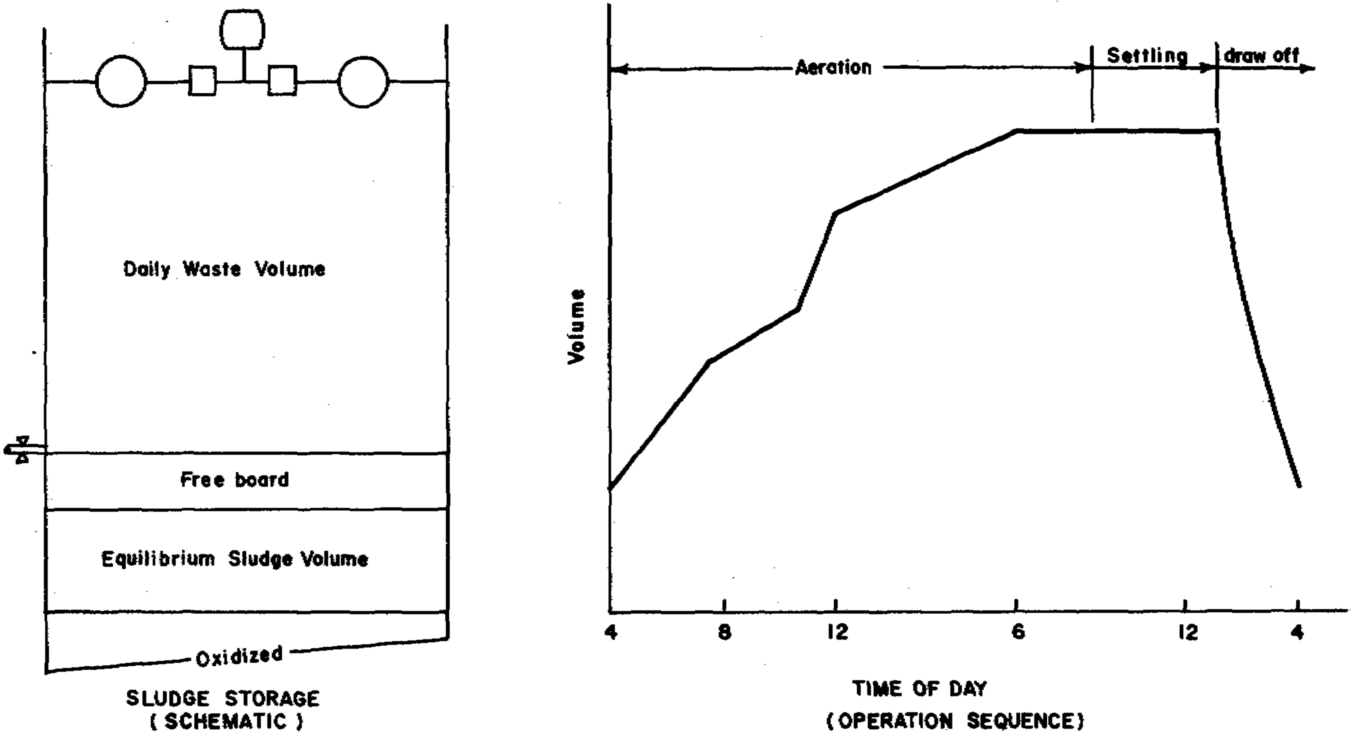


FIG. 3-8 Batch activated sludge process.

Dispersion – A biochemical reactor is classified as a completely mixed or a plug flow reactor depending on the longitudinal dispersion or mixing pattern. Conventional design of wastewater treatment plants aims to attain plug flow conditions as much as possible. It was shown by McKinney (1962) that the completely mixed activated sludge process has the same BOD removal efficiency as the conventional activated sludge and at the same time, exhibit better stability to shock load.

Based on computer modelling by Fan, Erickson, Shah and Tsai (1970) Milbury, Pipes, and Grieves (1965), the plug flow activated sludge process should have a higher BOD removal efficiency than the completely mixed activated sludge process. Since experimental results by McKinney (1962, 1974), and Toeber (1974) showed the contrary, it was assumed that the degradation constant k could vary with the mixing pattern. Although there have been some studies on the effect of turbulence on biological activity of bacterial flocs, the difference was not significant to account for the difference of efficiency between the plug flow and completely mixed system. Perhaps the major shortcoming of those earlier models was the assumption of a non-autocatalytic reaction.

Ouano and Dissayanake (1978) analyzed the effect of dispersion on biochemical wastewater treatment system from the standpoint of autocatalytic reaction. For very low removal efficiency (80%), the completely mixed system is more efficient than the plug flow system while at removal efficiencies higher than 95%, the reverse is true. In between, the two processes perform with similar efficiencies. Since, most biological wastewater treatment systems are operated at BOD removal efficiency between 80-95%, then the performance of the completely mixed and plug flow processes are almost similar as described above. Detailed discussion is made in the later section of this chapter on reactor size optimization.

Oxygen requirements – On the basis of oxygen requirements, a biological wastewater treatment reactor could be classified as aerobic, anaerobic or facultative. Anaerobic reactors are closed and provision for collection of methane gas is often made. Aerobic reactors are usually open and well ventilated except when pure oxygen is used.

Homogeneity – Although the wastewater and biological flocs or slime form a heterogeneous mixture, for simplicity of analysis, the biochemical reactions are assumed to take place in a homogeneous medium. This assumption is used even for anaerobic digestion of sludge.

REACTOR SIZING

The major capital cost of the wastewater treatment reactor is the civil works component for tank construction. In most instances, the limitation of the aerator size is not for oxygen transfer but for mixing the flocs. Since the mixing power requirements increase with the tank volume, the smaller the tank volume, the lower is the power cost. Power is the major operating cost.

For a given wastewater flow rate, q, the required wastewater treatment tank volume, V, is equal to:

$$V = q\Theta \tag{3-21}$$

If the incoming wastewater has a BOD concentration C_o and it is desired to have an effluent quality C_e, then integrating Eq. 3-15 yields:

$$\int_{C_o}^{C_e} \frac{dC}{f(X_a, C)} = \int_0^{\Theta} dt \tag{3-22}$$

or

$$\Theta = \int_{C_o}^{C_e} \frac{dC}{f(X_a, C)} \tag{3-23}$$

hence, the biological wastewater treatment tank volume is:

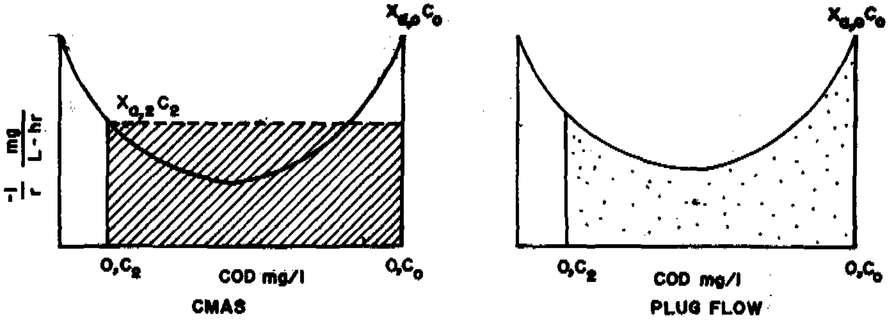
$$V = q\Theta = q\int_{C_o}^{C_e} \frac{dC}{f(X_a, C)} \tag{3-24}$$

Equation 3-24 is the general equation for determining the volume of the wastewater treatment plant reactor or aeration tank.

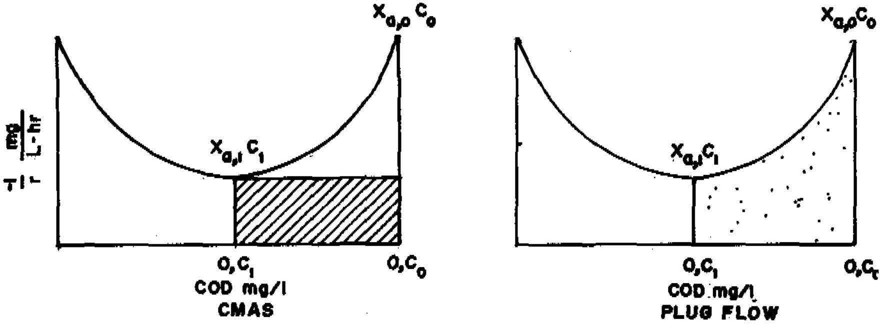
Dispersion – Consider a general case wherein the degradation rate is $f(X_a, C)$ instead of the first order term $k\,C$ used in Eq. 1-44. Then the general dispersion and biological reaction could be expressed as:

$$\Theta \frac{\partial C^*}{\partial t^*} = \frac{D_L}{\bar{V}L}\,\frac{\partial C^*}{\partial X^{*2}} - \frac{\partial C^*}{\partial X^*} - \Theta f\,(X_a, C^*) \tag{3-25}$$

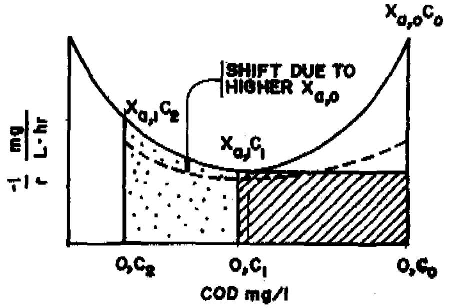
Eq. 3-25 is the generalized dimensionless form of Eq. 1-46.



a) PLUG FLOW MORE EFFICIENT THAN CMAS



b) CMAS MORE EFFICIENT THAN PLUG FLOW



c) OPTIMAL COMBINATION OF PLUG FLOW AND CMAS REACTOR

FIG. 3-9 Comparison of efficiency parameter.

Plug flow – For plug flow, D_L/VL is equal to zero. If the wastewater treatment system has attained steady state, then dC/dt is also zero. Hence, Eq. 3-24 simplifies to:

$$\frac{dC}{dt} = -\Theta f(X_a, C^*) \qquad (3\text{-}26)$$

Since $X^* = X/L$, $\Theta = V/q$, then

$$\int_{o}^{1} \frac{V}{q}\, dX = -\int_{C_o}^{C_e} \frac{dC}{f(X_a, C)} \qquad (3-27)$$

or

$$V = \int_{C_o}^{C_e} -q \frac{dC}{f(X_a, C)} \qquad (3-28)$$

Eq. 3-28 is the same as Eq. 3-24. The integral of Eq. 3-22 and 3-26 could be graphically visualized as the area bounded by C_o, C_2, C axis, and the loci of the curve $- 1/f(X_a, C)$. This is shown in Fig. 3-9 (a).

Completely mixed reactor – Since it is difficult to solve Eq. 3-25 when $D_L/VL \rightarrow \infty$ the completely mixed tank equation is solved through the mass balance approach. By definition, the effluent concentration C_e is equal to the concentration inside the tank since the solution is homogeneous all throughout the tank. The different mass flows are as follows:

mass in : $q\,C_o$
mass out: $q\,C_e + V\,f(X_a, C_e)$
accumulation = $V\,dC_e/dt$

From the law of mass conservation, mass in is equal to the sum of the outgoing mass and accumulation. Hence,

$$qC_o = qC_e + V\, f(X_a, C_e) + V\, dC_e/dt \qquad (3-29)$$

At steady state, accumulation is zero or $dC_e/dt = 0$. Hence, the volume V could be solved from Eq. 3-29 as:

$$V = q \frac{C_o - C_e}{f(X_a, C_e)} \qquad (3-30)$$

The reactor volume required in a completely mixed tank is equal to the product of the flow rate and area of the rectangle with width $(C_o - C_e)$ and length $f(X_a, C_e)$. This is shown graphically in Fig. 3-9 (b).

First order approximation – In general the microbial concentration is assumed to remain constant as X_a. Hence, Eq. 3-15 simplifies to

$$f(X_a, C) = \frac{K \bar{X}_a C}{K_m + C} \qquad (3-31)$$

When $K_m >> C$, and if $\bar{X}_a$ is not variable, then,

$$f(X_a, C) \approx f(C) = K'C \qquad (3-32)$$

where

$$K' = \frac{K X_a}{K_m} \qquad (3-33)$$

Eq. 3-33 is the first order approximation of the biological waste treatment process. This equation is commonly used in the design of wastewater treatment system since the solution to Eq. 3-24, 3-28 and 3-30 is easily derived.

Plug flow – For plug flow condition, Eq. 3-28 has the following solution:

$$V = q \int_{C_o}^{C_e} - \frac{-dC}{K'C} = -\frac{q}{K'} \ln (C_e/C_o) \qquad (3-34)$$

or

$$C_e = C_o \exp\left[\frac{(-K'V)}{q}\right] \qquad (3-35)$$

If the detention time Θ is equal to V/q, then Eq. 3-35 could be also expressed as:

$$C_e = C_o \exp(-K'\Theta) \qquad (3-36)$$

This type of equation is commonly encountered in trickling filter, and BOD degradation curve.

Completely Mixed Reactor – For a biochemical reactor which is completely mixed, the use of first order approximation yields

$$f(X_a, C) \approx K'C_e \qquad (3-37)$$

Hence, Eq. 3-30 has the following solution

$$V = q\,\frac{C_o - C_e}{K'C_e} = \frac{q}{k'}\left[\frac{C_o}{C_e} - 1\right] \qquad (3-38)$$

or

$$C_e = \frac{C_o}{1 + \dfrac{K'V}{q}} \qquad (3-39)$$

In terms of detention time Θ, Eq. 3-39 could be expressed as:

$$C_e = C_o/(1 + k'\Theta) \qquad (3-40)$$

This type of equation is commonly used in lagoon, completely mixed activated and trickling filter studies.

Completely Mixed Tanks in Series – If a number of completely mixed tanks of volumes $V_1, V_2, V_3, V_4, \ldots V_n$ are used, the mass balance at each tank are shown in Fig. 1–5. The effluent at the first tank using Eq. 3-40 is:

$$C_1 = C_o/(1 + K'V_1/q) \qquad (3-41)$$

At the second tank, the effluent is:

$$C_2 = C_1/(1 + K'V_2/q) \qquad (3-42)$$

Putting Eq. 3-41 into Eq. 3-42 for C_1, yields

$$C_2 = \frac{C_o}{(1 + K'V_1/q)\ (1 + K'V_2/q)} \tag{3-43}$$

After the nth tank, then the effluent BOD concentration C_n, could be expressed as:

$$C_n = \frac{C_o}{\prod_{i=1}^{n} (1 + K'V_i/q)} \tag{3-44}$$

If the tanks are all of equal volume, V/n, then Eq. 3-44 takes the form.

$$C_n = C_o / \left[1 + \frac{K'V}{nq}\right]^n \tag{3-45}$$

If the total tank volume is divided into n equal tanks each with a volume V/n, then Eq. 3-45 could be written as:

$$C_e = \frac{C_o}{\left[1 + \frac{K'V}{nq}\right]^n} \tag{3-46}$$

From Eq. 3-46, V could be solved as:

$$V = \frac{nq}{K'}\left[\left(\frac{C_o}{C_e}\right)^{1/n} - 1\right] \tag{3-47}$$

Eq. 3-39 is a special case of Eq. 3-47 when n = 1. It should be noted that as n increases, V decreases for the same values of C_o/C_e, q and K'. The decrease in V is shown graphically in Fig. 3-10.

Optimization – By means of geometric programming, Ouano (1974) proved that the minimum total volume is attained when the volumes at each stage are equal, that is:

$$V = V_1 + V_2 + V_3 + \ldots V_n \tag{3-48}$$

and

$$V_1 = V_2 = V_3 = V_4 = \ldots = V_n \tag{3-49}$$

Hence, the optimal equation which gives the minimum total volume for reactors in series is:

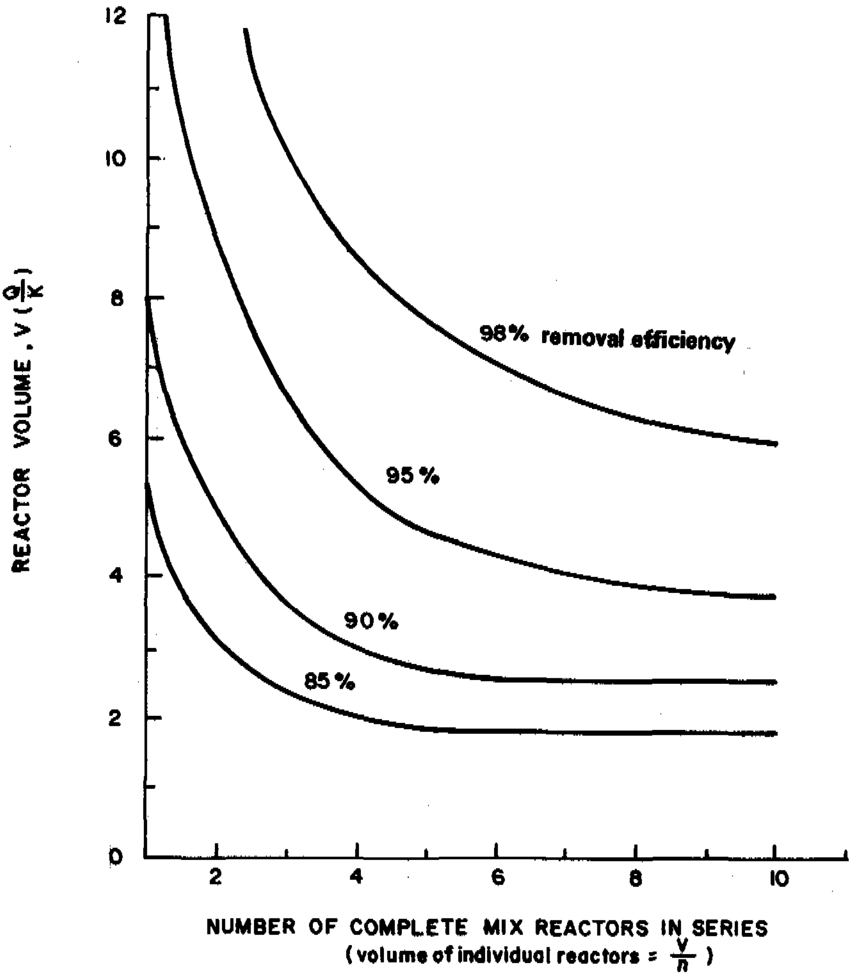


FIG. 3-10 Required reactor volume versus the number of complete mix reactors in series for various removal efficiencies.

$$V = \frac{nq}{K'}\left[\left(\frac{C_o}{C_e}\right)^{1/n} - 1\right] \qquad (3-50)$$

Marais (1970) showed that Eq. 3-47 and 3-50 define the minimum total volume of a bio-chemical reactor divided into *n* tanks in series using mathematical induction.

Example 3-1. Determine the facultative lagoon volume required to treat $1000 m^3$/day of sewage if K value is 0.4/day. The initial BOD is 250 mg/l and it is desired to have an effluent of 25 mg/l. The wastewater will be treated in a) 1 pond b) 2 ponds in series c) 4 ponds in series and d) 8 ponds in series.

Solution: From Eq. (3 – 50)

$$V = n \frac{1000}{.4}\left[\left(\frac{250}{25}\right)^{1/n} -1\right]$$

$$= n\, 25000 \times \left[(10)^{1/n} -1\right]$$

hence, for

a) $n = 1$
$V = 2500((10) - 1) = 22500 m^3$
b) $n = 2$
$V = 2500 \times 2 ((10)^{½} - 1) = 10{,}811 m^3$

The volume per pond is then equal to 10.811/2 or 5,405 m^3

c) $n = 4$
$V = 2500 \times 4 ((10)^{.25} - 1) = 7782 m^3$
The volume per pond is 7782/4 or 1,946 m^3

d) $n = 8$
$V = 2500 \times 8 ((10)^{.125} - 1) = 6670 m^3$.

It should be noted that reduction in volume required is very high when the lagoon is divided into two ponds. The first reduction from 22,500 to 10,811 m^3 is a saving of more than 50%. If the ponds were divided into 9 smaller ponds in series then the volume V is equal to 6560 or a saving of only 1.6% over eight ponds in series.

Hence, the extra cost of constructing the dividers to convert a pond into five smaller ponds in series could be higher than the incremental volume saved. It should be remembered that the effective number of tanks in series could be higher than the physical number of tanks depending on the pond configuration. One tank which is long and narrow with very slow cross sectional velocity could behave as if it were subdivided into five or seven tanks in series.

As the number of tank n approaches infinity Eq. 3-46 approaches Eq. 3-35. Hence, the behaviour of an n – completely mixed tank in series as n → infinity is the same as a plug flow tank. The proof is presented as follows:

$$\lim_{n \to \infty} \left(1 + \frac{K'V}{nq}\right)^{n} = \lim_{\frac{nq}{K'V} \to \infty} \left(1 + \frac{K'V}{nq}\right)^{\frac{nq}{K'V} \cdot \frac{K'V}{q}} \qquad (3-51)$$

Since from elementary calculus.

$$\lim_{x \to 0} \quad (1 + x)^{\frac{1}{x}} = \exp \qquad (3-52)$$

where exp is the base for the Napierian logarithm equal to 2.718, hence, the limit of Eq. 3-51 is:

$$\lim_{\frac{nq}{K'V} \to \infty} \left(1 + \frac{K'V}{nq}\right)^{\frac{nq}{K'V} \cdot \frac{K'V}{q}} = e\left(\frac{K'V}{q}\right)$$

(3-53)

Substituting Eq. 3-53 into Eq. 3-46, yields

$$C_e = C_o \exp(-K'V/q) \qquad (3-54)$$

Eq. 3-54 is equal to Eq. 3-35 which proves that as n → ∞, the performance of an n—completely mixed tank in series approaches that of a plug flow tank.

Example 3-2. Calculate the volume required to treat the wastewater with properties in Example 3-1 if the treatment tank is operated as a plug flow tank. b) Compare the volume required if n = 1,000 for the n-completely mixed tank in series.

Solution From Eq. 3-35,

$$\frac{C_e}{C_o} = \exp\left(\frac{-K'V}{q}\right)$$

substituting the values given in Example 3-1

$$\frac{25}{250} = \exp\left(\frac{-.4V}{1000}\right)$$

or $V = 5756.5 \text{ m}^3$

b) From Eq. 3-47

$$V = \frac{1000}{.4} \times 1000\left[\left(\frac{250}{25}\right)^{\frac{1}{1{,}000}} - 1\right]$$

$= 5763 \text{ m}^3$

The derivation of Eq. 3-50 for the n-completely mixed tank in series model shows that the plugflow reaction tank is more efficient than one completely mixed reaction tank of the same volume provided X_a is assumed *constant.*

Dispersion effect – At steady state dC/dt is equal to zero. Eq. 3-25 simplifies to:

$$\frac{D_L}{\bar{V}L}\frac{d^2C^*}{dX^{*2}} - \frac{dC^*}{dX^*} - KC^* = 0 \qquad (3-55)$$

Wehner and Wilhelm (1958) provided solution to the effect of dispersion on the efficiency of chemical reactors. The approach used by Wehner and Wilhelm was to divide the reactor into three sections, $X^* < 0$, $0 \leq X^* \leq 1$ and $X^* > 1$. The reactor is defined by the second region $0 \leq X^* \leq 1$. The two extremes were provided to differentiate the effect of $D_L/\bar{V}L \rightarrow \infty$ due to either $\bar{V}$ or L approaching zero. By defining the behavior of the reactor with X^* varying from $-\infty$ to ∞, it is imposed that L be greater than zero.

If the detention time, Θ, is greater than zero, then $\bar{V}$ or L/Θ is greater than zero. Therefore when $D_L/VL \rightarrow \infty$, the variation is due to increase in mixing or dispersion with $D_L \rightarrow \infty$.

Wehner and Wilhelm (1958) suggested the governing equations and boundary conditions for the three segments described above as:

For $X^* < 0$ there is no reaction, $K = 0$, hence Eq. 3-55 at this region is equal to:

$$\left(\frac{D_L}{\bar{V}L}\right)_a \frac{d^2C^*}{dX^{*2}} - \frac{dC^*}{dX^*} = 0 \qquad (3-56)$$

For $0 \leq X^{*} \leq 1$, Eq. 3-55 is fully applicable

$$\left(\frac{D_L}{\bar{V}L}\right)\frac{d^2C^*}{dX^{*2}} - \frac{dC^*}{dX^*} - KC^* = 0 \qquad (3-57)$$

And for $X^* > 1$, again $K = 0$, and Eq. 3-55 is simplified to:

$$\left(\frac{D_L}{\bar{V}L}\right)_c \frac{d^2C^*}{dX^{*2}} - \frac{dC^*}{dX^*} = 0 \qquad (3-58)$$

The boundary conditions to the three differential equations are:

$$C^*(-\infty) = 1 \qquad (3-59)$$

$$\bar{V}_a\, A_a\left[C^*(0-) - \left(\frac{D_L}{\bar{V}L}\right)_a \frac{d\,C^*(0-)}{dX^*}\right] =$$

$$\bar{V}_b\, A_b\left[C^*(0+) - \left(\frac{D_L}{\bar{V}L}\right)_b \frac{dC^*(0+)}{dX^*}\right] \qquad (3-60)$$

$$\bar{V}_a A_b = \bar{V}_b A_b = \bar{V}_c A_c \qquad (3-61)$$

$$C^*(0-) - \left(\frac{D_L}{\bar{V}L}\right)_a \frac{d\,C^*(0-)}{dX^*} = C^*(0+) - \left(\frac{D_L}{\bar{V}L}\right)_b \frac{C^*(0+)}{dX^*} \qquad (3-62)$$

$$C^*(0-) = C^*(0+) \qquad (3-63)$$

$$C^*(1-) - \left(\frac{D_L}{VL}\right)_b \frac{dC^*(1-)}{dX^*} = C^*(1+) - \left(\frac{D_L}{VL}\right)_c \frac{dC^*(1+)}{dX^*} \qquad (3-64)$$

$$C^*(1-^*) = C^*(1+) \qquad (3-65)$$

$$C^*(\infty) << \infty \qquad (3-66)$$

The subscript, a, denotes parameter of the section $X^* < 0$, while the subscript, b, for the section $0 \leq X^* \leq 1$ and subscript, c, for $X^* > 1$. The + and – signs denote the direction from which limits are taken.

For the region $0 \leq X^* \leq 1$, the general solution to the Eq. 3-57 is:

$$C^* = \frac{4\, a' \exp\left(0.5/\left(D_L/\bar{V}L\right)\right)}{(1+a')^2 \exp\left(.5a'/(D_L/\bar{V}L)\right) - (1-a')^2 \exp\left(-.5/(D_L/\bar{V}L)\right)} \qquad (3-67)$$

where $a' = \sqrt{1 + 4k\Theta(D_L/\bar{V}L)}$. The graph of Eq. 3-67 is shown in Fig. 1-2.

Fig. 3-11 shows the instantaneous values of C^* at different values of X^*. It should be noted that as $D_L/\bar{V}L$ increases, a pseudo – decrease in C^* takes place at $X^* < 0$ due to dispersion or back mixing.

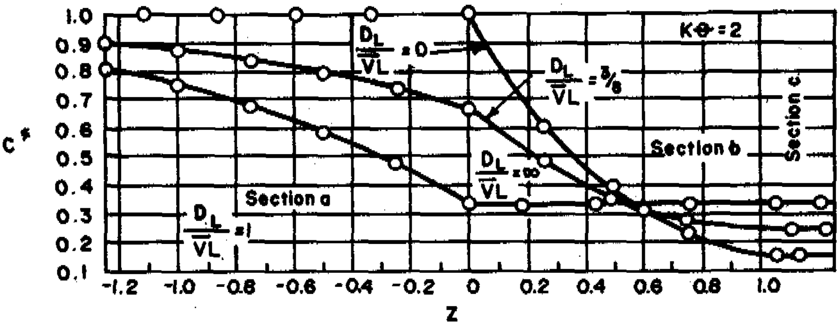


FIG. 3-11 Complete solution to Wilner – Wilhelm equation

The validity of Eq. 3-67 could be proven by comparing its limit when $D_L/\bar{V}L \to 0$ and $D_L/\bar{V}L \to \infty$ with those of Eq. 3-36 and 3-40 respectively. The reader will be tempted to use the following boundary conditions to Eq. 3-55 instead of those used by Wellner and Wilhelm:

$$\begin{aligned} X^* &= 0 & C^* &= 1 \\ X^* &= 1 & C^* &= C_e/C_o \leq 1 \end{aligned} \qquad (3-68)$$

The solution to Eq. 3-55 with Eq. 3-68 as boundary condition is:

$$C^* = \exp\left\{\frac{1 - \sqrt{1 + 4k\Theta\; D_L/\bar{V}L}}{2\,D_L/\bar{V}L}\right\} X^* \qquad (3-69)$$

When $D_L/\bar{V}L \rightarrow 0$, Eq. 3-69 approaches Eq. 3-36 at $X^* = 1$ but when $D_L/\bar{V}L \rightarrow \infty$, Eq. 3-69 does not define Eq. 3-40. Instead $C^* = 1$, at $X^* = 1$ when $D_L/\bar{V}L \rightarrow \infty$. This could mean that no reaction has taken place in the reactor because $L \rightarrow 0$ as $D_L/\bar{V}L \rightarrow \infty$. As discussed before as $L \rightarrow 0$, the reactor disappears.

For lagoons $D_L/\bar{V}L$ varies from 0.1 to 2.0, while for plug flow or conventional activated sludge it is below 0.2 For completely mixed activated sludge the dispersion number exceeds 4. The plug flow and completely mixed system are idealized conditions which could not be attained in practice.

> Example 3-3. Consider an experiment which shows that a facultative lagoon system reduces the influent BOD concentration from 250 mg/l to 25 mg/l after 20 days retention. Assume the rate of degradation to follow first order kinetics. Calculate K if the facultative lagoon a) is completely mixed b) has plug flow c) has a dispersion number of 0.1 d) has a dispersion number equal to 2.

Solution: The BOD remaining is calculated as:

BOD = 25/250 = 0.1 or 10%

a) From Eq. 3-38

$$\frac{C_e}{C_o} = \frac{1}{1+K\Theta}$$

or

$$\frac{25}{250} = \frac{1}{1+20K}$$

$$K = 0.45/\text{day}$$

b) From Fig. 1-2, for 10% BOD remaining, $K\Theta = 2.25$
Hence

$$K = 2.25/20 = 0.1125/\text{day}$$

The problem could be also solved using Eq. 3-34. Substituting values to Eq. 3-34, yields:

$$0.1 = \exp(-K20)$$

or

$$K = 0.115/\text{day}$$

c) From Fig. 1-2 for 10% BOD remaining and $D_L/\bar{V}L = 0.1$, $K\Theta = 2.8$
Hence,

$$K = 2.8/20 = 0.14/\text{day}$$

d) From Fig. 1-2 for 10% BOD remaining and $D_L/\bar{V}L = 2$,
$K\Theta = 6$. Hence,
$K = 6/20 = 0.3/\text{day}$

It should be noted that the K value varies drastically with the dispersion number. The ratio of the calculated K value for plug flow and completely mixed system varied from .115/day to .45/day or a deviation of almost 400%. Yet, it is very seldom that the dispersion number is reported in literature.

Autocatalytic reaction – For autocatalytic reactor the rate of organic waste degradation is not monotonically decreasing with the organic waste concentration. As discussed in the preceeding section and defined by Eq. 3-20, the rate of organic waste degradation attains a peak when a balance is reached between the microbial concentration X_a and the food concentration C.

Fig. 3-9 shows a graph of $-1/f(X_a, C)$. The graph is the reciprocal of Fig. 3-7. From the point $(X_{a,o}, C_o)$ to $(X_{a,1}, C_1)$ the completely mixed biochemical reactor volume required for a flow rate, q, is:

$$V = q \frac{C_o - C_1}{f(X_{a,1}, C_1)} \qquad (3-70)$$

Graphically the volume required is the area of the rectangle shown in Fig. 3-9 (a). For a plug flow reactor, the volume required is:

$$V = -q \int_{C_o}^{C_1} \frac{dC}{f(X_a, C)} \qquad (3-71)$$

The volume is equal to the area bounded by the points $(X_{a,1}, C_1)$, (O, C_1), (O, C_o) and $(X_{a,o}, C_o)$. Graphically the volume required is shown by the shaded portion of Fig. 3-9(b).

Hence under this condition, a completely mixed biochemical reactor will function more efficiently than a plug flow biochemical reactor.

On the other hand if the wastewater treatment efficiency requirement is upgraded such that the required effluent is equal to C_2 instead of C_1 as shown in Fig. 3-9 (a) then the volume of the completely mixed biochemical reactor becomes

$$V = q \frac{C_o - C_2}{f(X_{a,2}, C_2)} \qquad (3-72)$$

The volume required by the plug flow reactor is:

$$V = -q \int_{C_o}^{C_2} \frac{dC}{f(X_a, C)} \qquad (3-73)$$

The volume of the completely mixed reactor is the area of the rectangle as shown in Fig. 3-9 (a). The plug flow biochemical reactor in this case requires a smaller volume than the completely mixed biochemical reactor.

Optimization – The minimum reactor volume required to treat the wastewater from an initial concentration of organic pollutant C_o to C_2, is a complely mixed system followed by a plug flow biochemical reactor. The completely mixed biochemical reactor treats the wastewater from C_o to C_1, while the plug flow biochemical reactor treats the wastes from C_1 to C_2. The volume required by the combination is shown by the shaded portion of Fig. 3-9(c). Casual comparison will show that the shaded area of the combination is smaller than the area when either a completely mixed biochemical reactor or plug flow biochemical reactor is used.

As $X_{a,o}$ increases, the curve is shifted towards the right and it becomes flatter to the left. With increasing $X_{a,o}$, the time lag for the microbial mass and food combination to become ideal decreases. On the other hand, the microbial activity maintains its peak until the food becomes very low. Fig. 3-9(c) shows the effect of high initial microbial concentration in an activated sludge due to recirculation. The performances of the completely mixed and plug flow activated sludge are very similar. In fact the error inherent in the BOD test will result in a wider variation of errors than the difference in efficiency of the two processes.

CHAPTER IV

EQUALIZATION

Periodic and random fluctuation in wastewater quality and quantity is the major reason for overdesign of wastewater treatment facilities. While the design is based on a steady state assumption for simplicity, the variations in quality and quantity of the incoming waste continuously maintain the waste treatment plant system under transient or dynamic conditions. When reviewing design calculations it is worthwhile to note the concern by designers for minor factors which could affect the calculated equipment size by 1 to 2 % only, after which he multiplies the calculated equipment size by a factor of 150 to 200% to account for random fluctuations or uncertainties in the quality and quantity of the incoming waste stream. For periodic fluctuations, the designer could estimate the frequency of occurrence and amplitude as described in Chapter II. The fluctuations conform with the daily routine of the water consumers. In an industry those cycles could depend on the production cycle which may have periods shorter than a day. In terms of wastewater quality and quantity, they could have durations of less than an hour everyday. Yet, the designer has to size up the waste treatment system to meet the worst expected conditions in addition to the factor of safety described above.

While it is the waste treatment plant owner who pays for the overdesigned capacity, the designer stakes his reputation and professional license on the underdesigned operational periods when the effluent quality or the waste treatment plant performance is below the required standard. With the current state of the art for wastewater treatment plant design, the overdesign factors or factors of safety are invariably very high. To distinguish the essential overdesign factor from the unessential is almost difficult except to qualitatively consider the designer as too conservative. With the increasing cost of environmental management and its contribution to inflation, lower productivity and economic stagnation, it is worthwhile to reexamine and reevaluate overdesign cost.

The overdesign capacity is not only for the first equipment. It is also repeated and applied to the whole series of equipment in each unit process. If the waste treatment could be designed for the average quality and quantity, the waste treatment plant cost could be reduced by 50 to 80%. For example, a sewage treatment plant serving a community of less than 1,000 people has to consider peak flows five times the average daily flow. In large cities, the peak flows may be twice the average daily flow. In an industrial plant with batch processes such as in the pharmaceutical industry, the peak wastewater flow and BOD concentration could be hundred times the average.

Hence in cases where the fluctuations in wastewater quality and quantity are rather high, it would be more economical to build a tank to mix the wastewater discharge at various times to attain a homogeneous solution whose quantity is close to the average. The unit process for this purpose is known as equalization.

The process of equalization may call for the construction of a holding tank known as equalization basin or it may use existing structures such as sewer lines, grit tanks, and even the waste treatment tanks itself. As mixing is a primary requirement in equalization systems, other unit operations such as nutrient addition and coagulation could be done simultaneously using the same tank and equipment. In fact, in operations which are very sensitive to shock loads such as in pH adjustments of acid and alkaline solution with demineralized water, large equalization system which may serve at the same time as mixing tanks provides better damping and stability than small mixing tanks. In some instances where alternate discharges of antagonistic wastes are made such as in acid/alkaline processes, the addition of chemicals is minimized if not entirely eliminated when an equalization system is used.

Old wastewater treatment plants which are operating beyond their expected or design life need not have expensive expansion. The overdesign capacity of the wastewater treatment plant above the average operating condition could be utilized by incorporating an equalization system as recommended by USEPA (1974). As mentioned previously, the equalization process may not even require the construction of an equalization basin but may be done by simply regulating the flow in existing structures such as sewer lines, pumping station and grit tanks.

For this reason, an equalization system is perhaps one of the most important unit processes in the whole waste treatment system. Equalization basically consists of a tank or a hole in the ground with some mixing devices. The design of equalization tanks is too simple to encourage further researches on its principles. In fact, a wide range of wastewater engineering textbooks do not mention wastewater equalization at all. Some authors such as Nemerow (1971), mention equalization system design in passing. The full potential of the process has not been fully exploited.

DESIGN CRITERIA

The effectiveness of the equalization system on the performance of the other waste treatment processes which follow it is subject to positive claims and negative counterclaims. Ouano (1978) reviewed the design criteria for equalization basin currently used in practice and found them to vary by a factor of 3,600% from each other when applied to a waste stream with an eight hour period and a peak to average flow rate ratio of 3.0. The design criteria are often descriptive and where they are quantitative, they lack theoretical analysis to support it. Enumerated below are some of the suggested criteria:

1. An equalization tank should be sufficient to hold at least two hours of the hourly peak water flow, Paulson, (1972)
2. An equalization tank should be sufficient to hold waste generated in:
   a) one process cycle, Nemerow (1971)
   b) one-half process cycle, Pescod et al (1975)
   c) three process cycle, Herion and Roughhead (1964)
   d) one day flow,
3. An equalization tank should be completely mixed without short circuiting, (La Grega and Kennan, 1974)

4. For waste stream with long return periods, one completely mixed tank will give better performance than a series of completely mixed tanks. For wastewater streams with short return periods, the converse holds true. (Wallace, 1967)
5. An equalization tank could be mixed completely by a mechanical aerator or by baffles without any difference in its performance, Nemerow (1971)
6. As equalization tank should be completely mixed, Novothy (1978)

Example 4-1. To illustrate the divergence of the design criteria mentioned above, consider an industry with an 8-hour production cycle. The maximum wastewater discharge takes place between 6:00 and 8:00 A.M. when changes in personnel shift, breakfast, and peak production cycle coincide. The total waste discharge during the 8-hour period is 2,000 m^3 with the 2-hour peak period accounting for 800 m^3. Determine the equalization tank size.

Solution:

a) If the tank should be sufficient to hold the two hour peak flow, then the volume should be 800 m^3
b) If the tank should be sufficient to hold the waste generated for:
   i) one half process cycle
      $V = 0.5 \times 2{,}000 = 1{,}000\ m^3$
   ii) one process cycle
      $V = 1 \times 2{,}000 = 2{,}000\ m^3$
   iii) three process cycle
      $V = 3 \times 2{,}000 = 6{,}000\ m^3$
   iv) one day flow

$$V = 24\ \text{hrs/day} \times \frac{\text{cycle}}{8\ \text{hrs}} \times 2{,}000\ \frac{m^3}{\text{cycle}} = 6{,}000\ m^3$$

Should one build an 800 m^3 equalization tank or a 6,000 m^3 tank or any size in between? If one builds an 800 m^3 tank and finds out later that the performance of the succeeding waste treatment processes has not changed much, does it mean that an equalization system is worthless? It may be that the particular equalization system was badly designed; i.e. 800 m^3 was inappropriate. Should one use Delphic technique and take the average of what experts say or should one use 6,000 m^3 since by coincidence there are three cycles in a day? One may build a 6,000 m^3 tank and let the owner worry about the cost of unnecessary volume because he does not know the correct volume or he may build an 800 m^3 tank and risk his professional reputation.

After a thorough discussion with the design engineer and the business manager, one may finally decide to use a 6,000 m^3 equalization tank, thinking that anybody who will redesign the system will come up with the same uncertainty. The next question would be whether to divide the tank into a series of completely mixed tanks or just retain it as a single mixed unit.

The above scenario might not be as problematic in developed countries where the client could afford to pay for a pilot plant study. The designer could obtain answers by trial and error or intuition from pilot plant operations. In fact, after having commissioned three prototypes which are operating very well, he could sum up his numbers. Hopefully when there are 30 criteria or so, the average value will have some significance. The designer could calculate the confidence limits of his recommended equalization tank size from the variation of using the different design criteria.

Industrial wastewater quality and quantity fluctuate with time, depending on the industrial process and production cycle. Most waste treatment processes are very sensitive to changes in hydraulic loading, organic loading, pH, and temperature. An equalization basin serves to balance extreme quality and quantity fluctuations, hence minimizing the need for chemical additives, expensive control equipment and instrumentation.

An equalization basin is nothing more than a very large tank for mixing concentrated wastes with diluted wastes during the process cycle. In some instances the equalization basin may serve as a sedimentation tank, aerated lagoon, pH adjustment tank or chemical mixing tank. The derivation here is based on the assumption that the design of the balancing system is the governing objective.

An ideal equalization tank effluent should be of the same quality and quantity at all times. This is impractical, since it would require a tank with infinite volume. A reasonable effluent pattern is governed bv the wastewater plant tolerance and stability to shock loads.

For example, a trickling filter has a wider band of tolerance to organic shock loads, compared to an activated sludge process. Hence, for the same wastewater quality and quantity discharge pattern, the trickling filter will require a smaller equalization basin than an activated sludge.

Waste discharge patterns from three processes are shown in Figure 4-1. The three processes have equal durations of process cycle, volumes of waste discharge, and equal quantities of pollutant discharge per process cycle. Hence, if the methods proposed by Nemerow, Herion and Roughhead will be used, the three processes, will require an equalization tank with the same sizes, mixing patterns and dimensions. It is evident that the waste quality and quantity from process "A" falls, at all times, within the tolerance limit of the waste treatment plant, requiring little or no equalization system at all.

For process "B", the wastewater quality fluctuations within the process cycle tend to cancel out at short time interval. Hence, process B will require an equalization basin whose retention time is shorter than the process cycle duration.

In process "C", the wastewater quality and quantity are divided into two distinct zones during the process cycle. During the first half of the cycle, the pollutant concentration will increase continuously, at a rate inversely proportional to the equalization tank volume. A larger equalization basin will be required for process "C" than process "B."

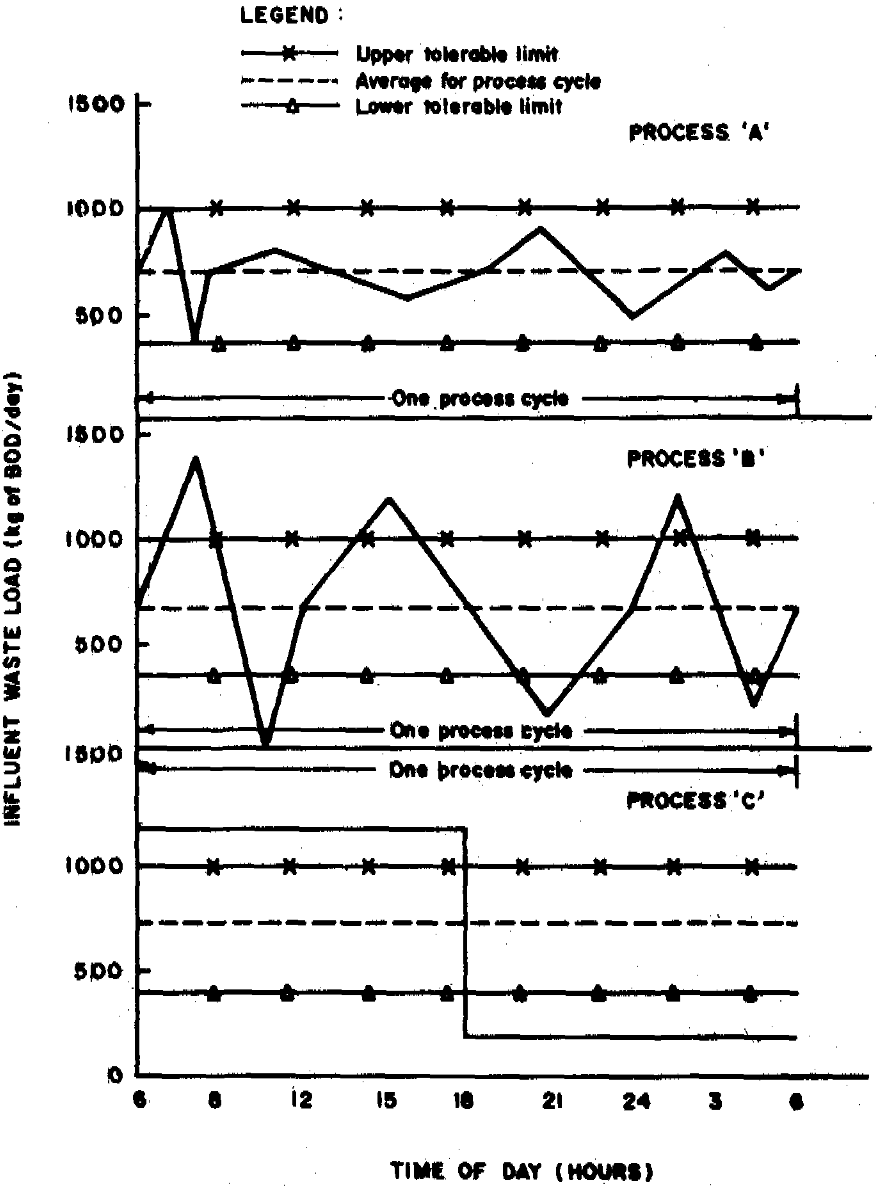


FIG. 4-1 Effluent of wastewater quality fluctuations and equalization tank requirements.

It is evident that the duration of the process cycle in itself is an inadequate design criterion. The design of equalization basin depends on the type of wastewater process used after equalization, the autocorrelation between the extreme wastewater quality and quantity fluctuations and mixing pattern of the equalization basins.

If the equalization system is used for damping the fluctuation in the wastewater flow alone then it behaves like water supply reservoir. Roy and Weston (1974) applied the same principles of reservoir sizing to equalization tank design.

FLOW EQUALIZATION

The wastewater flow rate into the equalization tank is a function of time which is hereafter designated as Q(t). The wastewater is pumped continuously from the equalization tank to the succeeding waste treatment processes, most likely a secondary biological treatment at a constant rate, q_p. If the flow fluctuations have a period of T hrs, (that means after a time T the whole cycle repeats itself), then within this period of time, the volume of the water removed is equal to the incoming volume, otherwise, the tank will eventually overflow if the outgoing volume is less than the incoming volume. There would be times when q_p is zero if the converse holds true. The schematic diagram of the system is shown in Fig. 4-2. The outgoing wastewater volume, V, is the integral of the pumping rate q_p over the period T. Similarly the incoming wastewater volume is the integral of Q(t)dt over the period T. Since the incoming volume of wastewater is equal to the outgoing wastewater volume, then,

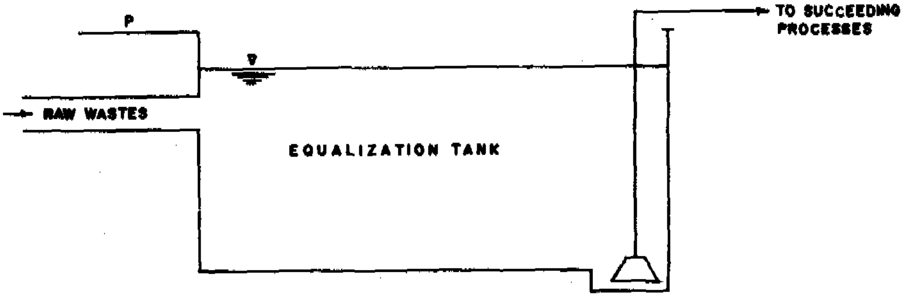


FIG. 4-2 Schematic diagram of an equalization tank.

$$V = \int_0^T q_p dt = \int_0^T Q(t)dt \qquad (4-1)$$

or

$$q_p = \frac{1}{T}\int_{o}^{T} Q\,(t)\,dt \qquad (4-2)$$

The rate of pumping q_p is then equal to the average flow rate within the time period, T. If the flow rate is measured intermittently, then Eq. 4-2 is approximated by the summation sign instead of the integral, or

$$q_p = \frac{1}{T}\sum_{i=1}^{n} Q\,(t_i)\ \Delta\, t \quad \text{where } n = \frac{T}{\Delta t} \qquad (4-3)$$

The required volume of the equalization tank, V_e, is the absolute value of the maximum difference between the volume of water that has entered into the equalization tank and the volume that has been pumped out. Hence,

$$V_e = \max \left|\int_{o}^{t^*} Q(t)dt \ - q_p t^*\right| \qquad (4-4)$$

where t* is the time when the maximum difference takes place. Since t* is unknown, Eq. 4-4 is difficult to solve mathematically. Eq. 4-4 is best solved graphically or by tabulation. By definition the total volume of incoming wastes, V_i, from time t = O, to time t* is

$$V_i = \sum_{i=1}^{n} Q\ (t_i)\,\Delta\, t, \ \text{where } n = t^*/\Delta\, t \qquad (4-5)$$

Similarly, the volume pumped out, V_o is equal to

$$V_o = q_p t^* \qquad (4-6)$$

Hence, the volume of the equalization tank, V_e, could be determined by solving the following equation for values of n = 0,1,2, . . . , T.

$$V_e = \max\left|\sum_{i=1}^{n} Q\,(t_i)\,\Delta\, t \ \ - q_p t\right| \qquad (4-7)$$

The use of Eq. 4-7 for numerical or graphical determination of the equalization tank volume is illustrated in the following example.

> Example 4-2. The wastewater flow rate fluctuation measured at hourly interval is shown in Table 4-1. a) Calculate the equalization tank

volume such that the wastewater could be delivered to the activated sludge process at a constant rate b) Determine also the equalization tank volume graphically. c) What is the wastewater flow rate into the activated sludge process?

Solution:

i) From table 4-1, the total flow in 24 hrs, is 23100 m^3. Hence, the pumping rate out of the equalization tank q_p is

$$q_p = \frac{23100}{24} = 962.5 \quad m^3/hr$$

ii) $q_p t^*$ are calculated as shown in the 6th column of Table 4-1 by multiplying q_p by column (4).

TABLE 4-1 Calculations of the Equalization Tank Volume

Time of Day (1)	Flow Rate (m^3/hr) Q_i (2)	(hr) Δt (3)	hr $\sum \Delta t = t^*$ (4)	$\sum_{i=1}^{n} Q_i \Delta t$ (m^3) (5	$q_p t^*$ (m^3) (6)	$\sum Q_i \Delta t - q_p t^*$ m^3 (7)
700	1000	1.0	1.0	1000	962.5	37.5
800	2000	1.0	2.0	3000	1925	1075.
900	1500	1.0	3.0	4500	2887.5	1612.5
1000	500	1.0	4.0	5000	3850	1150
1100	400	1.0	5.0	5400	4812.5	587.5
1200	1000	1.0	6.0	6400	5775.0	625.0
1300	800	1.0	8.0	7200	6737.5	462.5
1400	800	1.0	8.0	8000	7700	300.
1500	600	1.0	9.0	8600	8862.5	62.5
1600	1000	1.0	10.0	9600	9625	25
1700	1000	1.0	11.0	10600	10587.5	12.5
1800	1500	1.0	12.5	12100	11550	550
1900	1800	1.0	13.0	13900	12512.5	1387.5
2000	2000	1.0	14.0	15900	13475.0	2425
2100	1500	1.0	15.0	17400	14437.5	2962.5
2200	1200	1.0	16.0	18600	15400	3200.0*
2300	8 00	1.0	17.0	19400	16362.5	3037.5
2400	8000	1.0	18.0	20200	17325	2875.0
100	8 00	1.0	19.0	21000	182887.5	2712.5
200	400	1.0	20.0	21400	19250.-	2150.0
300	200	1.0	21.0	21600	20212.5	1387.5
400	200	1.0	22.0	21800	21175.	625
500	600	1.0	23.0	22400	22137.5	262.5
600	700	1.0	24.0	23100	23100.	—

iii) the difference between $Q_i \Delta t$ and $q_p t$ are calculated as shown in column 7 of Table 4-1.

iv) The maximum value of the difference between $Q_i \Delta t$ and $q_p t$ is determined by inspecting column 7 which yields 3200 m^3.

b) i) The cumulative volume of wastewater flow is plotted as shown in Fig. 4-3. The curve is a graph of $\Sigma Q_i \Delta t$ shown in Table 4-1.

ii) A line is drawn between the point (0,0) and the overall sum. This line represents the cumulative sum of the wastewater pumped out of the equalization tank. The equalization tank is empty at the start and it is empty after the end of the cycle since the cumulative sum of the incoming wastewater is equal to the cumulative sum of the wastewater pumped out. The pumping rate is the slope of the straight line since it is equal to the average rate at which the wastewater is drawn out of the equalization tank. Hence,

$$q_p = \frac{\Delta V}{\Delta t} = \frac{4000}{4.2} = 952 \text{ m}^3/\text{hr}$$

It should be noted that the calculated pumping rate from the previous solution is equal to 962.5 m^3/hr. The difference is primarily in the accuracy of reading the values of ΔV and Δt from the graph.

iii) Draw a line parallel to the $q_p t$ which is tangent to $Q_i \Delta t$. The easiest way is to move a ruler parallel to the line $q_p t$ and make further refinements using the slope of the line or the pumping rate q_p.

iv) Calculate V_e as the difference between the curve $Q_i \Delta t$ and $q_p t$ at the point of tangency. From Fig. 4-3, $\Sigma Q_i \Delta t$ is equal to 18200 m^3 at the point of tangency of the line parallel to $q_p t$. The value of $q_p t$ is equal to 15,150 from Fig. 4-3. Hence,

$$V_e = 18200 - 15100 = 3100 \text{ m}^3$$

v) Again it should be noted that the analytical calculation shows an equalization tank volume of 3,200 m^3 where the graphic solution yields 3100 m^3 due to errors in plotting and reading from the graph.

c) The flow rate into the activated sludge is equal to q_p, the wastewater pumping rate out of the equalization tank. From the calculations q_p = 962.5 m^3/hr or 952 m^3/hr if the graphical method is used.

Flow equalization in sewerlines – By controlling the water level in the sewer lines, a portion of the wastewater flow is stored inside the sewerlines. The

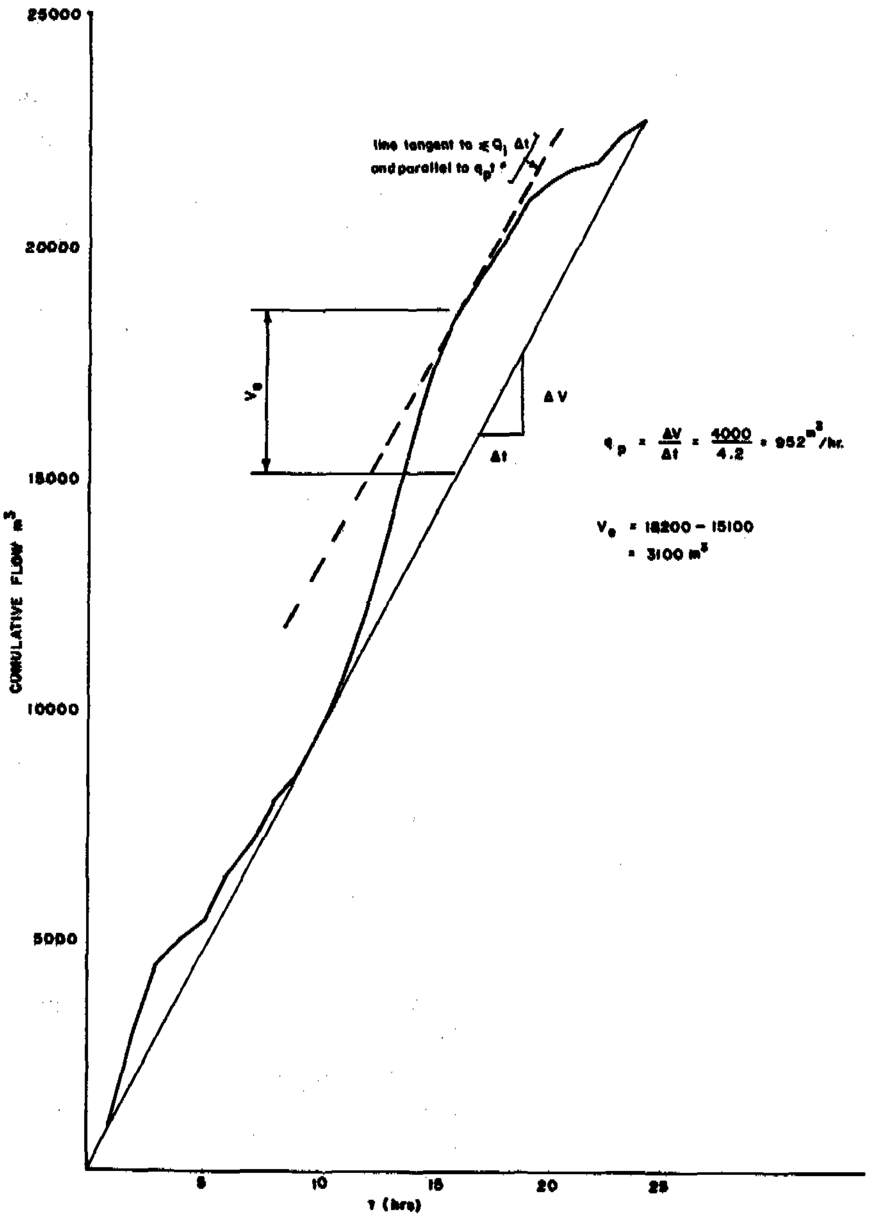


FIG. 4-3 Graphical calculation of flow equalization tank.

stored wastewater could be drawn at almost constant rate into the wastewater treatment plant. The total capacity of the main sewer trunk lines excluding the laterals could be equal to the total volume of the wastewater treatment system.

Although the process cuts down the cost for additional construction of the equalization tank, the practice has several shortcomings. The sewer slope could be very large that **overflowing takes place** after a small portion of the lines has been utilized. When the wastewater flow is blocked, the wastewater velocity inside the sewer lines could go down below the self cleansing velocity. Grit and other settleable solids will settle in the sewer lines, thus increasing the maintenance cost.

Hydrogen sulfide gases are formed from anaerobic decomposition of stale sewage. The hydrogen sulfide reacts with the water and oxygen in the sewer crown to form sulfuric acid which could corrode the sewer. The life span of the sewer lines could be **shortened unless the sewage is aerated during storage.**

> Example 4-3. Determine the length of sewer lines required to equalize the wastewater flow fluctuations given in Example 4-2. The sewer is 900 mm in diameter, designed to flow half full at peak load.
>
> Solution:
>
> i) The cross sectional area of the sewer line is:
>
> $$A = \frac{\pi}{4}\left(\frac{900}{1000}\right)^2 = .636\ m^2$$
>
> ii) From example 4-2, the volume of the equalization system required is 3200 m^3.
>
> iii) The length of the sewer line with the same volume as the equalization system is:
>
> $$V = AL \text{ or } L = 3200/.636 = 5031 \text{ m}$$
>
> iv) If the sewer line is designed to flow half full, then only 50% of the sewer line is available for equalization. Hence, the length of the sewer line required is:
>
> $$L = 5031 \times 2 = 10062 \text{ meters.}$$
>
> The reader may argue that the sewer is fully utilized at maximum flow which more or less coincides with the maximum equalization tank volume. Hence, the wastewater during peak flow is the same wastewater which requires equalization. The 50% utilization of the sewer line for equalization already includes sufficient factor of safety.

Gravity flow equalization tanks – In some instances, the wastewater treatment plant is constructed in a depressed area close to the river or the sea. The sewer line invert elevation just before entering the wastewater treatment plant could be higher than the primary treatment units. If an equalization tank is to be constructed between the sewer line and the primary treatment units, the wastewater could flow out of the equalization tank by gravity. The wastewater flow rate out of the equalization tank is not constant as it varies in proportion to the square root of the water depth inside the tank. However, if the surface area of the equalization tank is sufficiently large compared to its volume, the fluctuations in the water depth will be very small, making the wastewater flow rate out of the equalization tank relatively constant compared to the incoming wastewater flow rate fluctuation.

The rate of change of the stored wastewater volume inside the equalization tank could be expressed as:

$$\frac{dV}{dt} = Q_i(t) - q_o(h) \qquad (4-8)$$

where q_o is the flow rate out of the equalization tank, h is the wastewater depth in the equalization tank above the outlet pipe in m, and all other terms are as defined before. If the equalization tank has a surface area A_s and the outlet pipe has a cross sectional area, A_c, then Eq. 4-8 could be modified as:

$$\frac{dV}{dt} = A_s \frac{dh}{dt} = Q_i(t) - A_c\sqrt{2gh}\ C \qquad (4-9)$$

where C is the discharge coefficient of the pipe with values of approximately 0.6 to 0.8 and the other terms are as defined before. Equation 4-9 is a non-linear equation which would be difficult to solve even if $Q_i(t)$ could be expressed in simple linear equations. In most instances, $Q_i(t)$ is measured in the field and the resulting fluctuations may appear randomized and very complicated as discussed in Chapter II. Equation 4-9 is best solved by numerical techniques such as the Newton-Ralphson Method. Standard textbooks in numerical analysis should be consulted for a detailed discussion on numerical solution of differential equations such as Eq. 4-9.

Example 4-4. The incoming wastewater flow rate to an equalization basin could be expressed as:

$$q_i(t) = 4000 - 2000 \sin(24t) \quad m^3/hr$$

The wastewater flows out of the equalization basin by gravity through a 200 mm drain pipe. Determine the flow rate out of the equalization basin and the height of the equalization basin. The surface area of the basin is 100 m^2.

Solution:

$$q_o = C\sqrt{2g\,h}\,\frac{\pi}{4}\,d^2$$

$$= 0.80 \times \sqrt{2 \times 9.8\,h}\,\frac{\pi}{4}\,(.2)^2$$

$$= 0.53\sqrt{h}\ \text{m}^3/\text{sec} \times 3600\ \text{sec/hr}$$
$$= 1906\sqrt{h}\ \text{m}^3/\text{hr}$$

Substituting the other values given in the problem to Eq. 4-9, yields:

$$100\,\frac{dh}{dt} = 4000 - 2000\sin(24t) - 1906\sqrt{h}$$

or

$$\frac{dh}{dt} = 40 - 20\sin(24t) - 19.06\sqrt{h}$$

The differential equation is best solved using the Runge Kutta method. Standard programmable calculators are often sufficient to solve the above-mentioned differential equations.

The flow rate out of the equalization basin, q_O, is calculated once the water height above the effluent pipe h is known. In this particular case $q_o = 1906\sqrt{h}$. Fig. 4-4, shows the values of q_o, and q_i.

The influent flow rates vary from 6000 to 2000 m³/hr, whereas the effluent flow rate from the equalization basin varies only from 3000 to 4100 m³/hr.

Flow equalization in process tanks – The wastewater flow rate out of the different process tanks are controlled by weirs. Most often a triangular weir is used. The wastewater flow rate out of a triangular weir is dependent on the wastewater height, h, above the weir apex which could be expressed as:

$$q_o = C\sqrt{2g}\ h^{3/2} \qquad (4-10)$$

where C is a constant depending on the apex angle, and the other terms are as defined before. Since process tanks have very large surface areas, the increment in h for any change in the incoming flow would be very low. To prevent h from fluctuating drastically with the change in the incoming flow, very small weirs are used in parallel in the process tanks. Substituting Eq. 4-10 in Eq. 4-8 yields:

$$\frac{dV}{dt} = A_s\,\frac{dh}{dt} = Q_i(t) - n\ C\sqrt{2g}\,h^{3/2} \qquad (4-11)$$

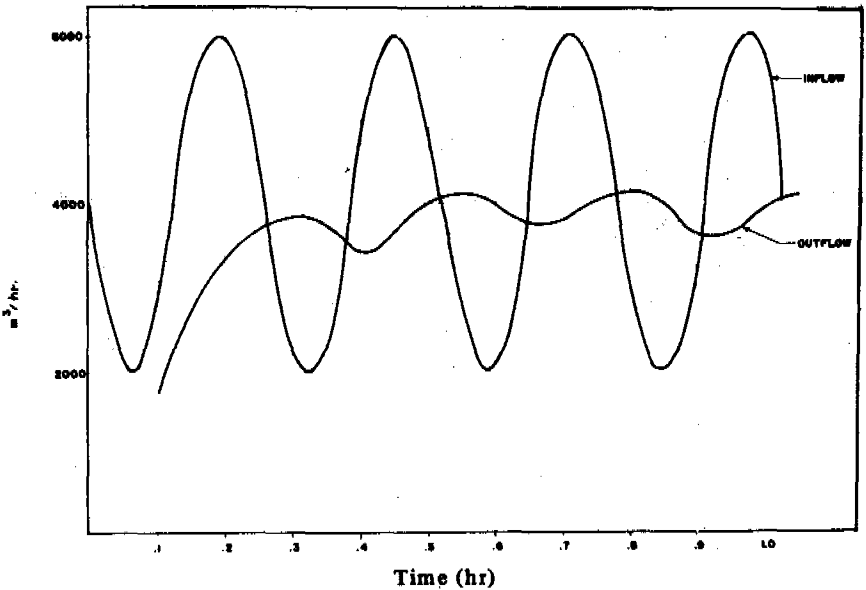


FIG. 4-4 Equalization effect of tanks and weirs.

where n is the number of weirs operated in parallel and the other terms are as defined before. Eq. 4-11 is a non-linear differential equation which makes it difficult to solve analytically. Numerical techniques such as those described in Example 4-4 are often used to solve Eq. 4-11.

WASTEWATER QUALITY EQUALIZATION

In the previous section, the equalization of wastewater flow rate or quantity fluctuation was discussed. Since wastewater is incompressible, the equalization of the wastewater quantity is simpler as it is independent of the mixing pattern. The concentration of pollutant in the wastewater is subjected to the dispersion or mixing phenomena in the equalization basin. In fact it is the mixing of the strong waste discharged at time, t, with the weak wastes discharged at $t+\Delta t$, and $t-\Delta t$, which is responsible for equalization of the wastewater quality. The strong wastewater is diluted by the weak wastewater while the weak wastewater is fortified by the strong wastewater resulting in a more or less uniform wastewater concentration in the effluent of the equalization tank.

The completely mixed constant volume equalization system is the simplest process for the equalization of wastewater quality fluctuations. The system involves a tank of fixed volume, V, which is highly mixed such that the effluent quality is equal to the wastewater quality inside the tank. A mass balance of the incoming, outgoing, and accumulated pollutant mass in the system yields:

$$V \frac{dC}{dt} = q\,C_i(t) - qC \qquad (4-12)$$

where V is the volume of the vessel, m^3, q is the flow rate of the waste, m^3/hr; C_i (t) is the incoming waste concentration which varies with time, t, in mg/l; and C is the pollutant concentration in the effluent, mg/l. As discussed in Chapter III, the term VdC/dt is the accumulation, q C_i (t) is the incoming pollutant mass rate, and qC is the outgoing pollutant mass rate.

Eq. 4-12 could be rearranged as:

$$V \frac{dC}{dt} + qC = qC_i(t) \qquad (4-13)$$

The complementary solution Y_C of Eq. 4-13 is equal to

$$Y_c = A \exp(-qt/V) \qquad (4-14)$$

It should be noted that as $t \rightarrow \infty$, $Y_c \rightarrow 0$. The complementary solution Y_c is also known as the transient response of the equalization system since it disappears as $t \rightarrow \infty$. The use of Eq. 4-12 is illustrated in the following example.

> Example 4-5. An equalization system with a volume of 100 m^3 is used to equalize the wastewater quality with a constant flow rate of 10 m^3/hr. The BOD concentration in the influent is defined as:
>
> $C_i(t) = 500\,(1\text{-}\sin t)$
>
> t = 0, and C= 0 since the equalization tank is initially filled with water used in the leak test. a) What are the minimum and maximum values of C? b) How do the minimum and maximum values of C compare with the minimum and maximum values of C_i (t)?
>
> Solution:
>
> a) The values of V C_i (t), and q are introduced into Eq. 4-12 which results to:
>
> $$100 \frac{dC}{dt} = 10 \times 500\,(1\text{-}\sin t) - 10\,C$$
>
> The complementary solution to the above equation is:
>
> $$Y_c = A \exp(-0.1\,t)$$
>
> where A is a constant.

The particular solution has the form $A_1 + A_2$ sint + A_3 cost. Substituting the particular solution to the differential equation yields:

$$A_2 \cos t - A_3 \sin t + .1\,A_1 + .1\,A_2 \sin t + .1\,A_3 \cos t = 50 \sin t$$

Hence,

$$.1A_1 = 50$$
$$A_1 = 500$$

from the sine term : $-A_3 + .1A_2 = -50$
from the cosine term : $.1A_3 + A_2 = 0$

Solving for A_2 and A_3 from the above equations results to:

$$A_2 = -4.95 \; ; A_3 = 49.5$$

Hence, the complete solution to the differential equation is:

$$C = A \exp(-0.1\,t) + 500 - 4.95 \sin t + 49.5 \cos t$$

Substituting the values of t= 0 and C = 0 to the above equation yields:

$$0 = A + 500 - 4.95 \sin(0) + 49.5 \cos(0)$$

or

$$A = -500 - 49.5 = -549.5$$

The exact solution to the differential equation is

$$C = -549.5 \exp(-0.1t) - 4.95 \sin t + 49.5 \cos t + 500$$

The graph of the influent concentration C_i (t) and C are shown in Fig. 4-5 to illustrate the effect of the equalization tank on damping the influent quality fluctuation.

b) The maximum and minimum values of C at steady state are determined by differentiating C(t) where exp $(-0.1t) \rightarrow 0$, and equating differential to zero. This results to:

$$\frac{dC}{dt} = 4.95 \cos t - 49.5 \sin t = 0$$

Hence,

$$\tan t = .1 \text{ or } t = -.1 \text{ hr}$$

since $t > 0$, the next cycle occurs when $t = 2\pi + t$ or $t = 6.183$ hrs.

The second derivative of C is derived to determine if t gives the maximum or minimum values of C. The second derivative of C is equal to:

$$\frac{d^2C}{dt^2} = 4.95 \sin t - 49.5 \cos t$$

at t = 6.183 hrs, the second derivative is equal to:

$$\frac{d^2C}{dt^2} = 4.95 \sin 6.18 - 49.5 \cos 6.18 = -49.74$$

Hence, t = 6.183 gives the maximum value of C. The maximum value of C is equal to:

$$\begin{aligned} C &= 500 - 4.95 \sin (6.183) + 49.5 \cos (6.18) \\ &= 549.75 \text{ mg/l} \end{aligned}$$

The minimum value of C occurs one half cycle apart, hence, t at minimum C is equal to 6.18 − π or 3.03. The value of C is calculated as:

$$\begin{aligned} C &= 500 - 4.95 \sin (3.03) + 49.5 \cos (3.03) \\ &= 450.25 \text{ mg/l} \end{aligned}$$

The minimum and maximum values of C could also be obtained from Fig. 4-5.

The maximum value of C_1 (t) takes place when $t = 3\pi/2$, when sint is equal to -1. At this point $C_i(t) = 1000$ mg/l. The minimum value of $C_i(t)$ (t) occurs at $t = \pi/2$ when sint is equal to 1. The minimum value of $C_I(t)$ is equal to zero.

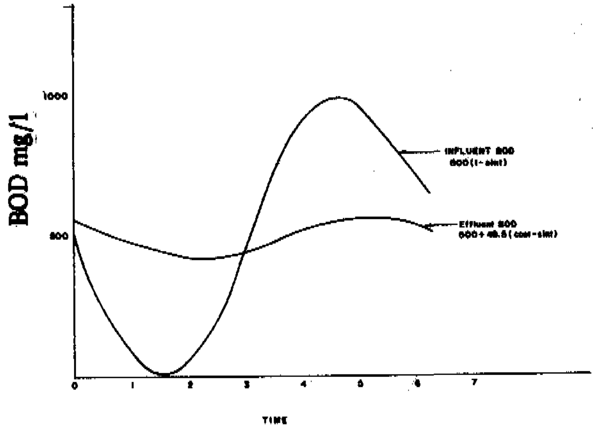


FIG. 4-5 Effects of equalization tank on influent BOD fluctuations excluding transient effects.

It should be noted that the minimum and maximum values of C fluctuates only by ± 50 mg/l around the average value of 500 mg/l whereas the influent BOD fluctuates by ± 500 mg/l around the average.

A sinusoidal input of the form A(1-sint) when used in Eq. 4-12 without the transient effect has the following general solution

$$C = A - \frac{A\ q\ \sin t}{V^2\left[1 + (q/V)^2\right]} + \frac{A\ \cos t}{V\left[1 + (q/V)^2\right]} \qquad (4-15)$$

where q, V and the other terms have been defined before.

It should be noted that C will be constant only when V → ∞. Hence, it is impractical to design an equalization system which gives a constant effluent quality. The required proximity of C to A will depend on the capability, of the succeeding process to absorb the wastewater quality fluctuation as discussed in the previous section.

AMPLITUDE DAMPING

Fig. 4-5 shows the damping effect of a completely mixed constant volume equalization tank. The two major points to consider are: 1) the output fluctuation is lower than the input fluctuation or the amplitude of the outflow is smaller than the amplitude of the inflow 2) the outflow follows a sinusoidal pattern as the inflow, but the maximum and minimum values are shifted by some values, φ. Of interest in the design of equalization tanks is the extent to which the maximum value is reduced close to the average or amplitude damping. As the ratio of the amplitude of the outflow to the amplitude of the inflow becomes smaller the more efficient the equalization system becomes. An ideal equalization system gives an amplitude damping ratio of zero, i.e. the outflow does not vary anymore but remains constant. The designer is more concerned with the maximum and minimum values. The values in between are damped to a degree almost similar to the damping of the maximum and minimum values. Hence, as the minimum and maximum values approach the average value, the values in between those two extremes approach the average simultaneously. From Fig. 4-5, one will right away conclude that equalization has taken place by merely inspecting the amplitudes of the functions. Hence, there is no need to solve the whole differential equation in order to evaluate the behavior of the system if amplitude damping could be defined.

Eq. 4-12 is a typical linear differential equation. In terms of the Laplace transform, the differential equation could be expressed as:

$$V\,s\,C(s) - C(O) = q\,C_i(s) - q\,C\,(s) \qquad (4-16)$$

where s is the Laplace transform variable and the other terms are as defined before. Eq. 4-16 could be rearranged as:

$$C\,(s) = \frac{q\,C_i(s)}{Vs + q} + \frac{V\,C(O)}{Vs + q} \qquad (4-17)$$

If the term $q/(Vs + q)$ is defined as $T(s)$, then Eq. 4-16 could be defined as:

$$C(s) = T(s)\ C_i(s) + V/q\ T(s)\ C(O)$$

If $C(O)$ is equal to zero, then Eq. 4-17 simplifies to:

$$C(s) = T(s)\ C_i(s) \qquad (4-18)$$

The term $T(s)$ is known as the transfer function operating on the Laplace transform of the input function, $C_i(s)$. Tedious mathematical exercise for determining the solution of the differential equation is converted to a simple multiplication operation through the use of the Laplace transform. Eq. 4-18 could be rearranged as:

$$T(s) = C(s)/C_i(s) \qquad (4-19)$$

From Eq. 4-19, the transfer function defines the ratio of the Laplace transforms of the outputs to the inputs. In general, the steady state sinusoidal output-input amplitude ratio and output phase shift with respect to the input can be determined by substituting $\imath\omega$ for s in the system's transfer function $T(s)$. The proof of this important concept is given in standard linear system theory textbook such as Takashi, Rabins and Auslander (1970). Hence, Eq. 4-19 is redefined as:

$$T(\imath\omega) = C(\imath\omega)/C_i(\imath\omega) \qquad (4-20)$$

Eq. 4-20 is known as the Fourier transform of Eq. 4-12. Fourier transform variation of the transfer function $T(\imath\omega)$ could be split further into the component $T'(\omega)$ and the imaginary component $\imath T''(\omega)$. Hence, Eq. 4-20 could be expressed as:

$$C(\imath\omega)/C_i(\imath\omega) = T'(\omega) + \imath T''(\omega) \qquad (4-21)$$

Eq. 4-21 could be expressed in exponential form as:

$$C(\imath\omega)/C_i(\imath\omega) = R(\omega)\exp(\imath I(\omega)) \qquad (4-22)$$

$$\text{where} \quad R(\omega) = \sqrt{T'(\omega)^2 + T''(\omega)^2} \qquad (4-23)$$

$$\text{and} \quad I(\omega) = \tan^{-1}\left\{T''(\omega)/T'(\omega)\right\} \qquad (4-24)$$

Since, $R(\omega)$ defines the amplitude of $T(\imath\omega)$ or the ratio of the output and input amplitudes, it is therefore known as the amplitude damping function. The imaginary term $I(\omega)$ defines the angle change of $T(\imath\omega)$ and it is called the phase shift function.

If the input function $C_i(t)$ is defined as $A\sin(\omega t)$ or $B\cos(\omega t)$, then the output function $C(t)$ will have the form $A\,R(\omega)\sin(\omega t + I(\omega))$ or $B\,R(\omega)\cos(\omega t + I(\omega))$.

Application – From Eq. 4-16, the transfer function of Eq. 4-12 is defined as:

$$T(s) = q/(Vs + q) \text{ or } 1/(\Theta s + 1) \qquad (4-25)$$

where $\Theta = V/q$. Substituting $\iota\omega$ for s yields

$$T(\iota\omega) = 1/(1 + \iota\omega\Theta) \qquad (4-26)$$

Multiplying the right hand side of Eq. 4-26 by $(1-\iota\Theta\omega)/(1-\iota\Theta\omega)$, results to:

$$T(\iota\omega) = \frac{1}{1+\iota\omega\Theta}\;\frac{1-\iota\Theta\omega}{1-\iota\Theta\omega} = \frac{1-\iota\Theta\omega}{1+\Theta^2\omega^2} \qquad (4-27)$$

$$= \frac{1}{1+\Theta^2\omega^2} - \frac{\Theta\omega}{1+\Theta^2\omega^2}\,\iota$$

From Eq. 4- 20 and 23

$$R(\omega) = 1/\sqrt{(1+\Theta^2\omega^2)} \qquad (4-28)$$

and from Eq. 20 and 24

$$I(\omega) = \tan^{-1}(\Theta\omega) \qquad (4-29)$$

Example 4-6. Calculate the maximum and minimum BOD values in the effluent of 100 m^3 equalization tank neglecting the transient effects. The equalization system is a completely mixed constant volume type. The incoming wastewater flow rate is 10 m^3/hr. The incoming BOD fluctuation is defined as 500(1-sint) b) calculate the time t when C is maximum.

Solution:

The BOD fluctuations could be split into two components as:

$$C_i(t) = 500 - 500 \sin t$$

The first term is a constant. Hence, the fluctuation is primarily due to 500 sin ωt with ω equal to 1.

The detention time Θ is equal to V/q or 100 $m^3/(10 m^3/hr)$ = 10 hrs. Hence, the amplitude damping function from Eq. 4-25 is equal to:

$$R(\omega) = 1/\sqrt{(1+10^2 \cdot 1^2)} = 1/\sqrt{(101)}$$

The maximum value of C is equal to the constant term plus the dampened amplitude of the fluctuating function or

$$C_{max} = 500 + 500\,R(\omega) = 500 + 500/\sqrt{(101)}$$
$$= 500 + 49.75 = 549.75$$

Similarly the minimum value of C is calculated as:

$$C_{min} = 500 - 500/\sqrt{101} = 450.25$$

The maximum and minimum values of C are the same as those calculated in Example 4-5. By using the amplitude damping function the calculation is facilitated.

b) The phase shift function $I(\omega)$ is equal to

$$I(\omega) = \tan^{-1}(\Theta\omega) = \tan^{-1}(-10) = 1.47 \text{ hr}$$

The output function C(t) has the same characteristics as the input function C_i (t) except that the output function is shifted by + 1.47. Since, the maximum value of the input function takes place when $t = 3\pi/2$ or sin(t) is equal to −1, then the maximum value of the output function takes place at $t = 3\pi/2 + 1.47$ or 6.18 hrs. It should be noted from Example 4-5, that the maximum BOD concentration in the output occurs at t = 6.18 hrs.

As discussed in Chapter II, the stochastic fluctuation of the wastewater quality or quantity could be approximated by the Fourier series or integrals. The wastewater quality and quantity fluctuations could be expressed in terms of sine and cosine functions. If the equalization system performance is defined by a linear equation such as Eq. 4-12, then the solution of the differential equation for composite inputs is a superposition or sum of the system's response to each individual input. The principle is illustrated in the following example.

Example 4-7. Consider the equalization tank given in Example 4-5, except that the influent BOD concentration fluctuates with time as

$$C_i(t) = 1000 - 300 \sin t - 200 \sin 2t - 100 \sin 3t$$

Neglect the transient effects. Determine the effluent BOD fluctuation.

Solution:

$$100 \frac{dC}{dt} = 10\left\{(1000 - 300 \sin t - 200 \sin 2t - 100 \sin 3t\right\} - 10\, C_l$$

or

$$\frac{dC}{dt} + .1C = 100\text{-}30 \sin t - 20 \sin 2t - 10 \sin 3t \qquad \text{(a)}$$

The above equation could be solved directly or C could be solved as the sum of C_1, C_2, C_3, C_4. The functions C_1, C_2, C_3, C_4 are solutions of the differential equation $dC/dt + .1C$ under individual inputs defined by the right side of Eq. (a). Hence, the linear equations defining C_1, C_2, C_3 and C_4 are as follows:

$$\begin{aligned}
\frac{dC_1}{dt} &+ .1C_1 = 100 \\
\frac{dC_2}{dt} &+ .1C_2 = -30\,\mathrm{sint} \\
\frac{dC_3}{dt} &+ .1C_3 = -20\,\sin 2t \\
\frac{dC_4}{dt} &+ .1C_4 = 10\,\sin 3t
\end{aligned} \qquad (b)$$

The direct solution to Eq. (a), has the form:

$$\begin{aligned}
C &= A_1 + A_2\,\mathrm{sint} + B_2\,\mathrm{cost} + A_3 \sin 2t + B_3\,\mathrm{cost}\,2t + \\
&\quad A_4 \sin 3t + B_4 \cos 3t \\
&= 100 - 30\,\mathrm{sint} - 20\,\mathrm{sin2t} - 10\mathrm{sin3t}
\end{aligned} \qquad (c)$$

Equating similar terms yields:

$$\begin{aligned}
.1A_1 &= 100 \\
A_1 &= 1000
\end{aligned}$$

For sint and cost term, the following equations are obtained

$$\begin{aligned}
A_2 + 1B_2 &= 0 \\
.1A_2 - B_2 &= -30
\end{aligned}$$

which yields

$$A_2 = -2.97, \quad B_2 = 29.7$$

For the sin2t and cos2t terms, the coefficients could be determined from the following equations:

$$\begin{aligned}
2A_3 + .1\,B_3 &= 0 \\
.1A_3 - 2\,B_3 &= -20
\end{aligned}$$

or

$$A_3 = -.499, \quad B_3 = +9.98$$

For the sin3t and cos3t terms, the coefficient A_4 and B_4 could be determined by equating the coefficients in the left and right hand side of Eq. (c). This results to:

$$\begin{aligned}
3A_4 + .1B_4 &= 0 \\
.1A_4 - 3\,B_4 &= -10
\end{aligned}$$

or $A_4 = -.022$ and $B_4 = .666$. Hence, the effluent BOD fluctuations C is defined as:

$$C = 1000 - 2.97 \sin t + 29.7 \cos t - .499 \sin (t) + 9.98 \cos (2t)$$
$$-.022 \sin (3t) + .666 \cos 3t$$

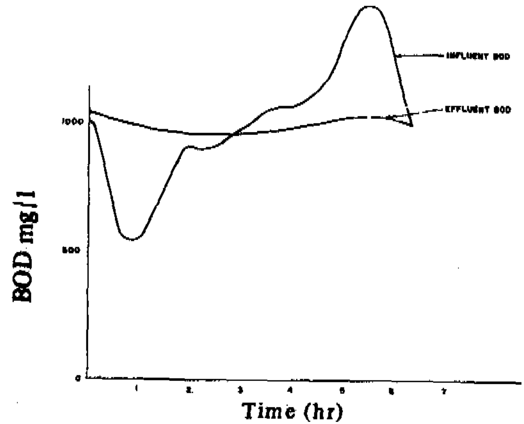


FIG. 4-6 Frequency simplication effect of the equation basin.

b) The functions C_1, C_2, C_3 and C_4 are derived as solutions to Eq. (b). The function C_1, has the general solution of the form $C_1 = A_1$, then

$O + .1\,A_1 = 100$ or $A_1 = 1000$. Therefore C_1 is equal to 1000.

For C_2, it has the general solution of the form $A_2 \sin t + B_2 \cos t$. The general solution for C_2 could be obtained by determining the values of the coefficient A_2 and B_2 from the equation $A_2 \cos t - B_2 \sin t + .1 A_2 \sin t + .1 B_2 \cos t = -30 \sin t$ or $A_2 + .1 B_2 = 0$ and $.1 A_2 - B_2 = -30$ which yields: $C_2 = -2.97 \sin t + 29.7 \cos t$. Similarly $C_3 = -.499 \sin 2t + 9.98 \cos 2t$ and $C_4 = -.022 \sin 3t + .66 \cos 3t$. If $C = C_1 + C_2 + C_3 + C_4$, then

$$C = 1000 - 2.97 \sin t + 29.7 \cos t - .499 \sin (2t) + 9.98 \cos (2t)$$
$$-.022 \sin (3t) + .66 \cos (3t)$$

which is exactly the same as the previous solution of C when the differential equation was solved with all the inputs considered simultaneously.

c) The maximum and minimum value of the C could be determined graphically from the loci of the function C as shown in Fig. 4-6. The maximum and minimum values of C could be approximated by neglecting the values of the functions sin2t, cos2t, sin3t and cos3t since the amplitudes of those functions are very small, in which case the maximum value of C takes place when, $t = 6.18$ or

$$C = 1000 - 2.97 \sin (6.18) + 2.97 \cos (6.18) = 1030.$$

It should be noted from the above example that amplitudes of large frequencies are damped very rapidly. In the influent function, the maximum effect of 20 sin(2t) is only 39% lower than the maximum effect of the term 30 sint, but in the effluent concentration the effect of the term 20 sin(2t) is 67% lower than the term 30 sint. This is readily apparent since the amplitude damping function for sinusoidal functions is inversely proportional to the frequency as defined by Eq. 4-28. The frequency, f, is equal to $2\pi\omega$.

For the Fourier series to converge, as $n \rightarrow \infty$, $|a_n|$ and $|b_n|$ should approach zero. Since f is proportional to n, as $n \rightarrow \infty$, $f \rightarrow \infty$. Aside from the rapid decline of the input amplitude $|a_n|$ and $|b_n|$, the amplitude damping function approaches zero rapidly as n becomes large. Hence, the effect of the large frequency term when C_i (t) is approximated by Fourier series could be neglected. Of interest are often the first four terms, where the constants a and b may attain their maximum values and contribution to the effluent fluctuations.

If the wastewater quality and quantity fluctuations are defined by the Fourier integral, C_i (t) is a continuous function of either sine or cosine or both. The amplitude of the Fourier integral, $A(\omega)$, is a continuous function of ω and the maximum amplitude of the effluent function could be determined from the maximum value of the product of the amplitude damping function, $R(\omega)$ and the amplitude of the input Fourier integral $A(\omega)$. Since $R(\omega)$ is a monotonically **decreasing function with respect to ω, the maximum effluent amplitude is contributed by input variables with low frequencies.**

For detailed discussion on amplitude damping equalization, standard textbooks on systems theory should be consulted. The subject is treated in the topic of noise filters. The original developments were made by communication engineers and the environmental engineer may require some familiarization with the terminology.

n-Tanks in Series – Consider the case when the equalization tank is divided in n-equal tanks in series. The volume of each tank is equal to V/n. For the first stage, Eq. 4-12 could be rewritten as:

$$\frac{V}{n}\frac{dC_1}{dt} = q\,C_i(t) - q\,C_1 \qquad (4-30)$$

The effluent from the succeeding tanks is defined as:

$$\begin{aligned} \frac{V}{n}\frac{dC_2}{dt} &= q\,C_1 - qC_2 \\ \frac{V}{n}\frac{dC_3}{dt} &= q\,C_2 - qC_3 \qquad (4-31) \\ \frac{V}{n}\frac{dC_n}{dt} &= q\,C_{n-1} - qC_n \end{aligned}$$

In terms of the Laplace transform Eq. 4-30 and 4-31 could be written as:

$$\frac{C_1(s)}{C_i(s)} = \frac{1}{\left[\frac{Vs}{nq} + 1\right]}$$

$$\frac{C_2(s)}{C_1(s)} = \frac{1}{\left[\frac{Vs}{nq} + \right]} \qquad (4-32)$$

The operation is repeated to the nth tank which yields:

$$\frac{C_n(s)}{C_{n-1}(s)} = \frac{1}{(Vs/nq + 1)}$$

Hence,

$$C_2(s) = C_1(s)\frac{1}{Vs/nq + 1} = C_i(s)\frac{1}{Vs/nq + 1}\,\frac{1}{Vs/nq + 1}$$

$$\frac{C_2(s)}{C_i(s)} = \frac{1}{(Vs/nq + 1)^2} \qquad (4-33)$$

If the procedure is repeated to the nth tank, this yields;

$$\frac{C_n(s)}{C_i(s)} = \frac{1}{(Vs/qn + 1)^n} = \frac{1}{\left[\frac{\Theta s}{n} + 1\right]^n} \qquad (4-34)$$

It should be noted that C_n defines the effluent concentration of the equalization tank. The amplitude damping function and the phase shift function of Eq. 4-34 for sinusoidal inputs are derived by substituting $\imath\omega$ for s in Eq. 4-34. It is difficult to segregate the real and imaginary terms of the transfer function $1/(1+\imath\omega\Theta/n)^n$.

The amplitude damping function and phase shift functions are derived by converting Eq. 4-34 to polar form first before raising it to the nth power.

The mathematical operation is described below. Multiplying and dividing the term $1/(1 + \imath\omega\Theta/n)$ by $(1 - \imath\omega\Theta/n)\,/\,(1 - \imath\omega\Theta/n)$

yields

$$\frac{1}{(1+\imath\omega\Theta/n} = \frac{1}{(1+\imath\omega\Theta/n)}\,\frac{1-\imath\omega\Theta/n}{1-\imath\omega\Theta/n} \qquad (4-35)$$

$$= \frac{1-\imath\omega\Theta/n)}{\left\{1+\dfrac{\omega^2\Theta^2}{n^2}\right\}}$$

The real term of Eq. 4-35 is equal to the square root of the sum of the squares of the real and imaginary terms or:

$$R(\omega) = \sqrt{\frac{1}{\left\{1+\dfrac{\omega^2\Theta^2}{n^2}\right\}^2} + \frac{\dfrac{\omega^2\Theta^2}{n^2}}{\left\{1+\dfrac{\omega^2\Theta^2}{n^2}\right\}^2}} \qquad (4-36)$$

$$R(\omega) = \sqrt{\frac{1}{\left\{1+\dfrac{\omega^2\Theta^2}{n^2}\right\}}}$$

Similarly the argument of Eq. 4-35 is equal to

$$\theta = \tan^{-1}\left(\frac{\omega\Theta}{n}\right) \qquad (4-37)$$

Eq. 4-35 could be written as:

$$\frac{1}{\left(1+\dfrac{\imath\omega\Theta}{n}\right)} = \frac{1}{\sqrt{1+\dfrac{\omega^2\Theta^2}{n^2}}}\ \exp\imath\left[\tan^{-1}\left(\frac{\omega\Theta}{n}\right)\right] \qquad (4-38)$$

The nth power of Eq. 4-38 is equal to:

$$\frac{1}{\left[1+\dfrac{\imath\omega\Theta}{n^2}\right]^n} = \frac{1}{\left[1+\dfrac{\omega^2\Theta^2}{n^2}\right]^{n/2}}\ \exp n\imath\left[\tan^{-1}\left(\frac{\omega\Theta}{n}\right)\right] \qquad (4-39)$$

The amplitude damping function is equal to the real term of Eq. 4-39 which is equal to:

$$R(\omega) = \frac{1}{\left[1 + \frac{\omega^2\Theta^2}{n^2}\right]^{n/2}} \qquad (4-40)$$

and the phase shift function is equal to the argument of Eq. 4-39 or

$$I(\omega) = n \tan^{-1}\left(\frac{\omega\Theta}{n}\right) \qquad (4-41)$$

Plug flow tanks – As shown previously, the performance of n-completely mixed tanks in series taken as a unit, is equal to the performance of a single plug flow tank as n approaches infinity. The amplitude damping function of an equalization tank operated at plug flow condition could be derived from Eq. 4-40 with n approaching infinity. Hence, for a plug flow equalization tank the amplitude damping function is equal to:

$$R(\omega) = \lim_{n\to\infty} \frac{1}{\left[1 + \frac{\omega^2\Theta^2}{n^2}\right]^{n/2}} \qquad (4-42)$$

$$= \lim_{n\to\infty} \frac{1}{\left[1 + \frac{\omega^2\Theta^2}{n^2}\right]^{\frac{n^2}{\omega^2\Theta^2} \cdot \frac{\omega^2\Theta^2}{2n}}}$$

Since by definition, as $\omega^2\Theta^2/n^2 \rightarrow 0$

$$\lim_{\frac{\omega^2\Theta^2}{n^2}\to 0} \left(1 + \frac{\omega^2\Theta^2}{n^2}\right)^{\frac{n^2}{\omega^2\Theta^2}} = \text{exp.} = 2.718 \qquad (4-43)$$

then, the limit of Eq. 4- 42 as n ⟶ ∞ is equal to

$$R(\omega) = \lim_{n \to \infty} \frac{1}{\exp \frac{\omega^2 \Theta^2}{2n}} = \frac{1}{\exp(0)} = 1 \qquad (4-44)$$

Therefore for an equalization tank operated at plug flow condition, the amplitude of the effluent quality fluctuations is the same as the amplitude of the influent quality fluctuation. No equalization could take place in an equalization tank operated under plug flow condition. A plug flow equalization tank is entirely useless in terms of wastewater quality equalization. If n exceeds 5, the equalization tank approaches plug flow.

It should be noted that Nemerow (1971) suggested the use of up and down baffles for mixing in equalization tank. Such arrangement introduce vertical and lateral mixing but the longitudinal mixing pattern is almost negligible. The equalization system suggested by Nemerow approaches a plug flow equalization system which is very ineffective as shown by Eq. 4-44.

The phase shift of a plug flow equalization tank is the limit of Eq. 4-41 as n approaches infinity. As $\omega\Theta/n$ approaches zero, the arctangent of $\omega\Theta/n$ is equal to $\omega\Theta/n$. Hence, the phase shift of a plug equalization tank could be derived from Eq. 4-41 as

$$\varphi = \lim_{n \to \infty} n \tan^{-1} \left\{ \frac{(-\omega\Theta)}{n} \right\} = \lim_{n \to \infty} \frac{(-\omega\Theta)}{n} (n) \qquad (4-45)$$

$$= -\omega\Theta$$

Dispersion Equation – The dispersion of pollutant in a tank is defined by the following equation whose derivation is described in Chapter I,

$$\Theta \frac{\partial C}{\partial t} = \frac{D_L}{\bar{V}L} \frac{\partial^2 C}{\partial x^2} - \frac{\partial C}{\partial x} \qquad (4-46)$$

The boundary conditions to Eq. 4-46 are:

$$\begin{aligned} &C(t, 0) = C_i(t) \\ &C(t, 1) = C(t) \\ &C(0, x) = 0.0 \end{aligned} \qquad (4-47)$$

Taking the Laplace transform of Eq. 4-47 yields:

$$\Theta sC(s, x) - sC(0, x) = \frac{D_L}{\bar{V}L} \frac{d^2C(s,x)}{dx^2} - \frac{dC(s,x)}{dx} \qquad (4-48)$$

Since sC(0,x) equals zero, Eq. 4-48 has the following solution

$$\begin{aligned} C(s,x) = A_1 \ &\exp\left[\left\{\frac{1-\sqrt{1+4s\Theta\,(D_L/\bar{V}L)}}{2\,D_L/\bar{V}L}\right\} x\right] \\ + A_2 \ &\exp\left[\left\{\frac{1+\sqrt{1+4s\,(D_L/\bar{V}L)\,\Theta}}{2\,D_L/\bar{V}L}\right\} x\right] \end{aligned} \qquad (4-49)$$

Since $C(s,x)$ should decrease with x, then A_2 is equal to zero.At x equals zero, $C(s,x)$ is the Laplace transform of C_i (t) or equal to C_i (s). Hence,

$$C(s,x) = C_i(s) \quad \exp\left[\left\{\frac{1-\sqrt{1+4s(D_L/\bar{V}L)\Theta}}{2D_L/\bar{V}L}\right\} x\right] \qquad (4\text{-}50)$$

Furthermore at x=1, $C(s,x)$ is the Laplace transform of the effluent pollutant concentration, C(s). Therefore Eq. 4-50 could be rewritten as:

$$\frac{C(s)}{C_i\,(s)} = T(s) = \exp\left\{\frac{1-\sqrt{1+4s(D_L\bar{V}L)\,\Theta}}{2\,D_L/VL}\right\} \qquad (4-51)$$

If the influent quality concentration fluctuates sinusoidally then the amplitude damping function and the phase shift function could be derived by substituting $\iota\omega$ for s in Eq. 4-51. This results to:

$$T(\iota\omega) = \exp\left\{\frac{1-\sqrt{1+4\iota\omega(D_L/\bar{V}L)\,\Theta}}{2\,D_L/\bar{V}L}\right\} \qquad (4-52)$$

To split the function $T(\iota\omega)$ into its real and imaginary components, the term $1 + 4\iota\omega D_L/VL$ is expressed in its polar form which yields:

$$1 + 4\,\omega\,D_L/\bar{V}L\ \Theta\,\iota =$$

$$\sqrt{1+16\omega^2\,(D_L\bar{V}L)^2\Theta^2} \quad \exp\left[\iota\,\tan^{-1}\left(\frac{\Theta 4\omega D_L}{\bar{V}L}\right)\right] \qquad (4-53)$$

To facilitate the calculation, let

$$\begin{aligned} \alpha &= 1+16\omega^2(D_L/\bar{V}L)^2\Theta^2 \\ \beta &= D_L/\bar{V}L \\ \varphi &= \tfrac{1}{2}\tan^{-1}\,(\Theta 4\omega D_L/\bar{V}L) \end{aligned} \qquad (4-54)$$

Eq. 4-52 then simplifies to:

$$T(\iota\omega) = \exp\left[\frac{1}{2\beta} - \frac{\sqrt{\alpha}}{2\beta}\exp(\iota\varphi)\right] \qquad (4-55)$$

$$= \exp\left[\frac{1}{2\beta}\left\{1 - \sqrt{\alpha}\cos\varphi - \iota\sqrt{\alpha}\sin\varphi\right\}\right]$$

since $\exp(\iota\varphi)$ is equal to $\cos\varphi + \iota\sin\varphi$

Eq. 4-55 simplifies further to:

$$T(\iota\omega) = \exp\left\{\frac{1}{2\beta}(1 - \sqrt{\alpha}\cos\varphi)\right\}\exp\left(\frac{-\iota\sqrt{\alpha}\sin\varphi}{2}\right) \qquad (4-56)$$

By examining Eq. 4- 56, it is apparent that $T(\iota\omega)$ is already expressed in its polar form. The amplitude damping function is the real term of Eq. 4-56 or equal to

$$R(\omega) = \exp\left\{\frac{1}{2\beta}(1 - \sqrt{\alpha}\cos\varphi)\right\} \qquad (4-57)$$

Similarly the phase shift function is the argument of Eq. 4-56 which yields:

$$I(\omega) = \frac{-\sqrt{\alpha}\sin\varphi}{2} \qquad (4-58)$$

Plug Flow Condition – The amplitude damping function and the phase shift function for plug flow condition could be derived from Eq. 4-57 and 4-58 as the limit when $D_L/\sqrt{V}L$ approaches zero. However, the solution is simpler if $D_L/\sqrt{V}L$ is equated to zero in Eq. 4-46. The Laplace transform of Eq. 4-46 at plug flow condition is equal to:

$$\Theta s\, C(s,x) = \frac{-dC(s,x)}{dx} \qquad (4-59)$$

The solution to eq. 4-59 with eq. 4-47 as boundary condition is equal to:

$$\frac{C(s)}{C_i(s)} = \exp(-s\Theta) \qquad (4-60)$$

If $C_i(s)$ is a sinusoidal function then $T(\iota\omega)$ could be determined from Eq. 4-60 by substituting $\iota\omega$ for s.which results to:

$$\frac{C(\iota\omega)}{C_i(\iota\omega)} = T(\iota\omega) = 1 \exp(\Theta\iota\omega) \qquad (4-61)$$

A cursory inspection of Eq. 4-61 will show that $T(\iota\omega)$ is already in its polar form. The amplitude damping function is the real term of Eq. 4-61 which is equal to one. Hence,

$$R(\omega) = 1 \qquad (4-62)$$

Eq. 4-62 confirms the conclusion made from the n-tank in series model that no equalization takes place if the equalization tank is designed for plug flow conditions. From Eq. 4-61, the phase shift function is the argument of $T(\iota\omega)$ which is equal to:

$$I(\omega) = \Theta\omega \qquad (4-63)$$

Eq. 4-63 is equal to the phase shift function for plug flow condition derived in Eq. 4-45. This proves further that behavior of the n-equalization tank in series approaches plug flow as the number of tanks in series approaches infinity.

Optimal Flow Patterns – Novotny (1976) suggested complete mixing as the ideal flow pattern in equalization basin. La Grega and Keenan (1974) proposed that an ideal equalization basin should be thoroughly mixed with minimal short circuiting. Ouano (1977) considered La Grega's conditions contradictory, as mixing proportionally increases short circuiting. It was suggested that a dispersion number between 5 to 50 is an ideal compromise between the two contradictory conditions proposed by La Grega.

A very large portion of the waste streams stay longer in the equalization basin if the dispersion number approaches zero as short circuiting becomes minimal. However, the mixing effect becomes minimal too, resulting in poor dilution. The ideal conditions for a balancing tank to be completely mixed without short-circuiting as suggested by La Grega and Keenan, are impossible to attain.

As the dispersion number approaches infinity, the completely mixed system is attained. Although mixing is at its maximum when the dispersion number is infinite, short-circuiting is also at its maximum resulting in instantaneous appearance of large quantities of fresh waste stream in the effluent. Dilution is only due to the waste streams which have been accumulated in the tank from the past.

When the dispersion number is approximately 0.5, a larger portion of peak waste input stays in the tank longer, doubling the dilution of the peak waste stream from the weaker concentration which has accumulated in the equalization basin and weaker waste which occurs after the peak load has passed.

Eq. 4-40 could be reduced to dimensionless form as:

$$\left[R(\omega)\right]^{1/\omega\Theta} = \frac{1}{\left[1+\left(\frac{\omega\Theta}{n}\right)^2\right]^{n/2\omega\Theta}} \qquad (4-64)$$

Eq. 4-64 could be also expressed in terms of the frequency of oscillation wherein ω is equal to $2\pi f$. Then Eq. 4-64 is equal to:

$$R(f)^{\frac{1}{2\pi f\Theta}} = \frac{1}{\left[1+\left(\frac{2\pi f\Theta}{n}\right)^2\right]^{n/4\pi f\Theta}} \qquad (4-65)$$

Similarly Eq. 4-57 could be expressed as:

$$R(f)^{1/2\pi f\Theta} = \qquad (4-66)$$

$$\exp\left[\frac{P_e}{4\pi f\Theta}\left\{1-\sqrt{\frac{1+\sqrt{64\left(\frac{\pi f\Theta}{P_e}\right)^2}+1}{2}}\right\}\right]$$

Where P_e is equal to $\bar{V}L/D_L$ commonly known as the Peclet number. Eq. 4-64 and 4-66 have a minimum value when

$$\frac{n}{2\pi f\Theta} = 0.5 \qquad (4-67)$$

and

$$\frac{P_e}{4\pi f\Theta} = 0.5 \qquad (4-68)$$

The graph of Eq. 4-64 and 4-66 are shown in Fig. 4-7.

It is apparent from Eq. 4-67 and 4-68 that the optimal mixing pattern for equalization basin lies between a completely mixed system and a plug flow system. At optimal mixing, a compromise is attained between mixing and short circuiting.

Walker and Cholette (1958) hypothesized that one large well mixed equalization tank is more efficient in damping amplitudes from low frequency fluctuations while a series of well mixed equalization tank is more efficient in damping amplitudes from fluctuations with high frequencies. The phenomenon is quantified by Eq. 4-67 since optimal amplitude damping takes place when

$$n = \pi f \Theta \qquad (4-69)$$

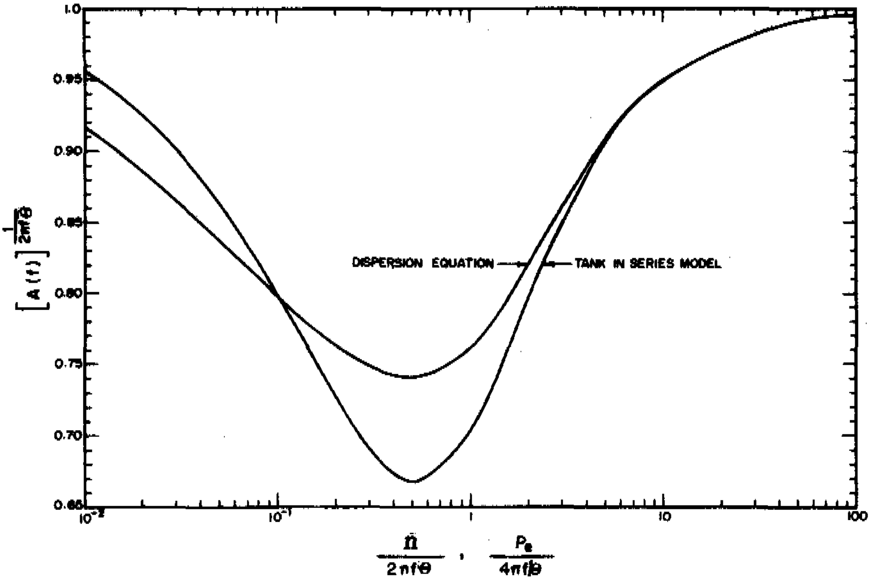


FIG.4-7 Effect of Mixing and Number of Tanks on Amplitude Ratio

From Eq. 4-67, as f increases, n increases proportionally to attain the maximum amplitude damping. Generally n should be less than five since amplitudes from fluctuations of high frequencies are damped very rapidly as described in the previous section.

Example 4-8. Consider an industrial process operating with a frequency of 3 cycles/day. The waste flow fluctuation may be composed of a number of frequencies depending on the production cycle, e.g., a diurnal wastewater flow (f=1/day), weekly washing (f=1/week), and annual washing (f=1/yr). Describe the behavior of the equalization tank.

Solution:

It would be impractical to consider equalizing the wastewater flow rates if the frequency of occurrence were very small as it would require an investment that could be utilized for a short duration only. If sufficient space were available to construct an equalization basin with a retention time of 24 hr, then using the dispersion equation, the required Peclet number of the tank would be:

$$Pe = 0.5 \times 4 \pi f \Theta = 0.5 \times 4 \times 3.14 \times \frac{3 \text{ cycles}}{\text{day}} \times 1 \text{ day}$$

$$= 19 \text{ with amplitude ratio of } 0.00253 \text{ or } (0.73)^{19}$$

If the n-tanks in series model were used, the number of completely mixed tanks required in series would be

$$n = \pi f \Theta = 0 \pi \times \frac{3}{\text{day}} \times 1 \text{ day} \approx 9.4$$

For nine tanks in series the expected amplitude ratio would be 0.000057.

If space for constructing the equalization basin were limited reducing the retention time 0.25 day, then the required Peclet number would be

$$Pe = 0.5 \times 4 \times \pi \frac{3 \text{ cycles}}{\text{day}} \times \pi \times 0.25 \text{ day} \approx 4.8$$

Using the n-tanks in series model, the required number of tanks would be

$$n = 0.5 \times 2 \times \pi \frac{3 \text{ cycles}}{\text{day}} \times 0.25 = 2.4$$

say three tanks.

The amplitude ratios would be 0.22 and 0.15, respectively. It is apparent using the graph in Fig. 4-7 that tradeoffs between constructing large equalization basins and mixer power requirements can be evaluated.

In most instances, a very low amplitude ratio would be unnecessary as the subsequent process after the equalization basin could absorb or tolerate a certain degree of shock loads. Considering the previous example, if an amplitude ratio of 0.1 were required, then using the dispersion model the required tank detention time would be

$$\Theta = \frac{1}{6} \frac{\log(0.1)}{\log(0.73)} \times 24 = 9 \text{ hr.}$$

and if the n-tanks in series model were used then

$$\Theta = \frac{1}{6} \quad \frac{\log(0.1)}{\log(0.67)} \quad x\,24 \quad = 7.3 \text{ hr.}$$

> The required Peclet number and number of tanks in series would be 7.0 and 3, respectively, using the procedure given in the previous example. The values 0.73 and 0.67 are the minimum values of Eq. 4-60 and 4-61.

The completely mixed tank has been accepted as a limiting case to the dispersion model when $P_e \rightarrow 0$ as shown by Livenspiel (1962). However, analysis of the amplitude damping function shows little correlation between the two models except when $P_e \rightarrow \infty$ and $n \rightarrow \infty$. The designer would have to make his own decision on the appropriate model to use.

As discussed in Chapter III as $D_L/\overline{V}L - L$ could approach zero instead of D_L approaching infinity as required in a completely mixed system. As L approaches zero, the equalization basin disappears hence, no equalization could take place. The deficiency of Eq. 4-66 as P_e approaches 0 could be overcome if the boundary condition used by Wellner and Wilhelm is used instead of Eq. 4-47.

Feeding Patterns – A single input/output configuration as shown in Fig. 4-2 is commonly used in equalization basin. This type of feeding pattern is simple to analyze and construct as shown in the previous sections. However, the total effect of the quality variation is localized in one section of the equalization basin. A step feed system shown in Fig. 4-8 is more effective as it spreads out the effects of the quality variation more uniformly throughout the system.

Khudenko (1979) studied such flow patterns and established the amplitude damping function of the system as:

$$R(\omega) = \sin \omega\Theta/\omega\Theta \qquad (4-70)$$

Other variations in feeding patterns and recirculation of the effluent to improve the equalization efficiency and process stability have not been quantified.

SIMULTANEOUS QUALITY AND QUANTITY EQUALIZATION

In the previous section, flow and quality equalizations were discussed separately. However, in practice, flow and quality could vary simultaneously and would require simultaneous equalization. Generally, the wastewater strength increases with increasing flow rates as described in Chapter II.

In a completely mixed system the incoming mass is defined as $q_i C_i$, the outgoing mass as $q_p C$ and the accumulation is $d(CV)/dt$. The mass balance of the input, output and accumulation yields:

$$\frac{d\,(CV)}{dt} = q_p(t)\;C_i(t) - q_p C\;(t) \qquad (4-71)$$

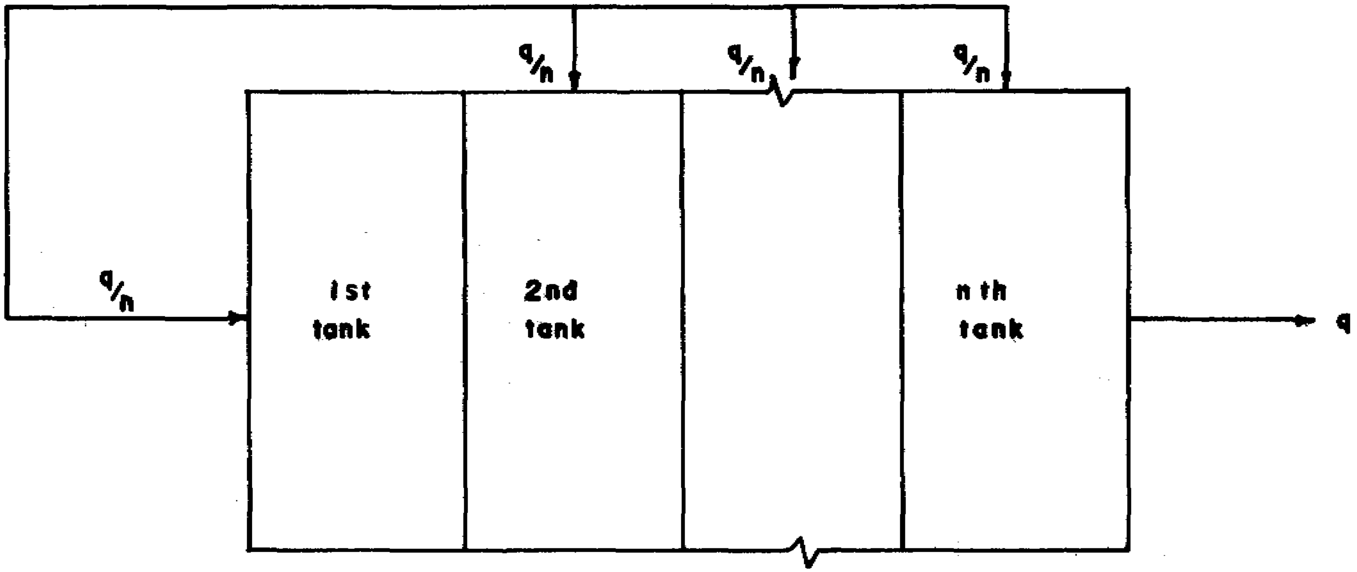


FIG. 48 Schematic diagram of a split flow equalization tank.

The accumulation term could be expanded as:

$$\frac{d\,(CV)}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt} \qquad (4-72)$$

Moreover, if evaporation in the basin is neglected then the rate of change of the wastewater volume inside the equalization tank is the difference between the incoming and outgoing flows. If the incoming flow is higher than the outgoing flow, then the wastewater volume in the equalization basin increases, hence, dV/dt is positive. The contrary holds true when the outgoing wastewater flow is higher than the incoming wastewater flow. The rate of change of the wastewater volume inside the equalization tank could be expressed as:

$$\frac{dV}{dt} = q_i(t) - q_p \qquad (4-73)$$

If the wastewater quantity fluctuation has a cycle, T, the rate of equalized flow out of the equalization basin is equal to

$$q_p = \frac{1}{T}\int_0^T q_i(t)\,dt \qquad (4-74)$$

as discussed in the previous section. After one cycle, T, the volume of the wastewater in the equalization basin returns to its original volume, V_o or residual wastewater volume. This is apparent since integrating Eq. 4-74 yields:

$$\int_{V_o}^{V} dV = \int_{0}^{T} q_i(t)\,dt - \int_0^T q_p\,dt \qquad (4-75)$$

or

$$V - V_o = \int_0^T q_i(t)\,dt - q_p T = 0$$

Finite Difference Approximation – Continuous monitoring of industrial wastewater quality and quantity is expensive. Most often, the samples are taken at an hourly interval or so. Eq. 4-71 and 4-73 could be approximated by finite difference as:

$$C(t)\left[q_i(t) - q_p\right] + V(t)\,\frac{C(t) - C(t - \Delta t)}{\Delta t} \qquad (4-76)$$

$$= q_i(t)\,C_i(t) - q_i\,C(t)$$

and

$$\frac{V(t) - V(t - \Delta t)}{\Delta t} = q_i(t) - q_p \qquad (4-77)$$

where C(t) is effluent pollutant concentration at time, t; $C(t-\Delta t)$ is the effluent pollutant concentration one time increment behind, V(t) is the wastewater volume in the equalization tank at time, t; $V(t-\Delta t)$ is the wastewater volume in the equalization tank one time increment behind and the other terms are as defined before. As $\Delta t \rightarrow 0$, Eq. 4-76 and 4-77 approach Eq. 4-71 and 4-73 respectively.

One of the objectives of the equalization basin is to deliver a constant hydraulic load into the waste treatment plant at the rate q_p. Hence, q_p could be calculated as the average wastewater flow rate based on actual measurements or estimated from the factory water consumption rates. To handle extreme wastewater flow conditions, the actual pumping specification should be greater than q_p by an appropriate factor of safety. The outgoing flow rate, q_p is calculated using Eq. 4-74, or it is approximated by Eq. 4-3.

To solve Eq. 4-76 and 4-77 the initial equalization tank volume and effluent concentration have to be established. The initial pollutant concentration at t=0, could be approximated from the weighted average of the pollutant concentration in the sample and the respective flow rate. Hence,

$$C(0) = \frac{\sum q_i(t)\,C_i(t)\,\Delta t}{\sum q_i(t)\,\Delta t} \qquad (4-78)$$

The initial residual volume V_0 is set at various values varying from 0.0 for the first trial to 20% of the total volume of one process cycle in the second trial. Normally V_0 is lower than the total volume generated in one process cycle. The ideal residual volume is determined from the graph which gives the required amplitude damping or effluent which could be tolerated by the succeeding process equipment. The procedure for calculating the equalization tank volume required for simultaneous equalization of wastewater quality and quantity is illustrated in the succeeding example.

Example 4-10. Consider a factory with a 24-hour process cycle whose wastewater flow rate and quality are shown in Table 4-2. The desired effluent quality from the equalization tank should fluctuate by not more than 60 percent of the average. Constant hydraulic rate should be applied to the waste treatment plant, as much as possible.

Solution:

1. Calculate q_p by summing up the $q_i(t)\Delta t$ and dividing it by the number of data available which gives $q = 1042\ m^3/hr$
2. Calculate $C(0)$ using Eq. 4-78. This results to $C(0) = 1000$ mg/l
3. Set $V_o = 0.0$
4. Calculate $V(t)$ by Eq. 4-77 and $C(t)$ by Eq. 4-76 for all values of t.
5. Set $V_o = q_p$; etc. and repeat Step 4 for each set of V_o on the waste stream as shown in Fig. 4- 9 .
6. Graph the maximum $C(t)$, average $C(t)$ and maximum, $V(t) - V_o$, as shown in Fig. 4-10. Determine the upper and lower tolerance limits by multiplying average $C(t)$ by 1.6 and 0.4, respectively.
7. From Fig. 4-10, the required residual volume (V_o) is 3700 m^3 and $(V_{max} - V_o)$ is 3100 m^3. Hence, V_{max} is equal to 6800 m^3. If a factor of safety of 20 percent is used, the recommended volume of the equalization tank is 8160 m^3.

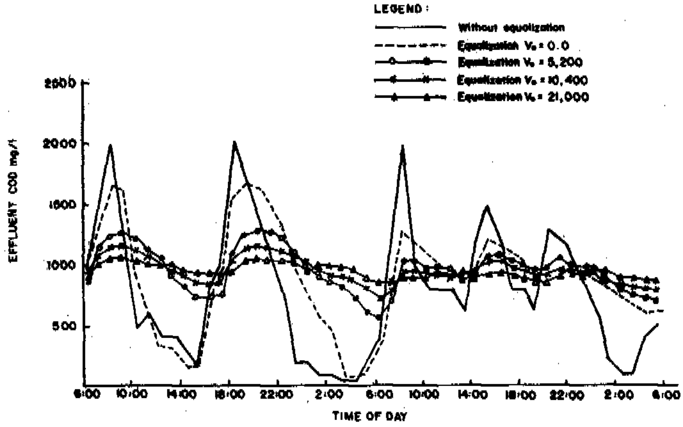


FIG. 4-9 Effect of residual volume on concentration equalization.

TABLE 4-2
Fluctuations of Wastewater Quality and Quantity with Respect to Time for Two Consecutive Process Cycle

Time of Day	Flow Rate m^3/hr	COD Conc., mg/1
7.00	1000	1000
8.00	2000	1500
9.00	1500	2000
10.00	500	1500
11.00	400	500
12.00	1000	600
13.00	800	400
14.00	800	400
15.00	600	300
16.00	1000	200
17.00	1000	800
18.00	1500	1000
19.00	1800	2000
20.00	2000	1800
21.00	1500	1500
22.00	1200	1200
23.00	800	800
24.00	800	200
1.00	800	200
2.00	400	100
3.00	200	50
4.00	200	50
5.00	600	50
6.00	700	200
7.00	1200	400
8.00	2500	1000
900	1800	2000
10.00	1500	1000
11.00	1000	800
12.00	600	800
13.00	600	800
14.00	800	600
15.00	1000	1200
16.00	2000	1500
17.00	1900	1200
18.00	1800	800
19.00	1200	800
20.00	800	600
21.00	800	1300
22.00	800	1200
23.00	2000	1000
24.00	1500	800
1.00	1000	600
2.00	400	200
3.00	400	100
4.00	300	100
5.00	600	400
6.00	500	500

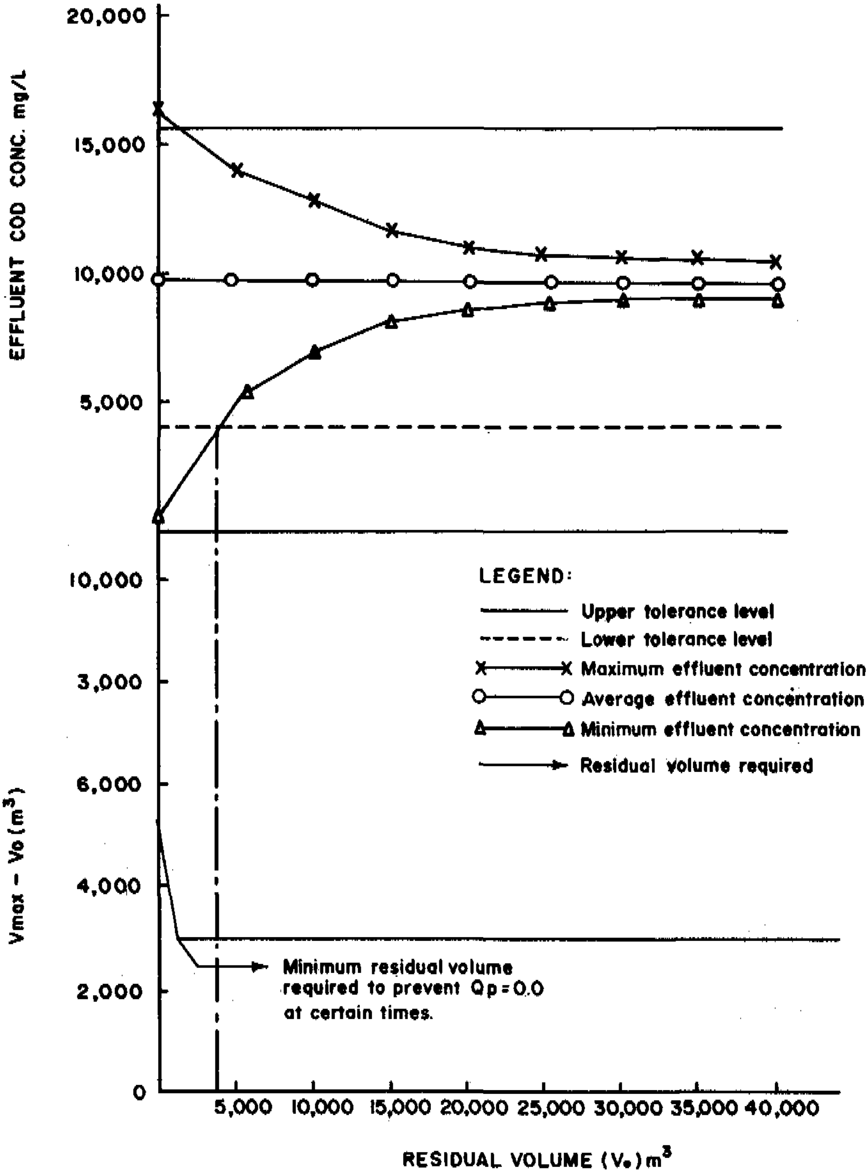


FIG. 4-10 Effect of residual volume on critical process parameters.

EQUALIZATION IN PROCESS TANKS

In the previous section the equalization effects of sewer lines were discussed. Equalization of the wastewater quality and quantity continuously takes place in the primary clarifier, grit tanks, activated sludge and in the final clarifiers. In processes which are very sensitive to shock loads such as the activated sludge process, plug flow condition is discouraged. Better process stability is attained with the internal equalization brought about by the mixing of the process tank content. Before, mixing and the subsequent short circuiting were considered detrimental to the process efficiency and were controlled just to maintain a specific level of stability. Since the biochemical stabilization of organic wastes was discovered to exhibit autocatalytic properties, mixing was found to improve the process efficiency in addition to increasing process stability. For this reason, current design of the activated sludge is for the *completely mixed type rather than the conventional* or plug flow system.

The principle discussed in the previous section could be used to determine the equalization in the different process tanks. In most instances the mass balance is modified by introducing the reaction rate constant in a similar manner to those discussed in Chapter III.

RANDOM FLUCTUATIONS

The equalization tank size required for random fluctuations could be determined by simulation. The process would involve the generation of random variates *which conform with the probability density function of the influent wastewater* quality and quantity. The finite difference approximation defined by Eq. 4-76 and 4-77 is used to simulate the effluent quality of the equalization basin and the residual volume required for equalization.

If the influent wastewater quality has a mean $\bar{C}_i$, and a standard deviation σ_i, then the random variate $C_i(t)$ is generated as:

$$C_i(t) = \left(\sigma \sum_{i=1}^{12} R_i\right) + (\bar{C}_i - 6\sigma_i) \qquad (4-79)$$

R_i is a random number obtained from tabulation or computer random number generation subroutines. The determination of $\bar{C}_i$ and σ_i values are discussed in Chapter II. In a similar manner, the wastewater quantity $\bar{q}_i(t)$ could be generated for Eq. 4-76 and 4-77 if mean $\bar{q}_i$ and σ are known. The wastewater flow rate is calculated as:

$$q_i(t) = \left(\sigma \sum_{i=1}^{12} R_i\right) + (\bar{q}_i - 6\sigma) \qquad (4-80)$$

Eq. 4-79 and 4-80 assume that the occurrence of a particular wastewater flow *rate $q_i(t)$ is independent of the occurrence of a particular wastewater quality C_i* (t). If a correlation exists between the wastewater quality and quantity, such that $q_i(t) = f(C_i(t))$, then $q_i(t)$ is calculated from $f(C_i(t))$.

CHAPTER V

LAGOONS

INTRODUCTION

There are five types of lagoon systems currently used for wastewater treatment. These are the aerobic, facultative, anaerobic, maturation and aerated lagoons. The applicability of a lagoon system is dependent on the strength of the waste, the land area available and the quality of the effluent required. Due to the long detention time in lagoons compared to that in other waste treatment systems, the pathogen removal is more extensive in the lagoon system. Lagoons require little operational and maintenance cost but require extensive land areas. Operation and maintenance means primarily desludging the pond once every four or even ten years, cutting the grass at the edge and removing the floating scums. Desludging is best carried out during the dry season by letting the sludge dry out rather than trying to remove the wet slurry. In some instances where large tracts of land are available, the cost of desludging could be more expensive than constructing a new series of lagoons.

A lagoon is basically a hole in the ground where the wastewater is stored to undergo natural purification which is brought about by the bacterial activity with symbiotic actions from algae and other living substances. The biological community in a lagoon system is more varied than that in other biological wastewater treatment processes. The living organisms present could vary from single cell microorganisms like bacteria to complex organisms such as fish. For this reason, the lagoon system is a very stable process. However, the complexity of the biological reaction and interaction makes it difficult to rationalize the design criteria.

Types of Lagoon System

Aerobic lagoon – An aerobic lagoon may be designed for maximum oxygen or algal production. Aerobic condition prevails throughout the pond depth which is very shallow at 0.3 to 0.4 m. About 80 to 95% of the soluble BOD could be removed in this system. However, the very high algal concentration in the unfiltered effluent could result in a very high BOD concentration. In fact, in some instances, the BOD equivalent of the algal mass could exceed the influent BOD concentration. Since the algal mass is not part of the organic pollutant load, it should be separated first before evaluating the system performance.

The aerobic lagoon system is used primarily for algal protein production since it requires very large tract of land per unit of BOD stabilized compared to the facultative or anaerobic system.

Facultative lagoon – A facultative lagoon is slightly deeper than the aerobic lagoon. The top portion of a facultative lagoon is always aerobic, while the bottom is anaerobic. Between those two layers, the dissolved oxygen concentration could vary from supersaturation in mid-afternoon to negligible level during the early hours of dawn. Fig. 5-1 shows a typical dissolved oxygen fluctuation in a facultative lagoon.

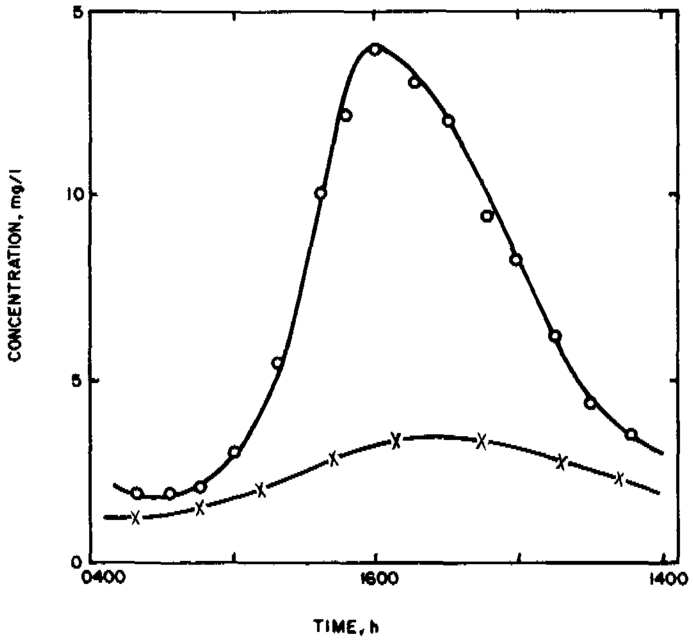


FIG. 5-1 Diurnal variation in dissolved oxygen; 0, top 200 mm of pond; 800 mm below surface.

The facultative lagoon system is more widely used than the aerobic or anaerobic lagoon system. It presents a good compromise between the very large area required for aerobic lagoons and the aesthetic and odor problems associated with anaerobic lagoons. For strong wastes or where land is limited, facultative lagoons are used to treat the effluent from anaerobic lagoons to effluent quality standards as will be discussed later. Although the facultative lagoon system is another "hole-in-the-ground" type of wastewater treatment system, the interaction of environmental factors such as sunlight, wind, temperature, topography, vegetation and the biological population in the process performance is so complex that quantitative evaluation of all the factors in the design is not worthwhile or is too complex to consider. As a result, a number of design misconceptions have developed through the years.

Anaerobic lagoons – Anaerobic lagoons are very much deeper than facultative lagoons. For this reason, the amount of oxygen supplied from the atmosphere is insufficient to maintain aerobic condition except for a very thin layer at the top of the water during start up. Later, scum layer is formed above the wastewater which prevents algae from growing due to the absence of sunlight and further oxygen diffusion from the air. The by-products of anaerobic degradation include hydrogen sulphide and methane gas. Hydrogen sulphide has a bad odor similar to decomposed eggs. Hence, the organic loading in anaerobic lagoon is controlled to minimize the evolution of the malodorous gases.

Previously, anaerobic lagoons were avoided as much as possible due to expected problems with odor. In the late 1940's, Parker (1950) demonstrated that even if an anaerobic condition exists in the lagoon, as long as the total weight of BOD loaded per unit volume per day does not exceed a certain specified value, the malodorous gases generated are not considerable as to cause any nuisance. For South African condition, a volumetric condition of less than 400 gm of BOD_5/m^3-day is specified to minimize odor problems (Marais, 1970).

Recirculation of facultative pond effluent to the anaerobic pond surface is also used to minimize the odor problems by providing an aerobic blanket. Addition of alkaline substances such as lime has been used to form salts with hydrogen sulphide gas, thereby preventing its release to the atmosphere. The allowable volumetric loading should depend on the alkalinity of the wastewater. The rate at which hydrogen sulphide is formed depends on temperature, BOD_5 strength of the wastes, and initial sulfate concentration in the wastes. The land to be used for anaerobic lagoon is downwind from built up areas. However, at present little has been done to quantify the effects of those variables on anaerobic pond performance to allow process optimization.

Anaerobic digestion is much slower than aerobic reaction. However, in anaerobic pond a large portion of the colloidal particles undergo coalescence to form the scum and sludge which hastens the removal of the suspended BOD_5.

Aerated lagoons – For facultative lagoon a large surface area is required for sunlight and oxygen diffusion. For anaerobic lagoon, the quantity of BOD loaded per unit volume is limited due to the evolution of malodorous gases. Where land is limited and strict odor control is required, oxygen may be supplied mechanically by means of diffusers or aerators. The process is known as an aerated lagoon system.

Due to shorter detention time, the biological community of aerated lagoon is not as diverse as in the facultative lagoon. Bacteria compose the predominant microbial species. To prevent the premature washout of the bacterial community from the aerated lagoon, the wastewater hydraulic retention time should be at least 1.5 days in the tropics and 3 days in cold climates.

Maturation ponds – The coliform content of sewage could be as high as 10^7 MPN/100 ml. After the BOD has been degraded to levels acceptable for discharge to a stream, the residual coliform is still very high. Although the lagoon system is more efficient in removing the coliform in the wastewater compared to the activated sludge or trickling filters, the effluent will require further treatment to remove the coliform organism. Further degradation of the residual BOD could take place in a maturation pond. A maturation pond is a lagoon up to 5 m. deep wherein the effluent from the facultative, anaerobic lagoon is treated to remove the coliform organism.

For industrial wastewater which are not contaminated with coliform organism or where the pathogenic organism is not expected to occur, maturation pond is not required. Chlorination of the contaminated effluent may be utilized instead of the maturation pond in cases where land is very limited.

Algal growth in the maturation ponds could take place rapidly. Hence in some instances the maturation ponds are also utilized as fishponds. The fish are often used for animal feeds although in most cases the fish are fit for human consumption.

DESIGN OF LAGOON SYSTEMS

This section will deal mostly with facultative, anaerobic and aerated lagoon systems. The aerobic lagoon is not commonly used in practice as previously described. The section closes with a discussion on the total optimization of the system.

Facultative Lagoon Design

The mechanics of organic waste stabilization in a facultative lagoon could be visualized by the schematic diagram shown in Fig. 5-2. The suspended, settleable solids and dead bacteria, algae and other organisms settle at the bottom and undergo anaerobic digestion. Carbon dioxide, methane, nitrogen and hydrogen sulphide as well as anaerobic bacterial cells are formed.

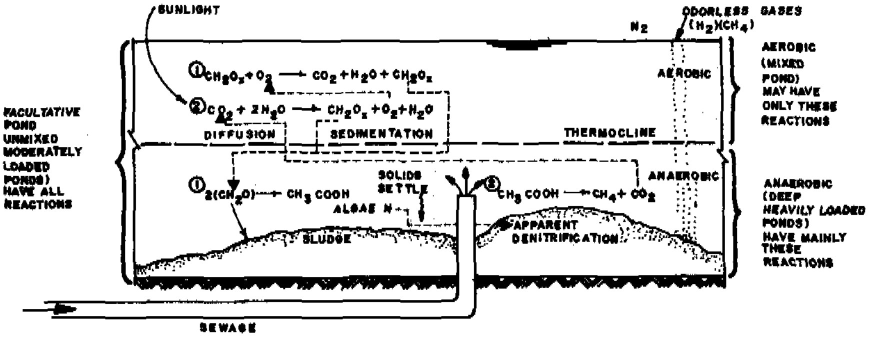


FIG. 5-2 Interrelationship of conditions and reactions in ponds.

The gases move upward towards the surface where the pH is in the alkaline range. During daytime when algal photosynthesis is very fast, the pH could be as high as 11. The carbon dioxide forms carbonates and the hydrogen sulphide forms sulphite with the excess oxygen. During the night time when the pH is just above neutral, bicarbonate and bisulphides are formed by carbon dioxide and hydrogen sulphide respectively. Methane and nitrogen are released into the atmosphere.

The carbonates are used by algae with the help of sunlight to form algal cells and oxygen in the process known as photosynthesis. During night time the algae undergo respiration in which case carbon dioxide is added to the water. Thus the dissolved oxygen declines and the whole pond could be almost anaerobic. Aerobic bacterial cells consume the colloidal and dissolved organic particles with oxygen, forming new bacterial cells and carbon dioxide. The carbon dioxide from aerobic degradation is used by algal cells in the same way as the carbon dioxide from anaerobic digestion. It can be seen that bacteria and algae form a symbiotic relationship.

The wind mixes the wastewater with the dissolved oxygen and organic waste in all direction. The mixing in the vertical and lateral direction improves the process efficiency by minimizing the dead volume, while the mixing in the longitudinal direction lowers the process efficiency by inducing short circuiting if the wastewater flow is in the same direction as the wind.

The wind creates waves on the pond surface which could increase the effective surface area for oxygen diffusion up to a factor of 10. Wind waves are created by the transfer of energy from the wind to the water surface. For very small ponds such as those in pilot studies oxygen transfer induced by the wind is negligible as contact length between the wind and the water surface is very low.

Marais (1966) conducted an experiment with the facultative pond protected by a wind breaker. The facultative pond turned septic and the efficiency went down. When the wind breakers were removed, the pond performance returned to normal. On the other hand, Oswald (1957) suggested the effect of wind on the facultative pond aeration is negligible and in some instance it is detrimental due to the turnover of the settled solids undergoing anaerobic decomposition.

The quantity of sunlight available for algal photosynthesis is proportional to the surface area of the lagoon, the location of the pond, and the season. The most active region for photosynthesis is the upper 0.3 m depth of the pond.

The available oxygen supply from photosynthesis or wind induced mass transfer is proportional to the pond surface area. The quantity of oxygen consumed for BOD removal is proportional to the quantity of BOD applied on the system. To maintain aerobic condition, a balance should exist between the oxygen supply and oxygen uptake. For similar operating conditions, the oxygen supply/demand balance should be constant.

If the oxygen uptake, O_r, is defined as

$$O_r = K_1 \text{ (wt of BOD applied/day)} \qquad (5-1)$$

and the oxygen supply O_i is defined as

$$O_i = K_2 A_s \qquad (5-2)$$

where A_S is the pond surface area, ha; then if O_r is equal to O_i,

$$O_r = O_i = K_2 A_s = K_1 \text{ (wt of BOD applied/day)} \qquad (5-3)$$

Eq. 5-3 could be rearranged as:

$$\frac{K_2}{K_1} = \frac{\text{wt of BOD}}{A_s} = A_L \qquad (5-4)$$

where A_L is a constant known as areal loading rate. The development of areal, loading rate as a design parameter was more intuitive than as described by Eq. 5-1 to 5-4. The oxygen uptake rate is proportional to the wt of BOD applied/day only when the effluent BOD is very low and the facultative pond is not divided into a number of smaller ponds operated in series.

A common practice today is to utilize the mid-depth area of the pond in calculating the areal loading rates. For very large areas, the difference between the surface area and the mid-depth area is very small. However, in pilot plants the error could be as high as 40%. As discussed before, the surface area is the region which controls the oxygen available to the system.

Using the mid-depth area in the calculation of the areal loading rates tends to overestimate the actual areal loading rates.

The allowable areal loading rates are highly dependent on the locality. As the area becomes closer to the equator, algal photosynthesis, oxygen production rates and bacterial oxygen consumption rates increase correspondingly. The state of Texas recommends areal loading rates of 77 kg of BOD/ha-day with a minimum detention time of 60 days. In Thailand, areal loading rate as high as 250 kg of BOD/ha-day with detention time as low as 15 days has been found acceptable.

In some instances, the effluent from the facultative lagoon is subject to further treatment in a maturation pond. Since only a portion of the BOD applied is degraded then, the oxygen consumption rate is not proportional to the weight BOD applied but it is proportional to the weight of BOD removed in the facultative lagoon. The BOD that goes out in the effluent does not consume any oxygen. Hence, Eq. 5-1 could be modified as:

$$O_r = K_1 (C_o - C_e) q \qquad (5-5)$$

where C_o is the initial BOD concentration, mg/l; C_e is the effluent BOD concentration, mg/l; q is the wastewater flow rate into the facultative lagoon, l/day; and the other terms are as defined before. Equating Eq. 5-5 and 5-2 results to:

$$O_i = K_2 A_s = O_r = K_1 (C_o - C_e) q \qquad (5-6)$$

or

$$\frac{K_2}{K_1} = q \frac{(C_o - C_e)}{A_s} = A_r \qquad (5-7)$$

The term A_r is known as areal removal rates. Usually A_r is expressed in terms of kg of BOD removed per hectare per day.

Regression Equations – McGarry and Pescod (1970) collated the experimental and operational results of more than 200 facultative lagoon systems. They conducted regression analysis to determine the average operational performance and efficiency of the facultative pond system under various operating conditions. Fig. 5-3 shows the effect of temperature on the areal loading and pond performance. The scatter of data is very wide as the effects of dispersion or mixing are often unaccounted for.

The maximum values of the areal loading has been regressed as:

$$A_L = 60.26 (1.099)^T \qquad (5-8)$$

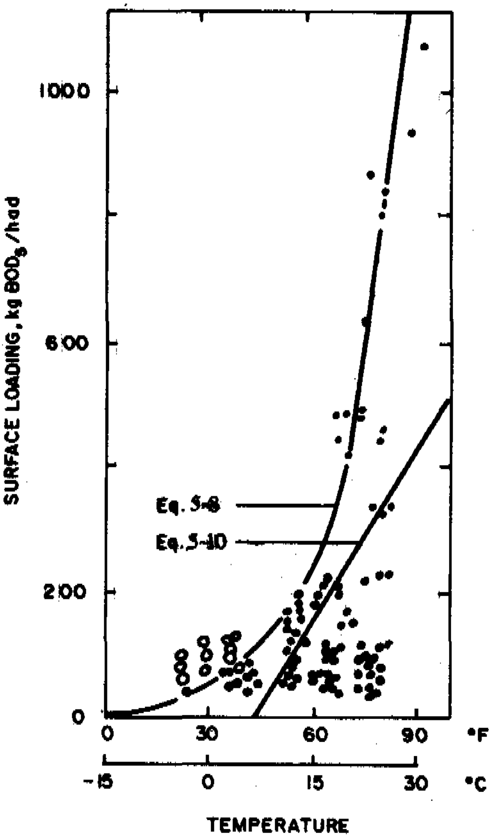


FIG. 5-3 Variation of maximum permissible and design loadings on facultative ponds with mean air temperature.

where the terms are as defined before. Mara (1976) suggested a more conservative formula which results to:

$$A_L = 40.35\,(1.099)^T \qquad (5-9)$$

Pescod and McGarry proposed a more conservative design criteria as:

$$A_L = 20\,T - 120 \qquad (5-10)$$

where the terms have been defined before. The original paper by Pescod and McGarry had temperature in degrees Fahrenheit. This equation has been recalculated in degrees Celsius for consistency.

The Indian Central Public Health Engineering Research Institute (now National Environmental Engineering Research Institute) (1976) recommended the following equation:

$$A_L = 375 - 6.15^{o}L \qquad (5-11)$$

where °L is the degree latitude. The lower the latitude of the facultative pond location the closer it is to the equator. At sea level, the lower the latitude the higher the ambient temperature is. Hence, Eq. 5-8, 5-9, 5-10 and 5-11 expressed the same condition.

However, temperature is also dependent on elevation and the surrounding environment. For example, there could be snow close to the equator as in Mt. Kilmanjaroo or the ocean currents could affect the ambient temperature.

McGarry and Pescod also conducted a regression analysis between the areal removal rate and the BOD loading rates at 30°C.They found that the areal removal rate A_r could be expressed as:

$$A_r = 0.725\,A_L + 10.75 \qquad (5-12)$$

Since by definition,

$$A_L = \frac{qC_o}{A_s} \qquad (5-13)$$

and

$$A_r = \frac{q(C_o - C_e)}{A_s} \qquad (5-14)$$

then introducing Eq. 5-13 and 5-14 to Eq. 5-12 yields

$$\frac{q(C_o - C_e)}{A_s} = 0.725\frac{q\,C_o}{A_s} + 10.75 \qquad (5-15)$$

or

$$\frac{C_o - C_e}{C_o} = 0.725 + \frac{10.75}{A_L}$$

From Eq. 5-15, the BOD removal efficiency of a facultative lagoon is limited to 73% when A_L exceeds 200 kg of BOD/ha-day. If the required wastewater treatment efficient is approximately 90%, then

$$A_L = \frac{10.75}{.90-.725} = 75.5 \text{ kg. of BOD/ha-day}$$

which coincides with the state regulation for facultative lagoon design in temperate climates.

Regression analysis is a powerful tool for establishing the quantitative relationship among the different parameters. However, regression analysis does not provide any insight into the mechanics involved in the degradation of organic wastes. The biggest problem with regression analysis is the blind acceptance of the results without evaluating the rate limiting conditions and other operational conditions. If the current practice is considered highly conservative, regression analysis can be viewed as giving the average conservative results rather than the possible optimal operating conditions. Optimal operating conditions could be attained only with an understanding of the mechanics involved in the waste stabilization.

Example 5-1. Domestic sewage with an initial BOD concentration of 250 mg/l and a flow rate of 10,000 m^3/day is treated in a facultative lagoon. The desired effluent quality is 25 mg/l. The ambient temperature is 30^{0}C.a) Calculate the area required by the facultative lagoon. b) If the facultative lagoon has a depth of 1.5 m, determine the detention time.

Solution:

i) The areal loading rate from Eq. 5-8 is equal to:

$$A_L = 40.26\,(1.099)^{30}$$
$$= 683 \text{ kg of BOD/ha-day}$$

ii) From Eq. 5-15, the effluent quality, C_e, is calculated.

$$\frac{C_e}{C_o} = 1 - .725 - 10.75/683$$
$$C_e = .26 \times 250 = 65 \text{ mg/l}$$

iii) The lagoon surface area is calculated using Eq. 5-13 as:

$$A_s = \frac{10{,}000 \text{ m}^3\text{/day} \times 250 \text{ mg/l} \times 10^{-3} \text{ (kg - l/mg-m}^3\text{)}}{683 \text{ kg/ha-day}}$$
$$A_s = 3.66 \text{ hectares}$$

Note: From Eq. 5-15 it would be impossible to attain the effluent quality of 25 mg/l in one pond unless the areal loading rate is reduced to very low levels. From Eq. 5-15, the areal loading rates required for one facultative pond is equal to:

$$\frac{250 - 25}{250} = .725 + \frac{10.75}{A_L}$$
$$A_L = 61 \text{ kg of BOD/ha-day}$$

Hence, the area required would be:

$$A_s = \frac{10{,}000 \text{ m}^3\text{/day} \times 250 \text{ mg/l} \times 10^{-3} \text{ (kg – l/mg-m3)}}{61 \text{ kg of BOD/ha-day}}$$

= 41 hectares if the facultative lagoon is designed as one pond

The area required is very large compared to the initial area given in step iii.

iv) For the second stage, the required areal loading is calculated from Eq. 5-15. The influent to the second stage is 66 mg/l. Hence.

$$\frac{66\text{-}25}{66} = .725 + \frac{10.75}{A_L} = .6$$

A_L =-103. A_L could be increased as high as possible without any effect on the final effluent quality. Hence, the areal loading rate defined by Eq. 5-8 governs.

The area of the second pond is equal to

$$A_s = \frac{66 \times 10{,}000 \times 10^{-3}}{683}$$
$$= 1 \text{ hectare}$$

use 100% factor of safety, then A_s = 2 hectares

v) The total facultative pond area is then equal to the sum of the area calculated in steps iii and iv.

A_s = 2 + 3.64 = 5.66 hectares.

vi) The effluent quality from the second pond is calculated using Eq. 5-15. This yields

$$\frac{66 - C_e}{66} = .725 + \frac{10.75}{683}$$

C_e = 17. mg/l $\ll$ 25 mg/l the required effluent quality

b) $V = Ah = 56600 \times 1.5 = 84{,}900 \text{ m}^3$
$\Theta = V/q = 84{,}900 / 10{,}000 = 8.49$ days

Experts such as Marais and Mara favor the use of the low areal loading rates defined by Eq. 5-9 or 5-10 in order to avoid anaerobic conditions in the first stage. However, the same experts recommend the use of the anaerobic/facultative combination. By using the higher areal loading rates, the facultative pond is merely converted to anaerobic/facultative pond combination, hence, the use of higher areal loading rates should not be discouraged.

Biochemical Kinetics Equations – If the bacterial population is very dilute and oxygen, nitrogen, phosporous, and other nutrient concentrations are non-rate limiting, the rate of organic waste stabilization could be approximated by a first order biochemical reaction. The reduction of the organic waste concentration expressed as BOD is defined as:

$$\frac{dC}{dt} = -kC \tag{5 – 16}$$

where C is the BOD concentration, mg/l; k is the degradation constant, day^{-1}, and t is time, in days. The degradation rate constant is approximately equal to 0.2 to 0.4/day at 20°C for domestic sewage. As discussed in Chapter III, k is calculated based on the assumption of complete mixing in the facultative pond. Complete mixing is an idealized condition which can be attained only when the mixer power approaches infinity. In practice, the wastewater mixing pattern in the facultative lagoon lies between completely mixed and plug flow conditions. As illustrated in Chapter III, the calculation of the k value from the influent and effluent BOD concentration of a facultative pond is highly dependent on the dispersion number. For this reason there is a wide variability of k values reported in literature for the facultative lagoons treating the same type of wastewater under similar operating conditions.

For mesophilic microorganisms, the enzymatic activity increases with increasing temperatures. As the rate of degradation k is proportional to the microbial activity, then k should also increase with temperature. Marais (1974) suggested the following equations:

$$k_T = k_{20}\,(1.05)^{T-20} \tag{5 – 17}$$

The value of k_{20} varies from 0.2 to 0.4 for domestic sewage with 0.3/day as average. In a completely mixed system, the mass balance of the incoming, outgoing, degraded and accumulated organic wastes in a facultative lagoon could be expressed as:

$$V\frac{dC}{dt} = q\,C_o - Cq - k\,C \tag{5 – 18}$$

At steady state, dC/dt is equal to zero. Hence,

$$C_e = C_o/(1 + kV/q) \tag{5 – 19}$$

If the facultative lagoon is divided into n- facultative lagoon of equal size operated in series, the effluent BOD, C_e, could be expressed as:

$$C_e = \frac{C_o}{\left(1+\frac{kV}{nq}\right)^n} \tag{5 – 20}$$

The principles for deriving Eq. 5-19 and 5-20 were discussed in Chapter III. For a first order biochemical reaction, the total lagoon volume required to remove a fixed quantity of BOD decreases as n increases. The economic comparison between the cost of building the lagoon dividers or dikes and the savings made from using smaller facultative lagoon in series dictates the choice of n. In practice, the facultative lagoon system is constructed as a series of three to five lagoons of equal sizes.

The areal loading rates in the first stage facultative pond could exceed the allowable limits. Hence, it is not advisable to divide the facultative pond into more than six stages by using Eq. 5-20 without checking the areal removal rates as well. This principle is best illustrated below.

Example 5-2. For domestic sewage,k has a value of 0.3/day at 20°C. Determine the area required for a facultative lagoon 1.5 m deep used to treat the wastewater whose characteristics are given in Example 5-1. The facultative lagoon is divided into seven stages.

Solution:

a) At 30°C,

$$k = 0.3\,(1.05)^{30-20} = .49/\text{day}$$

For $C_0 = 250$ mg/l, $C_e = 25$ mg/l, n = 7 and q of 10,000 m^3/day, then from Eq. 5-20,

$$V = \frac{nq}{k}\left[\left(\frac{C_o}{C_e}\right)^{1/n} - 1\right]$$

or

$$V = \frac{7 \times 10{,}000}{.49}\left[\left(\frac{250}{25}\right)^{1/7} - 1\right]$$

$$= 55{,}642\ \text{m}^3$$

ii) If the facultative lagoon is 1.5m deep, then the area per stage is equal to:

$$A = V/1.5\,n = 55{,}642/(7 \times 1.5) = 5300\ \text{m}^2$$

The volume per stage, V', is calculated as

$$V' = V/n = 55{,}642\ \text{m}^3/7 = 8000\ \text{m}^3$$

iii) The effluent and influent at each stage is calculated using Eq. 5-19. For the first facultative pond the effluent quality

$$C_{e,1} = 250/(1 + 8000 \times .49/10000)$$
$$= 179\ \text{mg/l}$$

The effluent from the first facultative pond $C_{e,1}$ is the influent to the second facultative pond. Hence, the effluent of the second pond is calculated as:

$$C_{e,2} = C_{e,1}/(1 + .49 \times 8000/10{,}000)$$

$$= 179/1.392 = 129 \text{ mg/l}$$

Similarly, the effluent from the third, fourth, fifth, sixth and seventh ponds are calculated. The effluent from the third facultative pond is 92.7 mg/1; the fourth, 66.6 mg/l; the fifth, 47.8 mg/1; and the sixth 34.36 mg/1. The effluent of the whole system is the effluent from the seventh pond which is equal to 24.7 mg/l.

iv) The areal loading rate at each stage is calculated, which yields,

$$A_{L,1} = \frac{10{,}000 \times 250 \times 10^{-3}}{.5300} = 4716 \text{ kg of BOD/ha-day}$$

$$A_{L,2} = \frac{179 \times 10{,}000 \times 10^{-3}}{.5300} = 3377 \text{ kg of BOD/ha-day}$$

$$A_{L,3} = \frac{129 \times 10{,}000 \times 10^{-3}}{.5300} = 2433 \text{ kg of BOD/ha-day}$$

$$A_{L,4} = \frac{92.7 \times 10{,}000 \times 10^{-3}}{.5300} = 1749 \text{ kg of BOD/ ha-day}$$

$$A_{L,5} = \frac{66.6 \times 10{,}000 \times 10^{-3}}{.5300} = 1256 \text{ kg of BOD/ ha-day}$$

$$A_{L,6} = \frac{47.8 \times 10{,}000 \times 10^{-3}}{.5300} = 901 \text{ kg of BOD/ ha-day}$$

$$A_{L,7} = \frac{34.36 \times 10{,}000 \times 10^{-3}}{.5300} = 648 \text{ kg of BOD/ ha-day}$$

From the areal loading rates, the six ponds will become anaerobic as the BOD applied is very much higher than the allowable limits normally used for domestic sewage at 30°C. Eq. 5-12 is only used for aerobic decomposition. For anaerobic decomposition, k has a different value and Eq. 5-20 does not hold true.

v) A reasonable design should divide the facultative pond into three ponds of equal sizes operated in series. Hence, from Eq. 5-20,

$$V = \frac{10{,}000 \times 3}{.49}\left[(10)^{1/3} - 1\right]$$

$$= 70{,}750.00 \text{ m}^3$$

For a facultative pond, 1.5m deep the area per stage is equal to:

$$A = 70{,}700/(1.5 \times 3) = 15{,}711 \text{ m}^2$$

The effluent from the first stage is equal to

$$C_{e,1} = 250/(1 + .49 \times 1.5 \times 15711/10{,}000)$$

$$= 116 \text{ mg/l}$$

$$C_{e,2} = 116/2.16 = 53.7 \text{ mg/l}$$

The areal loading rate for the first pond is equal to:

$$A_{L,1} = \frac{250 \times 10{,}000 \times 10^{-3}}{1.57} = 1667 \text{ kg of BOD/ ha-day}$$

For the second pond it is equal to:

$$A_{L,2} = \frac{116 \times 10{,}000 \times 10^{-3}}{1.57} = 731 \text{ kg of BOD/ ha-day}$$

which is already within the limits defined by Eq. 5-9. The areal loading rates for the third pond is equal to:

$$A_{L,3} = \frac{53.7 \times 10{,}000 \times 10^{-3}}{1.57} = 342 \text{ kg of BOD/ha-day}$$

Hence, the first facultative pond could become anaerobic while the succeeding two ponds will be operated as a facultative pond.

The areal loading rate is currently used to define the possible overloading and septicity of facultative ponds. The areal loading rate is valid only if the effluent quality is very low and the facultative pond system is designed as one big pond. If the pond is subdivided into stages to prevent short circuiting, the BOD removal efficiency at each stage could be as low as 10% Hence, only 10% of the applied BOD really consumes any oxygenThe areal removal rate is a more reasonable criterion than areal loading rate for defining the behavior of facultative pond under this condition. From Eq. 5-7, the areal removal rate is a balance between the oxygen supply and the oxygen required for BOD degradation.

> Examples 5-3. Compare the areal loading rates and areal removal rates for the three stage facultative pond whose characteristics are given in Example 5-2.
>
> Solution: For the first pond the areal removal rate is equal to:
>
> $$A_r = \frac{10{,}000 \times 10^{-3}\ (250 - 116)}{1.57} = 853 \text{ kg of BOD/ha-day}$$
>
> For the second stage it is equal to:
>
> $$A_r = \frac{10{,}000 \times 10^{-3}\ (116 - 53.7)}{1.57} = 397 \text{ kg of BOD/ha-day}$$
>
> For the third stage it is equal to:
>
> $$A_r = \frac{(53.7)\,(10{,}000)\,(10^{-3})}{1.57} = 182 \text{ kg of BOD/ha-day}$$
>
> If the oxygen supply is approximately equal to 683 kg of O_2 /ha-day, then the first facultative pond will be slightly septic only. The ratio of the areal loading rate to the areal removal rate is almost twice that in the first stage facultative pond.

The BOD removal $C_o - C_e$ could be expressed as:

$$C_o - C_e = C_o - \frac{C_o}{1 + k\,V/q} = \frac{kV/q}{1 + kV/q}\ C_o \qquad (5-21)$$

using Eq. 5-19. Multiplying both sides of Eq. 5-21 by q/A_S yields:

$$\frac{(C_o - C_e)\,q}{A_s} = \frac{C_o q}{A_s}\ \frac{k\,V/q}{1 + k\,V/q} \qquad (5-22)$$

Since,

$$\frac{kV/q}{1 + k\,V/q} = 1 - \left(\frac{q}{kV}\right) + \left(\frac{q}{kV}\right)^2 - \left(\frac{q}{kV}\right)^3 + \left(\frac{q}{kV}\right)^4 \qquad (5-23)$$

for small values of q/kV, Eq. 5-22 could be approximated as:

$$A_r = A_L\left[1-\left(\frac{q}{kV}\right)\right]+\epsilon \qquad (5-24)$$

Normally at 30°C k is equal to 0.49/day and V/q is 10 days. Hence, Eq. 5-24 could be expressed as:

$$A_r = A_L\left[1 - \frac{1}{.49 \times 10}\right] = .80\,A_L + \epsilon \qquad (5-25)$$

The term ϵ is the residual or sum of experimental errors. Eq. 5-25 is very similar to the regression curve given in Eq. 5-12. If A_L the area loading rate is in the range of 200 – 300 BOD/ha-day, the contribution of ϵ to A_r is very low which is normally in the range of 5 to 12 kg of BOD/ ha-day. For all purposes, ϵ could be neglected from Eq. 5-25 except when A_L is less than 40 kg of BOD/ ha-day.

A common design practice today is to calculate the theoretical facultative pond area based on the wastewater flow rate and quality, and then adopt a bigger area, which is normally twice the theoretical value, for construction purposes. Most often this results in oversizing, even if the areal loading concept is based on the balance between oxygen supply and oxygen consumption in the wastewater treatment system. Eq. 5-25 shows that the BOD is the rate limiting constituent for field operation instead of oxygen. Of significance is the failure point of Eq. 5-25 when oxygen becomes the rate limiting reactant in the wastewater treatment process. The break point of Eq. 5-25 determines the maximum possible areal loading rate in the facultative pond.

Effects of Dispersion – As discussed in Chapter III, the organic pollutant concentration along a wastewater treatment process is defined by the partial differential equation:

$$\frac{\partial C}{\partial t} = \frac{D_L\,\partial^2 C}{\bar{V}L\,\partial X^{*2}} - \frac{\partial C}{\partial X^*} - kC\Theta \qquad (5-26)$$

where the terms have been defined before. Eq. 5-26 is solved using the Wilner-Wilhelm boundary conditions which yields

$$\frac{C_e}{C_o} = \frac{4a\exp(0.5/N_D)}{(1+a)^2\exp(a/2N_D) - (1-a)^2\exp(-a/2N_D)}$$

where

$$a = \sqrt{1 + 4K\Theta N_D}$$

and $N_D = D_L/\bar{V}L$. If the facultative pond is divided into five or more equal ponds operated in series, the value of $D_L/\bar{V}L$ could be approximated by the following formula:

$$\frac{D_L}{\bar{V}L} \approx .5/n \tag{5-27}$$

The parameter n is the number of facultative ponds operated in series comprising the whole facultative pond system. The graph of Eq. 5-27 is shown in Fig. 1-2.

Example 5-4. Determine the area required for a facultative pond used to treat domestic sewage with a k value of 0.49/day, flow rate of the 10,000 m^3/day and influent BOD concentration of 250 mg/1. The facultative pond is to be divided into seven equal ponds operated in series. The pond depth is 1.5 m, the effluent BOD required is 25 mg/1,

Solution:

From Eq. 5-27

$$\frac{D_L}{\bar{V}L} = .5/7 = .071$$

The percentage of BOD remaining in the effluent is equal to 25/250 x 100 or 10%. From Fig. 1-2 the $k\Theta$ value defined by $D_L/\bar{V}L$ of 0.071 and 10% BOD remaining in the effluent is 2.75. Hence,

$$\Theta = 2.75/k = 2.75/.49 = 5.6 \text{ days}$$

The volume of the facultative pond system is

$$V = \Theta q = 5.6 \text{ days x } 10{,}000 \text{ m}^3/\text{day}$$
$$= 56{,}000 \text{ m}^3$$

The total area is equal to

$$A = V/\text{depth} = 56{,}000/1.5 = 37{,}400 \text{ m}^3$$

or 3.74 hectares.

It should be noted that the area and volume required are very close to those calculated using Eq. 5-21 as given in Example 5-1.

As discussed in Chapter III, complete mixing is an idealized condition which is not expected to occur in practice. A facultative pond divided into three smaller ponds operated in series could behave as if it were divided into five or more ponds operated in series depending on the wind direction, the geometric configuration of the pond and the wastewater flow rate.

Mechanistic Models – Eq. 5-1 is a very simplified mathematical expression for quantifying the amount of oxygen transferred to the facultative pond from the air and of oxygen produced from algal photosynthesis. The wind action creates waves on the facultative pond surface thereby increasing the effective surface area for oxygen mass transfer to ten times the area calculated by merely multiplying the top length and width of the pond. Oxygen production from algal photosynthesis is not consistent throughout the day.

The mechanistic model tends to study the different physical and biochemical mechanisms responsible for the degradation of organic pollutants in wastewaters. The rate controlling mechanism is evaluated and its effect on the total wastewater treatment performance is quantified. However, the rate controlling mechanism may vary at different wastewater treatment stages. For example, in the influent section of the wastewater treatment plant, the rate controlling mechanism could be the oxygen supply while at the effluent end it could be the supply of organic wastes as food to the microbial population.

The development of the areal loading concept is based on the rate controlling effect of oxygen supply on the facultative pond system performance. The quantity of oxygen produced from algal photosynthesis is dependent on sunlight energy passing through the facultative pond surface area. Similarly, the oxygen diffusion from the air is dependent on the pond surface area and the wind characteristics. Hence if the oxygen supply per unit area is rate controlling, then the organic waste introduction into the wastewater treatment system is regulated such that the system will not turn anaerobic.

As discussed in Chapter III, the first order BOD reduction equation is based on the condition that nutrients, oxygen and active bacterial mass are available in excess. The organic pollutant concentration must be the rate controlling parameter for the first order BOD reduction equation to be applicable. The reader should note from Examples 5-2 and 5-3 the failure of the first order kinetic equation to quantify the performance at the influent sections of the facultative pond operated in series. Similarly, from Example 5-1, the areal loading concept failed to quantify the behavior of the second stage facultative pond. This demonstrates the importance of establishing the applicability of the different design equations to the facultative pond operating environment.

Photosynthetic Oxygen – Algae convert 6% of the solar energy to microbial cells. For each kilogram of algal mass, 6 million calories of solar energy is required and 1.6 kg of oxygen is produced by photosynthesis. Hence, the quantity of oxygen produced per day in one hectare of land could be expressed as:

$$O_1 = \frac{\lambda_s \ \mathrm{x}\ 0.06 \ \mathrm{x}\ 1.6 \ \mathrm{x}\ 10^8 \ \text{cal/ha-day}}{6 \ \mathrm{x}\ 10^6 \ \text{cal/kg}} \qquad (5-28)$$

where λ_s is the solar energy expressed in langleys or cal/cm^2-day. Eq. 5-28 could be simplified as:

$$O_i = 1.6\,\lambda_s \qquad (5-29)$$

Table 5-1 shows the solar energy available at different times of the year at different latitudes. The minimum and maximum available amounts of solar energy take place at the north pole during winter and summer, respectively. During the polar summer, the sun never sets, hence, solar energy is available for almost 24 hours. Conversely in winter the sun may never rise. At the equator, the solar energy is distributed uniformly throughout the year.

A portion of the photosynthetic oxygen produced by algal cells is released to the atmosphere. Wind has positive and negative effects on the availability of the photosynthetic oxygen for bacterial activity. Wind action mixes the pond content which results in a more uniform distribution of dissolved oxygen throughout the pond depth. On the other hand, it also creates surface turbulence which increases oxygen dissipation to the atmosphere from the region with oxygen supersaturation.

In a very quiescent pond, at 3:00 PM, the dissolved oxygen in the pond surface could be as high as 30 mg/l with the pond bottom remaining anaerobic. Any disturbance on the pond surface which is supersaturated with oxygen will cause the release of oxygen to the atmosphere. To account for such oxygen release, Eq. 5-29 is modified further to:

$$O_i = 1.6\,\lambda_s \alpha \qquad (5-30)$$

where α is the utilization efficiency and the other terms are as defined before. The value of α could vary from 55% to 100% for Indian conditions and from 27% to 36% for American conditions. The current Indian practice is to utilize areal loading rates of 150 to 250 kg of BOD/ha-day while the American practice limits the value to 100 kg of BOD/ha-day. With very low areal loading rates there is a higher tendency for supersaturation to take place even if the pond content is continuously mixed. Therefore, the oxygen lost to the atmosphere is higher at lower areal loading rates.

In temperate climates, the facultative ponds are designed for winter conditions when the bacterial metabolism and algal photosynthesis are at their minimum. The wastewater detention time during this period is higher than in summer. During the summer months the facultative lagoons are grossly overdesigned which results in a very low algal photosynthetic oxygen utilization as described above,

If the facultative pond has a total surface area, A_S, then the quantity of oxygen produced per day is equal to:

$$O_i = 1.6\,\lambda_s A_s \alpha \qquad (5-31)$$

The maximum oxygen required to stabilize the organic pollutant is equi-

Table 5-1 PROBABLE VALUES OF VISIBLE SOLAR ENERGY AS A FUNCTION OF LATITUDE AND MONTH (METCALF AND EDDY, 19)

Latitude deg N or S	Month											
	Jan	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
0 max	225*	266	271	266	249	236	238	252	269	265	256	253
min	210	219	206	188	182	103	137	167	207	203	202	195
10 max	223	244	264	271	270	262	265	266	266	248	228	225
min	179	184	193	183	192	129	158	176	196	181	176	162
20 max	183	213	246	271	284	284	282	272	252	224	190	182
min	134	140	168	170	194	148	172	177	176	150	138	120
30 max	136	176	218	261	290	296	289	271	231	192	148	126
min	76	96	134	151	184	163	178	166	147	113	90	70
40 max	80	130	181	181	286	298	288	258	203	152	95	66
min	30	53	95	125	162	173	172	147	112	72	42	24
50 max	28	70	141	210	271	297	280	236	166	100	40	26
min	10	19	58	97	144	176	155	125	73	40	15	7
60 max	7	32	107	176	249	294	268	205	126	43	10	5
min	2	4	33	79	132	174	144	100	38	26	3	1

* Values of solar radiation λ in cal/cm^2/day.

To determine average value of λ, $\lambda_{av} = \lambda_{min} + P(\lambda_{max} - \lambda_{min})$ in which p is total hours of sunshine divided by total possible hours of sunshine.

valent to its ultimate BOD which is one and a half times the five-day BOD. The weight of ultimate BOD introduced into the facultative pond per day could be expressed as:

$$O_r = q\,C_o\,(1.5) \times 10^{-3}\,\text{kg-l/(mg-m}^3) \qquad (5-32)$$

where C_o is the organic waste concentration in terms of the five-day BOD. Equating Eqs. 5-31 and 5-32, yields:

$$1.6\lambda_s \alpha A_s = q\,C_o\,(1.5) \times 10^{-3}$$

or

$$\frac{10^{-3}\,q\,C_o}{A_s} = 1.07\lambda_s = A_L \qquad (5-33)$$

The right hand term of Eq. 5-33 is the areal loading rate.

The solar radiation available in a facultative pond is dependent on the cloud cover which may reduce the total number of hours of sunshine as compared to the total possible number of sunshine hours. The average solar radiation, λ_s, is calculated as:

$$\lambda_s = \lambda_{min} + p\,(\lambda_{max} - \lambda_{min}) \qquad (5-34)$$

where p is the fraction of sunshine hours compared to the total number of possible hours of sunshine

Example 5-5. Calculate the areal loading rates possible in the equator if the cloud cover reduces the sunshine hours to 80% of the total possible hours of sunshine.

Solution: a) From Table 5-1, the minimum solar radiation, λ_s, takes place in June when it is equal to 103 cal/cm^2 – day.

The average solar radiation is then equal to:

$$\begin{aligned} &= 103 + 0.8\,(236 - 103) \\ &= 209\ \text{cal/cm}^2 - \text{day} \end{aligned}$$

Substituting the above value in Eq. 5-33, yields:

$$A_L = 209 \times (1.07) = 223\ \text{kg of BOD/ha-day}$$

b) The highest possible loading rate takes place in January when the solar radiation is λ maximum at 225 cal/cm^2-min. Substituting this value to Eq. 5-33 to the average solar radiation.

$$\lambda_s = 210 + 0.8\,(225 - 210)$$

$\lambda_s = 222$

$A_L = 222 \times 1.07 = 237$ kg of BOD/ha-day

The results obtained have to be multiplied by the oxygen utilization efficiency. At 220 kg of BOD/ha-day, the value of α could be close to one.

Photosynthesis tends to reduce the carbonate, bicarbonate and carbon dioxide concentration in the wastewater. Carbon dioxide and water are converted to sugar and oxygen by algal cells during photosynthesis. The reduction of the carbon dioxide and carbonate ions results in an increase in the wastewater pH which could go up as high as 11 at the height of photosynthetic activity. A pH above 9 tends to inhibit bacterial activity, reducing further the carbon dioxide supply. The reduction of bacterial activity also reduces the BOD degradation rates. Hence, it is important to provide sufficient carbon dioxide in the facultative pond to maintain stability and this is attained by minimizing overdesign condition.

For very large overdesign, McGauhey (1968) observed a reduction of the BOD removal efficiency to 68% from a peak of 93% at optimal condition. The reduction is primarily caused by the increased wastewater pH. On the other hand, oxygen deficiency tends to retard the aerobic bacterial activity. Hence, it is important to balance the rate of oxygen supply and utilization carefully.

Wind Induced Oxygen Transfer – The oxygen transfer from the atmosphere to the facultative pond is enhanced by the turbulence on the pond surface created by the wind. As the wind moves along the pond surface, its energy is transferred to the water thereby creating waves. The wind waves increase the water surface area for oxygen diffusion and the mixing effect facilitates the deoxygenation process. This results in very low photosynthetic oxygen utilization if supersaturation exists as discussed before.

Contact time between the wind and the water surface should be sufficiently long enough before the waves generated are able to induce oxygen transfer from the atmosphere. In small pilot plants wherein the facultative lagoons may have an area less than 1000 m^2, the contact time between the wind and water surface is too short to induce sufficient oxygen mass transfer. For this reason, the effect of winds on facultative lagoon performance is subject to controversy.

Measurement by Oswald and Gotaas (1957) showed that algal photosynthesis is the major oxygen source in the facultative pond. The wind induced oxygen transfer accounts for less than 5% of the oxygen from algal photosynthesis. On the other hand, experiments by Marais (1970) showed that wind induced oxygen transfer to the facultative pond exerts significant influence on the system performance. From Example 5-4, the maximum areal loading rate possible in the equator is only 220 kg of BOD/ha-day on an assumption of 100% utilization of the algal photosynthetic oxygen. Fig. 5-3, shows that facultative pond areal loading rate in hot climates could go up to 1000 kg of BOD/ha-day before the pond turns anaerobic. The remaining 800 kg of BOD/ha-day is satisfied by oxygen transferred from the atmosphere to the facultative pond by the wind.

Wind induced oxygen transfer is dependent on the contact distance between the wind and the water surface known as "fetch", the wind velocity, the wastewater viscosity, density and surface area.

The wind waves in very large facultative lagoons have characteristics similar to wind waves in reservoirs. The relationship on the height, H_s, the fetch, F, wind velocity, $\overline{V}$, and the gravitational acceleration, g, is expressed by the following equation:

$$\frac{g H_s}{\overline{V}^2} = 0.0026 \left(\frac{gF}{\overline{V}^2} \right)^{0.47} \qquad (5-35)$$

The wave height H_s is the average of the highest one third of the waves in a particular series. The wave period, T, could be determined from the equation

$$\left(\frac{g T}{\overline{V}} \right) = 0.45 \left(\frac{g F}{\overline{V}^2} \right)^{0.28} \qquad (5-36)$$

The wind velocity $\overline{V}$ is the velocity above the water surface and could be greater than the wind velocity measured in land stations. The wind velocity over the water surface is approximately 10% higher than the wind velocity measured in land stations.

The wave length, L, in m is equal to:

$$L = 1.56\, T^2 \qquad (5-37)$$

where T is defined by Eq. 5-36.

Quano and Kumar (1981) conducted a wind tunnel experiment to measure the effect of the wind on the oxygen mass transfer rate. They found the oxygen mass transfer coefficient to increase in proportion to the square root of the wind velocity. For water at 30°C, the relationship between the oxygen mass transfer coefficient $K_L a$ and the velocity could be expressed as:

$$\frac{K_L a\ d\rho}{\mu} = a_1 \left(\frac{\overline{V} d \rho}{\mu} \right)^{0.5} + a_2 \qquad (5-38)$$

where d is the depth of the pond, μ, is the viscosity of the wastewater, ρ is the density of wastewater, a_1 and a_2 are constants. At wind velocities of 20 km/hr, the oxygen diffussion from the atmosphere to a facultative pond with near zero dissolved oxygen concentration is almost equal to the oxygen produced from photosynthesis. The oxygen transfer from the wind is important during night time when algal respiration competes with the bacteria for the dissolved oxygen.

Wind mixing -- Wind induces mixing of the facultative lagoon content. If the lagoon is oriented along the direction of the wind, the water molecules are transported at a faster rate to the outlet resulting to short circuiting. The wave velocity should not be used as a measurement for short circuiting since it is very much higher than the velocity of the water molecules.This phenomenon could be verified by putting a piece of paperon a lagoon. The water wave generated by a piece of rock thrown close to the piece of paper could cover the whole pond in less than 1 minute while the piece of paper merely moves in the vertical direction with the passing water waves.

On the other hand if the wind direction is perpendicular to the facultative pond length, the wind fetch will be very short, resulting in a very poor oxygen transfer efficiency from the atmosphere. Since the wind direction often shifts by 180° with the season it would be difficult to design a facultative pond wherein the wind velocity is always against the direction of the water flow from the inlet to the outlet section.

Anaerobic Degradation -- In a facultative pond the settleable solids, dead algal and bacterial cells are decomposed anaerobically at the bottom of the pond. The anaerobic degradation releases volatile fatty acids, methane, carbon dioxide and hydrogen sulphide. Carbon dioxide and hydrogen sulphide are trapped in the slightly akaline upper layers of the pond in the form of carbonates and sulphides. Thecarbonates are later on converted to carbohydrates and oxygen by algal cells in the presence of sunlight. The methane gas is released tc the atmosphere. The higher the areal loading rate the thicker is the anaerobic layer in comparison with the aerobic layer. Unless the mixing of the facultative pond content is very high the bottom of the pond will remain anaerobic all the time.

Siddiqui (1974) measured the methane gas released from facultative pond to determine the extent of anaerobic degradation of the organic wastes in the facultative pond. At areal loading rates of 767 kg of BOD/ha-day, the BOD removal efficiency was 78% out of which 65% was due to anaerobic activity. Due to the alkaline condition at the top layers of the facultative pond the malodorous hydrogen sulphide gases are trapped preventing serious nuisance problem associated with anaerobic decomposition. At areal loading rate of 70 kg/ha-day the BOD removal is entirely due to aerobic degradation. Fig. 5-4 shows the contribution of anaerobic reaction to the overall BOD degradation at various areal loading rates.

Closure -- Current design practice for facuitative pond is very conservative in that anaerobic degradation of organic waste is completely neglected. As shown by Siddiqui, in warm climates the BOD removal in the anaerobic zone could be much higher than in the aerobic zone. Even in the utilization of the aerobic zone, the effects of wind induced oxygen transfer is neglected. Due to the factor of safety introduced in the design, the facultative ponds are operated under the condition when BOD rather than oxygen is the rate limiting reactant.

Further research on the BOD stabilization mechanism may result in reduction of the land area required for facultative pond.

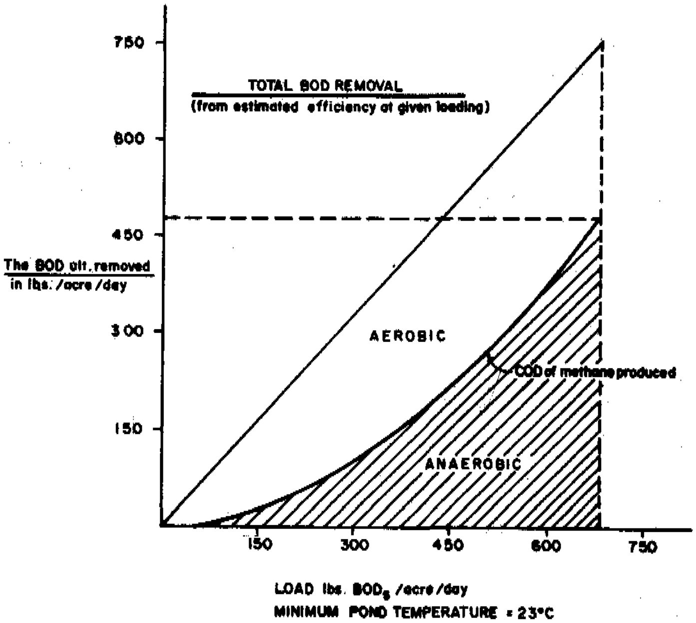


FIG. 5-4 Proportion of BOD removed anaerobically at different pond loadings.

Anaerobic Lagoon Design

Parker (1962) discovered that a facultative lagoon system overloaded to septicity would not release any foul odor as long as the volumetric loading was controlled in such a way that hydrogen sulphide concentration in the wastewater remained within its solubility limit. Mara (1976) suggested an areal loading rate of 400 gm of BOD/m^3-day or lower as ideal volumetric loading before hydrogen sulphide is released.

Anaerobic digestion is highly sensitive to pH and has to be carried out within a pH range of 6.0 to 8.0. At this level, the bicarbonates form the principal buffering species in water. Hence, for industrial wastes the quantity of hydrogen sulphide that exists in solution could be calculated from the solubility data of hydrogen sulphide in the operating pH range.

BOD removal— There has been very little quantification of the BOD removal rates in anaerobic lagoons. For sewage Mara (1976) recommended the following values of removal efficiency at various detention times.

TABLE 5-2 Removal Efficiency of Anaerobic Lagoons Treating Sewage

Detention Time	% Removal Efficiency
1	50
2.5	60
5.0	70

He further recommended a detention time of 5 days as a suitable compromise between overloading the pond which creates odor and underloading the pond which turns it facultative. If a first order kinetic equation is used, it should be noted that the first day removal efficiency conforms to a k value of 0.8/day which is almost twice the corresponding removal efficiency for a facultative lagoon system. In an anaerobic pond system a large portion of the BOD removal is due to sedimentation and scum formation. The scum provides a seal which, to some extent, controls the odor and at the same time prevents aeration of the anaerobic liquor.

When it comes to industrial wastewater, pilot plant testing has to be carried out. For strong wastes, the pond has to be acclimitized to establish a balance between the methane formers and acid formers. The acid formers are more hardy and reproduce faster than the methane formers. If a strong waste is introduced as a shock load into the anaerobic pond, the pH will drop due to the faster production of organic acids by acid formers. The methane formers are sensitive to pH change. They stop the conversion or destruction of organic acid to methane as soon as the pH drops below 6. This causes a further build up of acid until the acid formers themselves are inhibited at a pH below 4.0 during which the anaerobic digestion completely stops. Although anaerobic digestion does not require as much nutrients as aerobic digestion, in wastes with high BOD_5 and low nutrient concentrations, the acid formers tend to utilize most of the nutrients. Due to insufficient amount of nutrient, the growth and activity of the methane formers are inhibited, which could result in a lowering of the pH and the failure of the treatment system as discussed before. It is therefore a good practice to maintain a nutrient level that is at least 50% of the required level for aerobic treatment processes.

Once the pond has become acidic, alkali substances like lime, soda or soda ash can be added to raise the pH provided the alkali substance concentration does not reach the point of inhibiting the growth and activity of methane formers. If this happens, the pH will increase only temporarily after addition of the alkaline substance. The pH will drop again as the acid formers produce acids at a faster rate than the destruction of the volatile fatty acids to methane by the methane forming bacteria.

The effluent from anaerobic pond is often yellowish to dark brown in color with black suspended solids. Facultative lagoon is required to polish the effluent and maintain the level of the BOD_5 to desirable standards.

Desludging – Desludging is more frequent in an anaerobic pond than a facultative pond although the former is designed with a depth two to four times that of the latter. As explained before, the BOD removal in the system is primarily due to sedimentation and floatation of the solids. Anaerobic digestion of sludge is often limited to 60% of the total organic content due to the exhaustion of oxygen-containing substances in the sludge.

For sewage, Mara reports a range of 0.03 to 0.04 m^3/ person-year of sludge accumulation rate in anaerobic pond. For industrial wastes 20-30% of influent BOD_5 ends as sludge in the anaerobic pond. This mean that if the anaerobic pond removes 70% of the influent BOD, 50-40% of the BOD removed is converted to carbon dioxide and methane gas. The remaining 40-60% of the BOD removed is converted to sludge. The solid content of the settled sludge varies from 6 to 10% due to compaction.

Desludging is carried out when the pond is half filled with sludge. Hence, the time between desludging could be calculated as:

$$\Theta = \frac{0.5 \text{ x pond volume}}{\text{sludge accumulation rate x population}} \qquad (5-39)$$

where Θ is the desludging period in years. Eq. 5-39 differs from Mara (1976) equation in that the volume is divided by the sludge accumulated rate instead of being multiplied.

Although desludging seems easy, in practice drying the pond completely is better than attempting to remove the slurry. Draglines have been used for desludging but the operation is messy and inefficient. Bucket lines could be used manually but again it is very inefficient. Since for every kilogram of solids, another 9-9.5 kg of water has to be removed then to prevent the slurry from flowing a holding tank as big as the anaerobic lagoon is required. Thus desludging one hectare of anaerobic pond is like cleaning 5,000 septic tanks all at once.

Example 5-6. Design an anaerobic pond to treat domestic sewage with an initial BOD concentration of 250 mg/1 and a flow rate of 10,000 m^3/ day.

Solution:

i) Use detention time of 5 days. From Table 5-2, the BOD removal efficiency is 70%. Hence, the effluent quality is equal to 75 mg/1.

ii) The volume of the pond is

$$V = 5 \text{ days x } 10{,}000 \text{ m}^3/\text{day} = 50{,}000 \text{ m}^3$$

iii) The weight of BOD is calculated as:

$$\text{wt of BOD} = 250 \text{ mg/1 x } 10{,}000 \text{ x } 10^{-3} \text{ kg-1/m}^3 - \text{mg}$$

$$= 2500 \text{ kg/day}$$

Volume required to prevent hydrogen sulphide generation

$$V = \frac{2500 \text{ kg/day x } 1000 \text{ gm/kg}}{400 \text{ gm/m}^3 - \text{day}}$$

$$= 6250 \text{ m}^3 \ll 50{,}000 \text{ m}^3$$

Hence, malodorous generation of hydrogen sulphide will be minimal

iv) The weight of BOD removed is:

= 2500 x (0.7) = 1750 kg/day

The weight of sludge produced is approximately equal to 1750 x 0.6 = 1050 kg/day. The sludge is expected to undergo further anaerobic digestion. Hence, the residual sludge is estimated at 1050 x 0.4 = 420 kg/day.

At 6% solids concentration the sludge accumulation rate is 420 kg/day x 1 /kg/0.06 or 7000 1/day or 7 m^3 /day

v) The anaerobic lagoon will require desludging once in every 9.78 years as computed from;

$$\Theta = \frac{0.5 \times 50{,}000 \text{ m}^3}{7.0 \text{ m}^3/\text{ day} \times 365 \text{ days/year}}$$

Θ = 9.78 years

vi) The desludging rate could be estimated by dividing the flow rate of 10,000 m^3/day by average per capita wastewater generation rate of 150 lpcd The population is estimated at (10,000 m^3 /day x 1000 1 /m^3)/150 lpcd or 67,000 persons.

$$\Theta = \frac{0.5 \times 50{,}000 \text{ m}^3}{0.04 \text{ m}^3/\text{yr} - \text{person} \times 67{,}000 \text{ persons}}$$

= 9.3 years

Design of Aerobic Ponds

Aerobic ponds are designed with very shallow depths ranging from 0.2 to 0.6 m, to maximize the algal growth. At this depth pond lining will be necessary as vegetation tends to grow at the bottom of the pond.

Mixing may be done continuously to maintain the dissolved oxygen at a uniform level and to bring the motile algae to the surface for exposure to sunlight. The ponds normally have a detention time of 0.8 to 2 days although one day is a common value in the tropics. Pond areal loading rate of up to 450 kg/ha-day has been used in the tropics.

In very shallow ponds mixing might have to be carried out once or twice a day either manually or mechanically as the photosynthetic oxygen supply has to be synchronized with the BOD discharged. In the absence of mechanical aerator, the pond may become almost septic during the night since the algae undergo respiration and compete with the bacteria for the available oxygen. Consequently, wastes discharged in the evening are poorly stabilized if the detention time is less than one day.

As high as 95% BOD removal is possible in aerobic lagoons. However, the effluent contains very high concentration of algae. In fact if the algal oxygen demand is considered, the BOD_5 in the effluent could be higher than in the influent.

Harvesting of the algae by physico-chemical means is not economically attractive at present. However, fish culture to remove the algae or utilization of effluent for irrigation presents a possibility. In such cases, maturation ponds are often added to improve the removal of pathogenic organisms. The design procedure of aerobic pond is similar to that of facultative pond.

Design of Maturation Ponds

Maturation ponds are used after facultative, aerated lagoons, and in some instances, activated sludge and trickling filters to remove pathogenic organisms before the reuse of the treated wastewater for irrigation. The maturation pond has an influent where most of the organic wastes have been removed in the preceeding treatment processes.

Maturation ponds are rather deep ponds, with depths ranging from 3 to 5 m. Like in anaerobic ponds, economics dictates the maximum pond depth although in no case should the pond be made shallower than 3 m. This minimum value allows for siphoning of the effluent below the algal growth layer without disturbing the settled solids. As most of the algae in the maturation pond congregate at the upper 0.5 m where sunlight is available, the effluent is siphoned at 1.5 to 2.0 m depth in order to minimize the amount of algae going to the effluent. In this way, only the motile algae are carried out because most of the mobile algae stay on the top 0.4 m.

The normal detention time in maturation ponds depends on the required bacterial removal efficiency using fecal coliform as an indicator. It is interesting to note that due to the simplicity of the lagoon system, higher microbial effluent quality is required than in most secondary treatment processes such as the activated sludge process or the trickling filter. However, the coliform removal efficiency in the facultative and anaerobic lagoon is higher. Table 5-3 shows the range of microbial removal rates in secondary biological treatment processes, indicating that the lagoon system has superior removal efficiency due to its longer detention time as compared to activated sludge, trickling filter and anaerobic digester.

Bacteriological standards are often very stringent to a point that they require better effluent quality than the surrounding and receiving water bodies. The bacteriological standards in developing countries brought in by foreign consultants ultimately defeat the whole sanitation program as the people and government are unable to finance and implement the standards. Sanitation and the willingness to pay and maintain the waste treatment facilities depend on the habits of the users and their exposure to the convenience of using the sani-

Table 5-3 Removal of Bacteria by Various Treatment Processes

Process	% Removal
Coarse Screen	0- 5
Fine Screen	10-20
Grit Climbers	10-25
Plain Sedimentation	25-75
Chemical Precipitation	40-80
Trickling Filters	90-95
Activated Sludges	90-98
Facultative Lagoons	95-99

tation facilities. Hence, the widespread utilization of waste treatment facilities of lower efficiency may be more useful in the initial phase. The users could pay and then improve the system as their standards of living improve.

The influent to the maturation pond contains very low concentration of organic wastes, to cite,it may have filtered BOD values ranging from 20 to 60 mg/l only. But the balance between the oxygen supply and demand is not important in the design of maturation ponds. Hydraulic retention time is currently used as the major design criterion for maturation ponds. As discussed in Chapter III, surface area may be an important design parameter if future researches could confirm the effect of solar radiation on coliform removal.

Maturation ponds are operated in series of two or more ponds, each with a detention time of 5 to 10 days. First order bacterial die off kinetics is applied in maturation pond design. The rate of change of the coliform concentration could be expressed as:

$$\frac{dC}{dt} = -k_b C \qquad (5-40)$$

where k_b is the coliform die-off constant, day^{-1}, t is time in days, and C is the coliform concentration in MPN/100 ml. MPN stands for most probable number.

A mass balance of the incoming, outgoing, accumulation and removal of coliform in a completely mixed maturation pond could be expressed as:

$$V \frac{dC}{dt} = q\, C_o - q\, C - k_b CV \qquad (5-41)$$

where V is the maturation pond volume, m^3; q is the wastewater flow rate in and out of the maturation pond, m^3/day and the other terms are as defined before. At steady state, V dC/dt is equal to zero. Hence, Eq. 5-41 could be expressed as:

$$C = C_o/(1 + k_b\ V/q) \qquad (5-42)$$

If the wastewater detention time, Θ, is equal to V/q, then Eq. 5-42 could be rewritten as:

$$C = C_o/(1 + k_b\Theta) \qquad (5-43)$$

The coliform organisms are continuously removed from the wastewater in all the treatment processes preceeding the maturation ponds. Applying the principles of a n-completely mixed tanks in series with unequal volume as discussed in Chapter III, the final coliform concentration could be determined from the following equation:

$$C_e = \frac{C_o}{(1+k_b\theta_1)(1+k_b\theta_2)(1+k\,\theta_3)\ldots(1+k_b\theta_n)} \qquad (5-44)$$

where C_o is the influent coliform concentration, MPN/100 ml; C_e is the effluent coliform concentration, θ_1, and θ_2, θ_3 are the respective hydraulic detention times in the facultative, anaerobic, and maturation ponds comprising the whole wastewater treatment system. For sewage, C_o is usually taken at 10^6 MPN/100 ml.

Temperature Effects – At 20°C k_b has a value of 2.6/day. Marais (1974) proposed an exponential increase of the k_b value with temperature. Marais equation for k_b is as follows:

$$K_{b(T)} = 2.6\ (1.19)^{T-20} \qquad (5-45)$$

where $k_{b(T)}$ is the k_b at temperature T expressed in °Celsius. Considering that coliform organisms are typical mesophiles, the decay rate should decrease with increasing temperature provided the temperature does not exceed 40°C. As the temperature approaches 37°C, the coliform growth and reproduction rates reach their optimum. In fact in the presence of food, the coliform population tends to increase rather than decrease.

Franzmathes (1970) found the average coliform concentration in the effluent from three facultative ponds operated in series during summer to be 16 times compared to the average coliform concentration from the same pond, during winter. Parker (1962) found similar effects for facultative ponds although his results from anaerobic pond studies were on the contrary.

There is a serious need to reexamine Marais formulation of Eq. 5-45 as it does not have any theoretical basis.

Example 5-7. Design the facultative and maturation pond to treat sewage with an initial BOD of 250 mg/l and coliform concentration of 10^6 MPN/100 ml. The required effluent BOD and coliform concentration should be lower than 25 mg/l and 100 MPN/100 ml. The sewage flow rate is 10000 m^3 / day. The operating temperature is 20°C.

Solution:

1) Assume coliform die off is the rate limiting condition for design. From Chapter III , the optimal condition for Eq. 5-42 takes place when the fractions removed per stage are equal. Hence,

$$C_e = C_0/(1+k_b\theta)^n$$

where θ is the detention at each maturation and facultative pond. From Eq.5-45 k_b is equal to 2.6/day.

The detention times at each maturation pond are from 5 to 7 days. Select a detention time of 6 days. Then, substituting values into the above equation yields:

$$100 = 10^6/\ (1 + 2.6 \times 6)^n$$

or

$$(16.6)^n = 10000$$

or

$$n = \log 10{,}000/\log 16.6 = 3.27 \text{ say } 4$$

The effluent coliform concentration is then equal to:

$$C_e = 10^6/(1 + 2.6 \times 6)^4$$

$$= 13 \text{ MPN}/100 \text{ ml} \ll 100 \text{ MPN}/100 \text{ ml}.$$

At 20°C, the k value for BOD degradation is equal to 0.3/day. The effluent BOD concentration is equal to:

$$C_e = 250/(1 + .3 \times 6)^4$$

$$= 4.06 \ll 25 \text{ mg/l}$$

The volume of each pond is equal to $q\Theta$ or 6 days x 10,000 m^3/day or 60,000 m^3. The first two ponds will serve as facultative ponds. The effluent BOD from the first pond is calculated as :

$$C_1 = 250/(1 + .3 \times 6) = 89 \text{ mg/l}$$

ii) The required areal loading is calculated from Eq 5-15,

$$\frac{C_o - C_e}{C_o} = .725 + \frac{10.75}{A_L} = \frac{250 - 89}{250}$$

which shows that A_L could be as high as the oxygen supply would allow. From Eq.5-8, A_L is calculated as:

$$A_L = 60\,(1.099)^{20} = 396 \text{ kg/ha-day}$$

Hence, the area required is equal to the weight of BOD removed divided by the equivalent oxygen supply, or

$$A = 1610/396 = 4 \text{ hectares.}$$

The lagoon depth is equal to V/A or 60,000/40000 or 1.5 m. The first pond may turn anaerobic in some instances. Refer to a later section on discussion of anaerobic/facultative pond combination.

From the second pond, the effluent BOD is equal to :

$$C_2 = 89/\ (1 + .3 \times 6) = 32\ \text{mg/1}.$$

The weight ot BOD removed is equal to:

$$= 10{,}000 \times 10^{-3} \times (89\text{-}32) = 570\ \text{kg/day}.$$

For ease in construction, the second pond is constructed with the same dimensions as the first pond. This provides 100% factor in the second pond to cover up for deficiencies if the first pond turn anaerobic.

$$A_L = 570\ \text{kg/day}\ /\ 4\ \text{ha} = 142\ \text{kg/ha-day}$$

which is very low compared to the 396 kg/ha-day allowable areal loading rates.

iii) The third and fourth ponds are strictly maturation ponds since almost all the BOD are removed in the second pond. The effluent BOD from the third pond is equal to:

$$C_3 = 32\ /\ (1 + .3 \times 6) = 11.4\ \text{mg/1}$$

Therefore the weight of BOD removed is equal to:

$$\text{wt of BOD} = q\ (C_2 - C_3) \times 10^{-3} = 10{,}000 \times 10^{-3}\ (32\text{-}11)$$

$$= 210\ \text{kg of BOD/day}.$$

Using a maturation pond 6m deep, the area required is:

$$A = q\Theta/6 = 60{,}000\ \text{m}^3/6 = 10{,}000\ \text{m}^2$$

$$= 1\ \text{hectare}$$

The areal loading rate is:

$$A_L = 210/\ 1\ \text{hectare} = 210\ \text{kg of BOD/ha-day}$$

which shows that the maturation pond will remain aerobic.

The fourth pond will have the same dimensions as the third pond to facilitate construction. Hence, the total area required is 10 hectares.

Design of Aerated Lagoons

The major drawback of anaerobic, facultative and aerobic lagoon is the large tract of land required. An additional disadvantage is the high cost of land near the wastewater generation site so that the wastewater will have to be pumped up to some 40 km. away in the suburbs where land is usually available. Consequently, the sewage could become septic before it reached the treatment plant site thereby creating serious nuisance and odor problem. Also the cost of pumping and laying out the sewer transmission lines could far exceed the savings in construction, operation and maintenance of the facultative lagoon system over other wastewater treatment methods.

As explained in the previous section, the surface area of the lagoon system is for the transmission of the oxygen from the atmosphere and of sunlight for photosynthesis. The land area could be reduced if oxygen is introduced into the lagoon by mechanical aerators or diffusers. This type of lagoon system is known as aerated lagoon.

Oxygen Mass Transfer – The transfer of oxygen from the atmosphere to the wastewater is defined by:

$$V \frac{dO}{dt} = K_L A \alpha (O_s - 0) \qquad (5-46)$$

where V is the wastewater volume, m^3; A is the surface area for oxygen diffusion, m^2; O is the dissolved oxygen concentration in the wastewater, mg/1; α is the oxygen transfer efficiency which depends on the concentration of surface active agents in the wastewater; t is time in hrs; K_L is oxygen mass transfer coefficient, m/hr., and O_s is the dissolved oxygen concentration at saturation, mg/1. The value for K_L depends on the turbulence and temperature as described in Chapter I.

Diffusers and aerators are calibrated at 20°C at sea level using tap water as the absorbing medium. The dissolved oxygen concentration at saturation will have to be corrected to account for variation in salt concentration, atmospheric pressure and temperature. Fig. 5-5 shows the dissolved oxygen concentration at saturation for different temperatures.

Diffusers – Diffusers are classified as coarse bubble and fine bubble diffusers. Fine bubble diffusers are porous media such as wire screens and woven synthetic fabrics. The surface area for oxygen diffusion is the sum of the surface area of the air bubbles. Hence, the fine bubble diffusers have higher surface area than the coarse bubble diffusers for the same air volume. Even when the diffuser is new, the fine diffusers have much higher headloss than the coarse bubble diffuser. Bacterial slimes, dirt, dust and ferric oxide easily clog the fine diffuser holes thereby increasing the headloss.

The weight of oxygen transferred to the wastewater for the same amount of energy is almost equal for both types of diffuser. Due to the operation and maintenance problems with fine bubble diffusers, coarse bubble diffusers are more popular today.

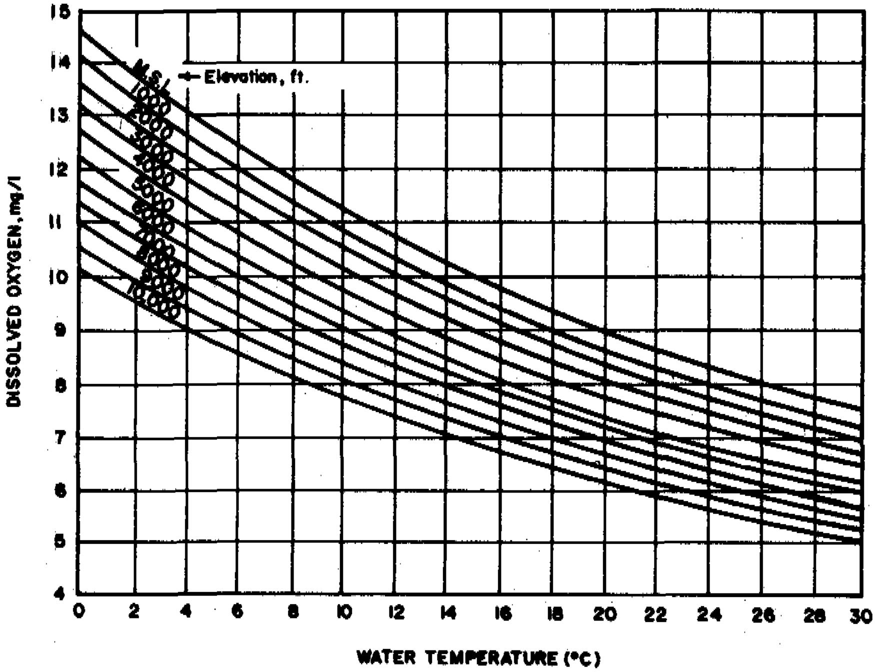


FIG. 5-5 Saturation concentration for atmospheric oxygen.

Blowers are used to pump the air into the diffusers as it is important to continuously maintain sufficient air pressure in the diffuser, even for the coarse bubble types, to prevent clogging. The headloss in the diffuser increases with the flow rate and it is unique with each type of diffuser. Manufacturers' catalogue should be consulted in calculating the headloss through the diffusers.

The aeration efficiency of diffusers is dependent on the immersion depth, the bubble size and air flow rate. The deeper the immersion of the diffuser, the higher the aeration efficiency becomes, as the contact time between water and air bubbles is longer. At very high air flow rates, the air bubbles have higher chances of colliding with each other thus increasing the bubble size which results in smaller surface area and shorter detention time in the liquid. Typical values of diffuser aeration efficiency are shown in Fig. 5-6.

Once the air flow rate is known, the required blower size is calculated using the following equation:

$$\text{bhp} = \frac{wRT_1}{550\,ne}\left[\left(\frac{p_2}{p_1}\right)^n - 1\right] \qquad (5-47)$$

where bhp is the brake horsepower of the blower, R is the gas constant for air, 53.5, w is the weight of air, lb/sec; n is a constant which is equal to 0.283 for air; T_1 is the inlet temperature in °R; p_2 is the outlet absolute pressure and p_1 is the inlet absolute pressure.

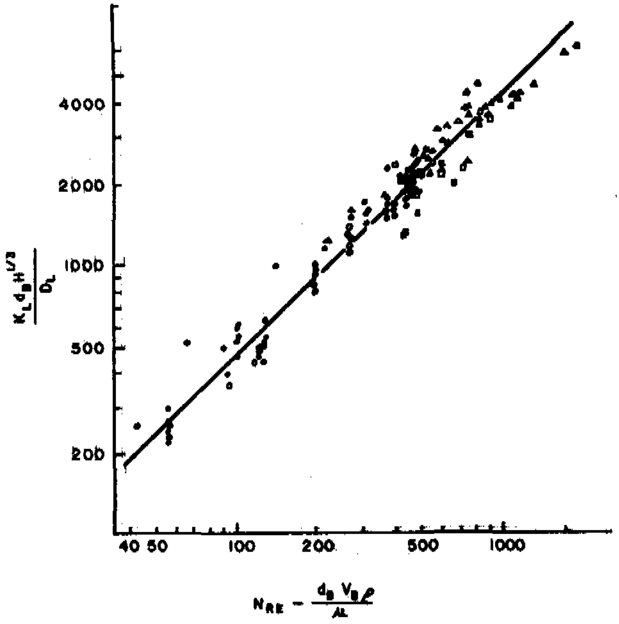


FIG.5-6 Correlation of bubble-aeration data.

When the gas is compressed, its termperature increases. The temperature increase ΔT, could be estimated from the following formula:

$$\Delta T = T_1 \left[\left(\frac{p_2}{p_1} \right)^n - 1 \right] \qquad (5-48)$$

where all the terms have been defined before.

The headloss in the piping is calculated from the nomograph shown in Fig. 5-7. The total outlet pressure p_2 should be sufficient to overcome the headloss in the diffuser, and in the pipings and the static water pressure above the diffuser. The specified p_2 is made 10-20% higher than the theoretical p_2 to account for other losses and increased headloss in the diffuser due to clogging.

The diffusers in aerated lagoons are attached to flexible tubes, hence, they are movable in contrast to the fixed ones used in activated sludge. The flexible tubes are continuously pushed from one end of the lagoon to the other end by the reaction force of the released gases. The movement of the diffuser allows better mixing and distribution of the dissolved oxygen over a wide area The aerated lagoon surface area could be five to ten times the area of an activated sludge process but the oxygen demand per unit area is much smaller due tolonger detention times and low volatile suspended solid concentration. The use of fixed diffusers will be very expensive if the entire lagoon area is to be covered.

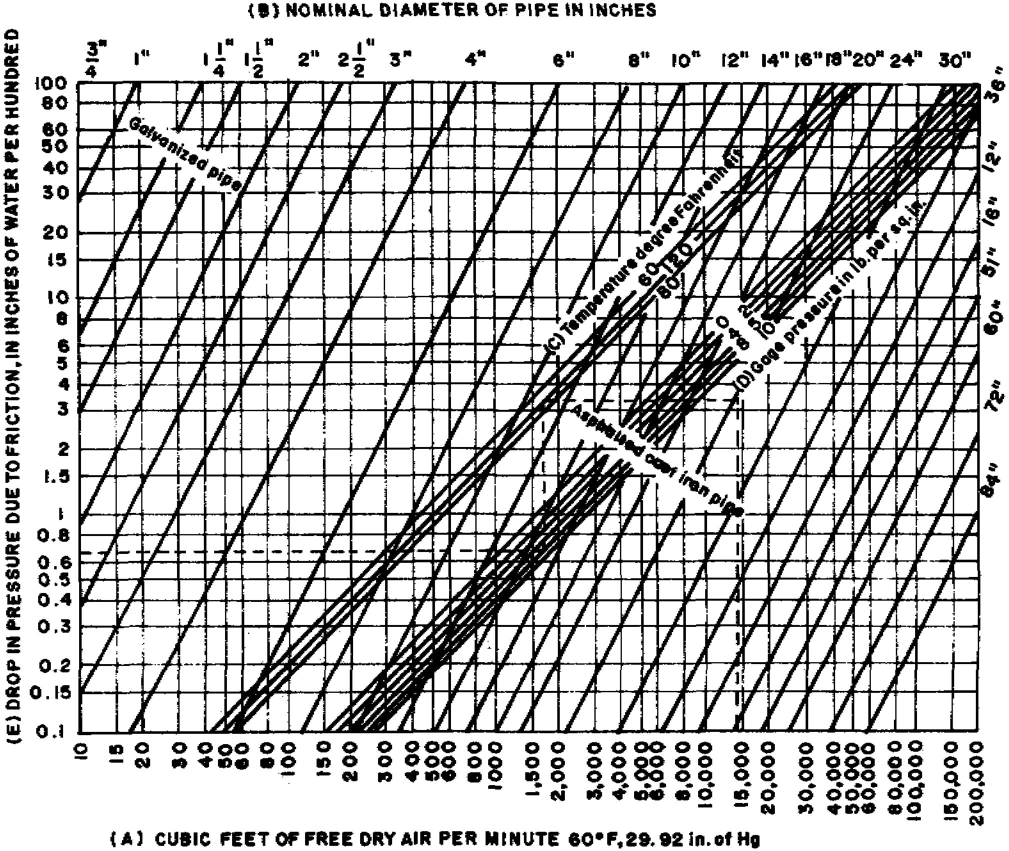


FIG. 5-7 Flow of air in pipes.

Mechanical Aerators – Mechanical aerators are classified as high speed or low speed aerators. As discussed in Chapter I, the power consumption of mechanical aerators increases in proportion to the third power of the speed of revolution while the oxygen transfer rate increases in linear proportion to the speed of revolution. The mass of oxygen transferred per unit of power consumed is lower for high speed aerators than for low speed aerators. With the increasing cost of power, the low speed aerator has become popular.

However, for aerated lagoon the mixing power required may exceed that required for oxygen transfer. The high speed aerator is used in most instances specially for very large aerated lagoon systems. The mixing power requirement for aerated lagoon varies from 0.01 to 0.02 kw/m^3 of tank volume. Mixing for surface aerators is often limited to 4m deep but if draft tubes are added, the mixing depth can be increased to 5.5. m.

The mass of oxygen transferred to the aeration tank at 20°C using tap water varies from .5 kg of O_2/kg-hr to 1 kg of O_2/kw-hr for high speed aerators. Under ideal condition slow speed aerators could transfer up to 2 kg of O_2/kw-hr. Manufacturer's guaranteed performance should be consulted when specifying mechanical aerator size.

Oxygen Requirements at 20°C – The operating temperature of the aeration tank could differ from the 20°C test temperature used for calibrating diffusers and mechanical aerators performance. The equivalent oxygen requirements at 20°C is calculated using the formula:

$$N_{20} = N_{(T)} \frac{\alpha\, 9.17\, (1.024)^{T-20}}{\beta (O_s - O)} \qquad (5-49)$$

where N_{20} is the oxygen required at 20°C, kg of O_2/kw-hr; N_T is the oxygen required at the aeration tank operation temperature; O_s is the saturation dissolved oxygen concentration at temperature, T, in mg/1; O is the operating dissolved oxygen concentration, mg/1; β is the correction factor to account for the effects of salinity and aeration tank unit elevation, on the saturated dissolved oxygen concentration, α is the oxygen transfer correction factor, and T is the temperature in °C.

The oxygen transfer correction factor depends on the concentration of surface active agents such as detergents in the wastewater and the mixed liquor suspended solids. The surface active agent tends to lower the α value to as low as 0.4 while MLVSS increases the α value to as high as 1.3. Since aerated lagoons are operated with very low MLVSS concentration, α could be as low as 0.6 for laundry wastes. For conventional activated sludge α could vary from 0.85 to 0.9. For extended aeration process with very high MLVSS concentration, α could go up to 0.95. The value of β is very close to one.

BOD Removal – The organic pollutant in the wastewater is converted to carbon dioxide, water and microbial cells during the treatment process. The rate of stabilization of the organic pollutant in the aerated lagoon would be expressed by the first order kinetic equation.

$$\frac{dC}{dt} = -k'C \qquad (5-50)$$

A mass balance of the incoming, outgoing, accumulation and degradation of the organic pollutant in the aerated lagoon yields:

$$V\frac{dC}{dt} = q\,C_o - q\,C - k'C\,V \qquad (5-51)$$

where the terms are defined before. At steady state dC/dt is equal to zero, hence, Eq. 5-51 simplifies to:

$$C = C_o / (1 + k'\Theta) \qquad (5-52)$$

where Θ is the hydraulic detention time, V/q; and the other terms have been defined before.

An aerated lagoon may be designed as a facultative aerated lagoon or a completely mixed aerobic aerated lagoon. A facultative aerated lagoon must be deeper than the mechanical aerator mixing depth. As most mechanical aerators have mixing depths of 3-4 meters, most facultative aerated lagoons are designed at 6 m depth. The 5-6 m depth serves as a sedimentation section for the suspended solids. Anaerobic degradation of the settled solids takes place in this region thereby extending the time interval required between desludging operations. For a facultative aerated lagoon, the effluent BOD is approximated by Eq. 5-52.

In aerobic aerated lagoon, the bacterial cells remain in suspension and are carried out with the effluent. The bacterial concentration produced per unit of BOD reduced could be expressed by the formula:

$$\frac{dX_a}{dt} = -Y\frac{dC}{dt} \qquad (5-53)$$

where X_a is the bacterial cell concentration, mg/1; Y is the yield coefficient; and the other terms are as defined before. The derivation of Eq.5-53 and further explanation on microbial growth are given in Chapter III.

The actual change in the microbial concentration is the net effect of the bacterial respiration and growth or

$$\frac{dX_a}{dt} = -Y\frac{dC}{dt} - k_bX_a \qquad (5-54)$$

A mass balance of the incoming, outgoing, accumulation, growth and respiration of bacterial cells in a completely mixed system yields:

$$V \frac{dX_a}{dt} = q\,X_o - q\,X_a - qY(C - C_o) - k_b X_a V \qquad (5-55)$$

Normally, X_o is very low compared to X_a. At steady state, dX_a/dt is equal to zero, hence, Eq. 5-55 simplifies to:

$$X_a = \frac{-q\,Y(C - C_o)}{q \;\; + k_b\;\; V} = \frac{(C_o - C)Y}{1 + k_b\Theta} \qquad (5-56)$$

To completely stabilize 1 gm of bacterial cell, the oxygen consumption is equivalent to the oxygen consumed by 0.95 gm of BOD_5. Hence, the total effluent BOD from an aerobic aerated lagoon is:

$$C' = C + 0.95\,X_a \qquad (5-57)$$

Substituting Eq. 5-56 into Eq. 5-57 yields:

$$C' = C + \frac{0.95\;(C_o \; - C)}{1 \; + \; k_b\Theta}$$

$$C' = \frac{0.95\;C_o}{1 + k_b\Theta} + \left(1 - \frac{0.95}{1 \; + \; k_b\Theta}\right)C \qquad (5-58)$$

Substituting Eq. 5-52 into Eq. 5-58 yields;

$$C' = \frac{0.95\;C_o}{1 \; + \; k_b\Theta} + \left(1 - \frac{0.95}{1 \; + \; k_b\theta}\right)\frac{C_o}{1 \; + \; k'\Theta} \qquad (5-59)$$

$$C' = C_o\left[\frac{1 \; + \; k_b\Theta + 0.95\;k'\Theta}{(\,1 \; + k'\Theta\,)\;(\,1 \; + \; k_b\Theta)}\right]$$

Normally, an aerobic aerated lagoon is followed by a maturation pond or sedimentation pond to remove the dispersed bacterial cells.

Kinetic Parameter – Eq. 5-50 is an approximation of the general equation

$$\frac{dC}{dt} = -k\;X_a C \qquad (5-60)$$

as discussed in Chapter III. Due to mixing created by the aerators in an aerated lagoon the microbial mass stays in suspension for a longer period than in a simple lagoon Hence, the microbial cells participate more actively in the biochemical reaction in an aerated lagoon than in a simple lagoon. The degradation constant kX_a at 20°C for common sewage is 5/day. For other temperatures kX_a is calculated: (Marais, 1974) as

$$k' = kX_a = (5)\ (1.035)^{T-20} \qquad (5-61)$$

Eq. 5-61 shows that the bacterial cell activity during biochemical reaction for an aerated lagoon is 16.7 times higher than in a conventional facultative lagoon. The yield coefficient Y for sewage varies from 0.4 to 0.6 gm of bacterial cells/gm of BOD_5 removed. For industrial wastewater, pilot plant studies have to be carried out to measure the values of k', k_b, and Y.

Oxygen Requirements – The ultimate BOD is approximately 1.5 times the 5-day BOD. Hence, the total oxygen uptake rate to reduce the organic pollutant from concentration C_o to C, when C_o and C' are expressed in terms of BOD_5 is:

$$O_r = 1.5\,(C_o - C')\,q \qquad (5-62)$$

Substituting Eq. 5-59 into Eq. 5-62 yields:

$$O_r = 1.5\ q\ C\left[1 - \frac{1 + k_b\Theta + 0.95\,Y\,k'\Theta}{(1 + k'\Theta)(1 + k_b\Theta)}\right] \qquad (5-63)$$

$$= 1.5\ q\ C_o\left[\frac{k'\Theta\,(1 + k_b\Theta - 0.95\,Y)}{(1 + k\,\Theta)\,(1 + k_b\Theta)}\right]$$

Since q is normally expressed in m^3/day, Eq. 5-63 is multiplied by 0.001 to express O_r in kg of O_2/day , or

$$O_r = 0.0015\,q\,C_o\ \frac{k'\Theta\,(1 + k_b\,\Theta - 0.95Y)}{(1 + k'\Theta)\,(1 + k_b\Theta)} \qquad (5-64)$$

Example 5-8:

a) Calculate the weight of oxygen required in an aerated lagoon to reduce an influent BOD of 250 mg/l to 25 mg/l for domestic sewage with a flow rate of 10,000 m^3/day. The ambient temperature is 20°C.

b) Calculate the effluent BOD for a facultative aerated lagoon.
c) Calculate the effluent BOD for an aerobic aerated lagoon.
d) What is the effect on the oxygen uptake rate if the lagoon is designed as facultative aerated lagoon instead of aerobic aerated lagoon?

Solution: From Eq. 5-52, the required detention time is equal to:

$$\frac{C}{C_o} = .10 = \frac{1}{1 + 5\Theta} \qquad \Theta = 1.8 \text{ days}$$

Using a value of $Y = 0.5$, and $k_b = 0.07/\text{day}$, then from Eq. 5-64

$$O_r = 0.0015 \times 10{,}000 \times 250 \left\{ \frac{5 \times 1.8\,(1 + 0.07 \times 1.8 - .95 \times 0.5)}{(1 + 5 \times 1.8)\,(1 + .07 \times 1.8)} \right\}$$

$$= 1951 \text{ kg of } O_2/\text{day}$$

It should be noted that the theoretical oxygen requirement if the bacterial cells are completely oxidized is:

$$O'_r = 0.0015 \times 10{,}000 \times (250 - 25)$$

$$= 3375 \text{ kg of } O_2$$

O'_r is 72% higher than O_r. For the design of facultative lagoons, the areal loading rates are determined by equating the photosynthesis oxygen production rate with the oxygen uptake rate on the assumption that all the BOD degraded is converted to carbon dioxide and water. The bacterial cells are degraded anaerobically at the bottom of the facultative pond. This proves the very conservative design approach for facultative lagoons.

If the design approach for aerated lagoons were utilized to design a facultative pond, correction for the bacterial cells should be considered. The theoretical areal loading could be 70% higher than the oxygen supply from the wind and photosynthesis.

b) The effluent for a facultative aerated lagoon would be equal to:

$$C = C_o / (1 + k'\Theta) = 250 / (1 + 5 \times 1.8)$$

$$= 25 \text{ mg/l}$$

The bacterial cells would settle at the bottom of the pond. In practice up to 30% of the solids is carried out. From Eq. 5-46, the microbial cell concentration is calculated which yields:

$$X_a = \frac{0.5\ (250 - 25)}{1 + 0.07 \quad x\ 1.8} = 100 \text{ mg/1}$$

Substituting the value of 0.3 into Eq.5-57, results to:

$$C' = 25 + 100 \ x\, 0.3 \ x \ 0.95$$
$$C' = 53.5 \text{ or } 54 \text{ mg/1}$$

c) The effluent BOD from the aerobic aerated lagoon could be calculated directly from Eq5-59 or from Eq. 5-56 and 5-57. Hence, substituting the value of X_a from part (b) into Eq.5-57 yields

$$C' = 25 + 100 \, x \, 0.95 = 120 \text{ mg/1}$$

d) The oxygen uptake rate remains the same for facultative and aerobic aerated lagoons. The bacterial cells in a facultative aerated lagoons undergo anaerobic degradation at the bottom of the lagoon.

In an aerated lagoon system the mixing power requirements for aerators often exceed the power required for oxygen transfer. To maintain a well mixed lagoon system, the mixing energy must vary from 17 to 35 Hp (.013 to .026 kw) per 1000 m^3 of wastewater volume. To determine the exact mixing power required, the radius and depth of influence of the aerator should be considered.

Example 5-9. a) Determine the mechanical aerator required for the facultative aerated lagoon to treat the sewage whose characteristics are given in Example 5-8. b) Determine the aerobic aerated lagoon size to treat the same wastewater.

Solution: a) The minimum DO which does not retard the biological activity is 0.5mg/1 For design purposes assume a residual dissolved oxygen of 1.5 mg/1 . From Eq. 5-49 the weight of oxygen required per day at 20°C is equal to:

$$N_{20} = 1951 \frac{(9.17)}{(9.17 - 1.5)} = 2332 \text{ Kg./day}$$

The required surface aerator for mixing must have a power rating of:

$$Hp = \frac{35 \text{ Hp}}{1000 \text{ m}^3} \quad x \ 1.8 \text{ days} \ x \ 10{,}000 \text{ m}^3\text{/day}$$

$$= 630 \text{ Hp}$$

The transfer rating of the aerator at 20° should be

$$= 2332/(630 \times 24)$$

$$= 0.15 \text{ kg of } O_2/\text{bHp-hr}$$

The power requirement for mixing exceeds that for oxygen supply. Use a high speed aerator with oxygen transfer rating of 1.0 kg of O_2/bHp-hr. This would supply six times the oxygen required.

If the aerobic depth is 4 meters, the lagoon surface area is equal to

$$A_s = 1.8 \times 10{,}000/4 = 4{,}500 \text{ m}^2$$

Add another 2 meters for the solids and anaerobic degradation zone, hence, the pond depth is equal to 6 m. To minimize short circuiting use a length to width ratio of 3:1. Hence,

$$A_s = L \times W = 3W \times W = 4500 \text{ m}^2$$

$$W = 38.7 \text{ or } 39 \text{ m}$$

$$L = 117 \text{ m}$$

Use 12 – 60Hp mechanical aerators, located 10m from each other and 5 m from the lagoon walls.

b) The aerobic aerated lagoon has a depth of 4 m. The length, width and mechanical aerator requirements will remain the same.

Effects of Staging – The aerated lagoon could be operated as a series of n-smaller lagoons in series. The optimal condition is to maintain equal volume per lagoon. Operation of smaller aerated lagoon in series does not only reduce the lagoon volume requirements but also the power required for mixing. By applying the principles of mass balance to the aerated lagoon operated as a series of n-completely mixed aerated lagoon of smaller volume, Eq. 5-52 could be modified as:

$$C = C_o / (1 + k\Theta)^n \tag{5-65}$$

The oxygen requirements O_r, X_a, and C' are calculated for each stage of the aerated lagoon individually.

Example 5-10. Design the aerated lagoon and surface aerator required to treat sewage with an influent BOD of 250 mg/1 and flow rate of 10,000 m^3/day. The ambient temperature is 20°C. The aerated lagoon is operated as a series of 3 smaller aerated lagoons. The required soluble BOD in the effluent is 25 mg/1.

Solution: 1) From Eq. 5-65, the detention time per stage is equal to:

$$\frac{25}{250} = \frac{1}{(1 + 5\Theta)^3}$$

$$\Theta = \frac{1}{5}\left[\left(\frac{250}{25}\right)^{1/3} - 1\right] = 0.23/\text{day}$$

The volume per aerated lagoon is equal to

$$V = 0.23 \times 10000 = 2{,}300 \text{ m}^3$$

The soluble BOD in the effluent from the first lagoon is equal to:

$$C = 250/\ (1 + 5 \times 0.23) = 116 \text{ mg/l}$$

The microbial mass X_a is calculated from Eq. 5-56 as

$$X_{1,a} = \frac{0.5\,(250 - 116)}{1 + 0.23 \times 0.07} = 66 \text{ mg/l}$$

Therefore,

$$C' = C + 0.95\ X_a = 126 + .95 \times 66$$

$$= 178.7 \text{ say } 179 \text{ mg/l}$$

From Eq. 5-62,

$$O_r = 1.5\,(250 - 179)\ 10{,}000 \times 10^{-3}$$

$$= 1065 \text{ kg of } O_2/\text{day}$$

For 1.5 mg/l residual dissolved oxygen

$$O_r' = 1065 \frac{9.17}{(9.17-1.5)} = 1273 \text{ kg of } O_2/\text{day}$$

The power required for mixing is calculated as:

$$Hp = \frac{35 \text{ Hp}}{1000 \text{ m}^3} \times 2300 \text{ m}^3 = 80.5 \text{ Hp}$$

Required aerator rating for oxygen supply is equal to

$$= \frac{1273 \text{ kg of } O_2/\text{day}}{24 \text{ hr/day}} \times \frac{1}{80.5 \text{ Hp}}$$

$$= 0.658 \text{ kg of } O_2/\text{hr} - \text{Hp}$$

Use one high speed aerator. Using a square lagoon with an aerobic zone of 4m deep, the length s is then equal to:

$$s = \sqrt{2300/4} = 24 \text{ m.}$$

Use 1–80 Hp surface aerator. Actual pond depth is 6m to allow an additional 2m for the anaerobic zone.

ii) For the second stage

$$C = 166/(1 + 5 \times 0.23) = 53.95 \text{ say } 54 \text{ mg/l}$$

$$X_{2,a} = \frac{0.5\,(116 - 54) + X_{1,a}}{1 + 0.23 \times 0.07}$$

where $X_{1,a}$ is the microbial mass carried out from the first stage. If a facultative aerated lagoon is used, then $X_{1,a}$ is equal to 33% of 66 mg/l or 22 mg/l.

Hence,

$$X_{2,a} = \frac{0.5\,(62) + 22}{1 + 0.23 \times 0.7} = 52 \text{ mg/l}$$

$$C' = C + 0.95\,X$$

$$= 54 + 0.95 \times 52 = 103 \text{ mg/l}$$

$$O_r = 1.5 \times 10{,}000 \times 10^{-3}\,(116 - 103)$$

$$= 510 \text{ kg of } O_2/\text{day}$$

From part (i) the mixing power required is 80 Hp. Since O_r is smaller than in the first stage, the mixing requirement controls the aerator specification. The lagoon dimension is the same as calculated in step (i)

iii) For the last stage, the BOD in the effluent is calculated as:

$$C = 54/ \ (1 + 5 \times .23) = 25 \text{ mg/l}$$

$$X_{3,a} = \frac{6.5\,(54\text{-}25) + .3 \times 52}{(1 + .07 \times .23)} = 30 \text{ mg/l}$$

$$C' = 25 + 30\ (.95) = 54 \text{ mg/l}$$

$$O_r = 1.5\,q \times 10^{-3} \quad (C_2 - C')$$

$$= 1.5 \times 10{,}000 \times 10^{-3} \left[(54\text{-}54) + 52 \times 0.3 \times 0.95\right]$$

$$= 223 \text{ kg of } O_2/\text{day}$$

Since, O_r is smaller than O_r in the first stage, the power requirement for mixing controls the surface aerator specification. It should be noted that the total surface aerator power requirements is reduced from 660 Hp to 240 Hp by dividing the aerated lagoon into three smaller aerated lagoons of equal sizes operated in series. Secondly, the area required is reduced from 4,500 m^2 to 1728 m^2 when compared to example 5-9.

PROCESS OPTIMIZATION

The large land area required for the lagoon system is the major constraint in the application of this wastewater treatment process. As reviewed in the previous section the optimum utilization of the pond surface area has not been exploited thoroughly. The development of the concepts of areal loading rate and areal removal rate was more intuitive than as described in the previous section. The oxygen supply is equated to the ultimate BOD removed even for the mechanistic models whereas in aerated lagoon and activated sludge process, the oxygen supply is equated to the BOD converted to carbon dioxide excluding those converted to cells and given in Eq (5-33). The oxygen requirement criterion for the facultative lagoon is almost twice than in aerated lagoon as shown in the previous section. Future trends would be for higher areal loading rates and optimal utilization of the lagoon surface area.

Anaerobic/Facultative Lagoon Combination – The anaerobic pond could be dug as deep as economically possible. The major constraint on the volumetric loading is controlled by the hydraulic detention time in the pond since the volumetric loading rate is defined as:

$$V_r = \frac{q\,C_o}{V} = C_o/\Theta \qquad (5-66)$$

To minimize the surface area required for the facultative lagoon, an anaerobic lagoon is utilized to remove the suspended solids, and settleable solids to 70% of the influent soluble BOD. The facultative lagoon polishes the effluent from the anaerobic lagoon thus minimizing the oxygen requirements in the facultative lagoon for BOD degradation.

Although the principle is rational, the effect of the anaerobic lagoon on the anaerobic/facultative lagoon combination has been exaggerated. The anaerobic/facultative lagoon combination requires a minimum of two lagoons operated in series. The first lagoon is the anaerobic pond and the second is the facultative lagoon. Hence, the land area requirements for the anaerobic/facultative combination should not be compared with that for a single facultative lagoon but rather with two or or more smaller facultative lagoons operated in series. The effects of lower short circuiting when using two or more ponds in series should be the same for the anaerobic/facultative pond system and the facultative pond system in order for the comparison to be accurate.

Example 5-11. Compare the performance of a facultative lagoon and anaerobic facultative lagoon in treating 10,000 m^3/ day of domestic sewage with an influent BOD_5 of 630 mg/1. The design temperature is 20°C and the required BOD_5 in the effluent is 60 mg/1.

Solution :

i) The volume required for a facultative lagoon using a k value of 0.3/day is:

$$\frac{260}{630} = \frac{1}{1 + 0.3V / 10{,}000}$$

$$V = 316{,}666 \ m^3$$

The area required is:

$$A = 316{,}666/1.5 = 211{,}111 m^2 \text{ or } 21.1 \text{ hectares}$$

The areal loading rate is

$$A_L = \frac{q\ C_o}{A} = \frac{10{,}000 \times 10^{-3} \times 630}{21.1} =$$

298.5 kg of BOD/ha-day

ii) Use an anaerobic pond 3m deep with 5 days detention time. From table 5-2, the BOD removal efficiency is estimated at 70%. The area of the anaerobic pond is then calculated as.

$$A = \frac{5 \text{ days} \times 10{,}000 \ m^3 \ / \text{ day}}{3 \text{ m deep}}$$

$$= 16{,}666 \ m^2 \text{ or } 1.67 \text{ hectares}$$

The effluent is equal to 630 x 0.3 or 189 mg/1. The facultative pond volume required to reduce the BOD from 189 to 60 mg/1 is equal to:

$$60 = 189/(1 + 0.3V/10000)$$

or

$$V = 71{,}667 \text{ m}^3$$

Hence, the area required for the facultative pond is

$$A = 71{,}667/1.5 = 47{,}778 \text{ m}^2 \text{ or } 4.78 \text{ hectares}$$

The combined area for the anaerobic/facultative pond is 6.45 hectares which is only 30% of the area required for a purely facultative pond system. The areal loading rate of the facultative pond is:

$$A_L = \frac{189 \times 10^{-3} \times 10{,}000}{4.78} = 395 \frac{\text{kg of BOD}}{\text{ha-day}}$$

The volumetric loading for the anaerobic lagoon is equal to:

$$V_r = \frac{630 \text{ mg/1} \times 10^{-3} \ (\text{kg} - \text{1/mg-m}^3) \times 10{,}000 \text{ m}^3\text{/day}}{5 \text{ days} \times 10{,}000 \text{ m}^3\text{/day}}$$

$$= .126 \frac{\text{kg}}{\text{m}^3\text{-day}}$$

iii) Proper comparison should be between the performance of the anaerobic/facultative pond system with the facultative pond sytem operated as two smaller ponds in series. The detention time per facultative pond could be calculated as:

$$60 = 630/(1 + 3\Theta)^2$$

$$\Theta = \frac{1}{3}\left[\left(\frac{630}{60}\right)^{1/2} - 1\right]$$

$$= 7.46 \text{ days}$$

Hence, the volume per facultative pond is equal to:

$$V = q\,\Theta = 7.46 \text{ days} \times 10{,}000 = 74{,}600 \text{ m}^3$$

The area required per facultative pond is equal to:

$$A = 74{,}600/1.5 = 49{,}786 \text{ m}^2 \text{ or } 5 \text{ hectares}$$

For the two ponds the total area is 10 hectares. The savings from the anaerobic/facultative pond system is only 35% not 70% as calculated in step i and ii.

The effluent BOD from the first stage is equal to

$$C_1 = 630/\ (18.3 \times 7.46) = 195 \text{ mg/l}$$

Hence, the areal removal rate is equal to:

$$A_r = \frac{(630 - 195)\ 10^{-3} \times 10,000}{5.0} = 870 \text{ kg of BOD/ha-day}$$

The areal removal rate at the second lagoon is

$$A_r = \frac{(195\text{-}60)\ (10^{-3})\ (\ 10,000)}{5.0} = 270\ \frac{\text{kg of BOD}}{\text{ha-day}}$$

The reader might complain that the areal removal rate in the first facultative pond is quite high and the tendency to turn anaerobic is very high. There is no sense in condemning the high areal removal rates in the first pond at the same time recommending the anaerobic/facultative pond system. From step (ii) the areal removal rates in the anaerobic pond is equal to $0.7 \times 630 \times 10^{-3} \times 10,000/1.67$ or 2640 kg of BOD/ ha-day which is almost three times the areal removal rate in the first stage facultative pond.

Pollutant – In lagoon system design, the anaerobic and facultative lagoons are designed for BOD removal while the maturation ponds are designed for coliform removal. In Chapter III, it was shown that the minimum total volume of a number of lagoons operated in series takes place when the detention times at each lagoon are equal provided the pollutant degradation conforms to the first order biochemical kinetics. The residual pollutant should control the design since the other pollutant species are reduced to acceptable level during the treatment operation well ahead of the residual pollutant. The persistent pollutant is the pollutant with either the highest concentration or the lowest k value. This principle is illustrated in the succeeding example.

Example 5-12. The domestic sewage described in Example 5-11 has an initial coliform concentration of 4×10^7 MPN/100 ml. Compare the result when the system design is based on the conventional functions of each process, and if the design is based on the residual pollutant. The desired effluent should have a BOD of less than 25 mg/l and fecal coliform of less than 100 MPN/100 ml.

Solution: The conventional function of the anaerobic and facultative pond is to remove the BOD. The anaerobic pond and facultative pond have areas of 1.67 and 4.78 hectares respectively as calculated in the previous example. The detention time in the anaerobic pond is 5 days while in the facultative pond, is 7.17 days.

At 20°C, k_b is equal to 2.6/day. If the maturation pond has a detention time of 6 days, then the number of maturation ponds required could be calculated from Eq. 5-42 as:

$$100 = 4 \times 10^7 / [(1 + 2.6 \times 5) (1 + 2.6 \times 7.17) (1 + 2.6 \times 6)^n]$$

$$100 = \frac{4 \times 10^7}{14 \times 19.64 \times 16.6^n}$$

$$16.6^n = 1455$$

$$n = \log 1455 / \log 16.6$$

$$= 2.59 \text{ say } 3 \text{ ponds}$$

The volume of each maturation pond is equal to

$$V = 6 \times 10{,}000 \text{ m}^3/\text{day} = 60{,}000 \text{ m}^3$$

The area of each maturation pond for a depth of 4m is

$$A = 60{,}000/4 = 15{,}000 \text{ m}^2$$

For the 3 ponds, the area required is 15,000 x 3 = 45,000 m^2

The total area for the system is then equal to the sum of the anaerobic, facultative and maturation pond areas or 1.67 + 4.78 + 4.5 or 10.95 hectares

ii) The same problem could be designed using the coliform organism as the residual pollutant. Since the anaerobic pond detention time is fixed at 5 days to prevent overloading or from turning facultative, then, the detention time at each stage is fixed at 5 days. The number of ponds in series is calculated as:

$$100 = 4 \times 10^7 / (1 + 2.6 \times 5)^n$$

or

$$16.6^n = 400{,}000$$

$$n = \log 400{,}000 / \log 16.6 = 4.59 \text{ say } 5.$$

The volume of each pond is equal to (5 days x 10,000 m^3/day) or 50,000 m^3.

The anaerobic lagoon from Example 5–10 is 3m deep and the facultative lagoon is 1.5 m deep. Hence the area for the anaerobic lagoon is:

$$A = 50{,}000/3 = 16670\ m^2 = 1.67\ \text{hectare}$$

And using two ponds for the facultative lagoon,

$$A = 2 \times 50{,}000/1.5 = 33.300\ m^2 \text{ or } 3.33 \text{ hectares}$$

The two maturation ponds will require an area equal to

$$A = \frac{2 \times 50{,}000}{4} = 25{,}000\ m^2 \text{ or } 2.5 \text{ hectares}$$

The total land area required is 7.5 hectares which is 25% lower than when the design is based on the conventional function calculated in step (i).

The BOD removal efficiency is checked. For the anaerobic lagoon the effluent BOD is .3 x 630 mg/1 or 189 mg/1. The volumetric loading

$$V_r = \frac{10{,}000\ m^3/\text{day} \times 630\ \text{mg/1} \times 10^{-3}}{50{,}000} = .126\ \text{kg/m}^3\ \text{day}$$

which is less than the .400 kg/m^3 –day limit for odor control.

The areal loading rate to the first facultative pond is:

$$A_L = \frac{189\ \text{mg/1} \times 10{,}000 m^3/\text{day} \times 10^{-3}}{3.33}$$
$$= 567\ \text{kg/ha–day}$$

Hence, the effluent BOD is equal to:

$$\frac{C_o - C_e}{C_o} = 0.725 + \frac{10.75}{A_L}$$

$$= .725 + 10.75/567 = .743$$

$$C_e = 189\ (1 - .743) = 48\ \text{mg/1}$$

For the second facultative pond the areal loading rate is:

$$A_L = \frac{48 \times 10{,}000 \times 10^{-3}}{3.33} = 150 \ \frac{\text{kg}}{\text{ha-day}}$$

The second facultative pond could be converted into a maturation pond since the BOD load is very low. Using a depth of 4 m, then the area is adjusted to 2.5 hectares.

The corrected areal loading rate is:

$$A_L = \frac{48 \times 10{,}000 \times 10^{-3}}{2.5} = 193 \ \frac{\text{kg}}{\text{ha-day}}$$

The effluent from this pond is:

$$\frac{48 - C_e}{48} = .725 + \frac{10.75}{240}$$

$$C_e = 48\left(1 - .769\right) = 11 \text{ mg/l}$$

The effluent quality as far as BOD is concerned has satisfied the standards already. For the 1st and 2nd maturation ponds, the BOD is expected to decline further.

CLOSURE – The major deficiency of the lagoon system is the large land area required. However, due to the simplicity of the system in terms of construction, operations and maintenance, there seems to be very little interest in optimizing the operations of the system. The conservative practice prevailing today tends to lower the system efficiency.

CHAPTER VI

DISPERSED BACTERIAL REACTORS

The major constraint on the application of the lagoon systems described in the previous section is the large tract of land required. Water pollution takes place when the wastewater generation rate exceeds the self-purification capacity of the receiving water body. Water pollution occurs in areas around large cities and in human settlements where land is expensive. The cost of land or wastewater transport to areas with low land values lowers the viability of using lagoon systems for wastewater treatment.

As discussed in the previous chapter, the rate of organic waste stabilization in aerated lagoons is almost 15 *times faster than in the conventional* facultative or anaerobic lagoon due to the suspension and longer participation of the dispersed bacterial cells in the biochemical reaction. In the facultative lagoon, the bacterial cells are almost immediately removed due to settling.

The rate of biochemical reaction or wastewater stabilization could be improved further by concentrating the bacterial mass in a clarifier or thickener and recirculating the concentrated solids in the aeration system. In this way the active microbial mass could be ten times higher than in the aerated lagoon. The required retention time for wastewater treatment is further reduced from 20 days in facultative ponds to four hours in a wastewater treatment system where the bacterial mass is recirculated. The land area required for the wastewater treatment plant is reduced drastically.

The wastewater treatment processes which utilize high microbial mass concentration to hasten the stabilization of organic wastes are the activated sludge, the high rate anaerobic digestion, and the oxidation ditch. An oxidation ditch is often classified as a variation of the aerated lagoon but the operating and design criteria are closer to those of the activated sludge variation known as extended aeration system.

Some bacterial cells form colonies or flocs which separate more easily from the aeration tank effluents than the unicellular bacterial cells due to their bigger sizes. The bacterial colonies are recycled to the aeration tank where they gain dominance over the unicellular bacteria. The settled bacterial colonies could be very dense with a concentration of up to 1% solids in a standard clarifier and 4-5% in thickeners. The settled bacterial colonies look like the sewage sludge which are normally removed in primary clarifiers. However, unlike sewage sludge, the settled and concentrated bacterial colonies are biologically active in degrading the organic wastes in the presence of oxygen and nutrients. For this reason, the concentrated bacterial colonies and the wastewater treatment process are known as activated sludge.

In the aeration tank, the concentrated bacterial colonies are diluted by the incoming raw wastewater by a factor of 10 to 50, which reduces the bacterial colony concentration to the range of 1,500 to 5,000 mg/1. The diluted bacterial colonies are easily resuspended by the turbulence created during the aeration process. The diluted bacterial cells are similar in physical properties to the ordinary suspended solids found in sewage and other industrial wastewaters. As bacterial cells are primarily organic in composition, they are destroyed when heated to temperatures above 600°C in the presence of oxygen.

Since the bacterial colonies are too numerous to count and their sizes are highly variable, the bacterial colony concentration in the aeration tank is approximated by the mixed liquor volatile suspended solid concentration.

The same principle is utilized in anaerobic digestion. In a conventional anaerobic digester, the anaerobic bacterial cells are washed out in the effluent. Since the methane formers have a very slow growth rate, the detention time in the conventional anaerobic digester could be as long as 60 days in temperate climate. The anaerobic bacterial cells are concentrated in a clarifier or thickener after degasification. Degasification is carried out to remove the methane, nitrogen, carbon dioxide, hydrogen sulphide and other gases normally trapped in the anaerobic sludge. The gases often reduce the density of the anaerobic sludge which makes solid/liquid separation difficult. Flotation could not be utilized to concentrate the floppy anaerobic sludge since oxygen kills the anaerobic bacteria.

The anaerobic digestion process which utilizes recirculation of the anaerobic sludge is known as high rate digestion to differentiate it from the conventional process. The high rate anaerobic digester could stabilize and remove the same quantity of organic wastes almost 10 times faster than the conventional anaerobic digester. The high rate anaerobic digesters are commonly used in degrading or stabilizing the excess activated sludge.

For domestic sewage, the excess activated sludge flow rate is 0.1 and 0.02% of the influent wastewater flow rate when it is concentrated in clarifiers and gravity thickeners, respectively. Since the excess sludge flow rate is very low compared to the wastewater flow rate, the use of high rate anaerobic digesters is not very popular due to the more sophisticated mixing and process equipment required as compared to the conventional anaerobic digesters with very long retention time. However, with the possibility of recovering methane gas as an energy source, the high rate anaerobic digester is becoming more popular not only for sludge digestion but also for the treatment of strong industrial wastewater.

The principles of the activated sludge and the high rate anaerobic digester are very similar. Both processes are able to stabilize organic matter in wastewater at a high rate compared to conventional processes due to the build up of activated sludge in the reaction tank. The biochemical reaction is brought about by dispersed bacterial growth or colonies in the wastewater. The major difference between the two processes is the presence of oxygen and the large energy consumption of the activated sludge process while in a high rate anaerobic digester, the reaction takes place in the absence of oxygen and there is more energy released by the reaction, in the form of methane gas, than energy consumed Due to the similarity in the two processes, they are discussed together in this chapter. Hopefully, a large number of principles currently used in activated sludge process could be applied to the high rate anaerobic digester in the future. The activated sludge process has been extensively studied since its development in the early 1900 by Ardern and Lockett, while the high rate anaerobic digester is primarily in its early stage of development.

ACTIVATED SLUDGE PROCESS

The basic version of the activated sludge process consists of a biological reactor or aeration tank and a solid-liquid separator arranged as shown schematically in Fig. 6–1. Some of the process variants, however, involve different equipment configuration and/or waste contacting patterns and will be discussed in a subsequent section.

In the basic version, the influent is brought into contact with activated sludge held in suspension inside the reactor vessel in an aerobic environment. Here, the bio-degradable organic matter in the waste is assimilated by the microorganisms present in the sludge. A portion of the organic matter is oxidized into organic end products with the release of energy for cellular functions, while the remainder is synthesized into protoplasmic constituents leading to growth and multiplication of microorganisms. Since dissolved oxygen is continuously removed in the oxidation reactions, diffused or mechanical aeration is used to replenish oxygen supply and ensure the maintenance of aerobic conditions in the reactor. The contents of the reactor, termed mixed liquor, flow into the solid-liquid separator which is usually a gravity settler, where the sludge particles settle down and the clarifier supernatant is suitably disposed of. A portion of the concentrated sludge in the underflow from the settler is recycled back into the reactor in order to maintain the desired sludge concentration (commonly expressed as mixed liquor suspended solids-MLSS-or mixed liquor volatile suspended solids – MLVSS). The remainder of the sludge is removed from the system to be disposed of after further treatment.

The microbial community in activated sludge is heterogenous in character, i.e., different types of microorganisms coexist in the sludge. These include bacteria, fungi, protozoa and some metazoa such as rotifers. The engineer concerned with the design and operation of the process should have an understanding of the role played by each type of organisms in bringing about the overall purification of the waste.

Removal of organic impurities contributing to oxygen demand (COD or first stage BOD) is the primary objective in most activated sludge systems. The decomposition of the organic matter is usually effected by aerobic heterotophic bacteria which constitute the bulk of activated sludge microorganisms. It will be recalled that energy is required for the oxidation of organic carbon to CO_2 as well as for cellular synthesis of organic compounds. In addition, nutrients sucn as nitrogen, phosphorous and sulfur will also be removed by these organisms to an extent consistent with the requirements for cellular synthesis. If the waste is deficient in any of these, the deficiency should be made up by external nutrient supplementation if maximum efficiency with respect to carbon removal is to be maintained. In other words, the process is usually operated under carbon-limiting conditions.

Occasionally, an activated sludge process is designed to achieve nitrifica-

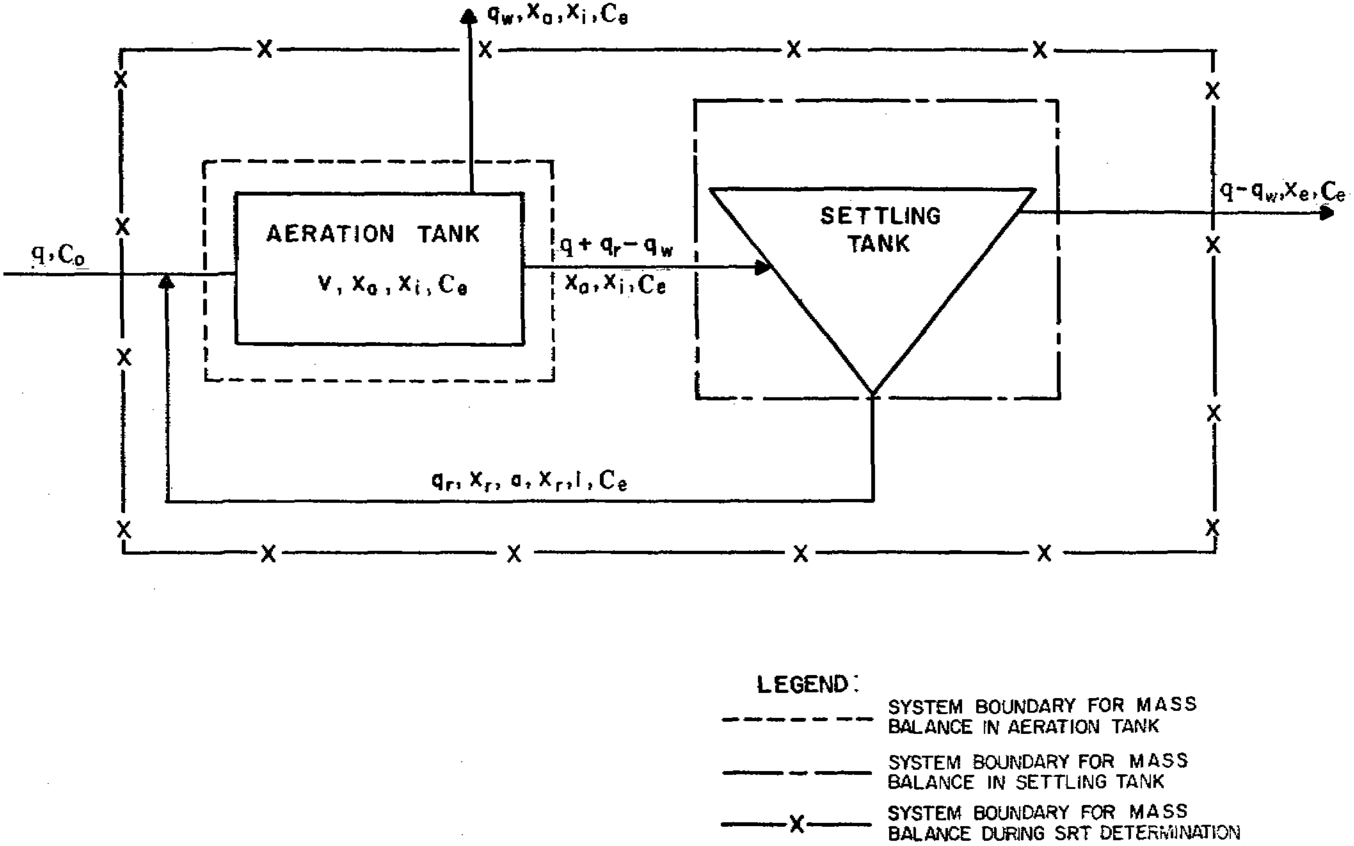


FIG. 6-1 Process flow diagram with aerator MLVSS desludging.

tion so as to minimize the concentration of ammonia nitrogen (either originally present in the waste or resulting from the decomposition of nitrogeneous organic mattter such as protein) escaping in the effluent as it could exert a demand on the oxygen resources of the receiving water. The sludge would then contain a significant number of nitrifying bacteria – Nitrosomas and Nitrobacter. These organisms derive energy from the oxidation of ammonia to nitrite and nitrate while carbon dioxide serves as the carbon source.

Fungi which are generally aerobic heterothrops sometimes constitute, the predominant group of microorganisms in activated sludge either through natural selection or by design. While the optimal pH for most bacteria is between 6.5 and 7.5, fungi can tolerate lower values with the optimum at around 5.6 (Metcalf and Eddy, 1972). In addition, they have lower nitrogen requirements compared to bacteria. These characteristics frequently favor their use in place of heterotrophic bacteria in the treatment of wastes requiring pH neutralization and/or nutrient supplementation. Inputs of industrial wastes having a low nitrogen content and/or low pH into municipal sewage treatment plants have often been found to give rise to the growth of filamentous fungi called moulds through a natural adjustment of dominant organisms to suit the environmental conditions. Such changes in organism predominance have often been cited as a cause of sludge bulking and associated operational difficulties. These will be discussed in more detail under process operation. Protozoa and metazoa, although present in the sludge in relatively small number, nevertheless perform a significant role in enhancing the quality of effluent from the process. These organisms prey upon dispersed bacteria and tiny sludge flocs which settle poorly and are therefore liable to escape in the clarifier effluent impairing its quality.

An understanding of the kinetics of substrate removal and microbial growth in the activated sludge process aids in identifying parameters of importance in process design and operation as well as in defining quantitatively the influence of these parameters on process performance. Susbtrate removal and its metabolic utilization resulting in growth is accomplished by microorganisms through a complex sequence of interdependent transfer and transformation steps involving mass transfer across cell membrane and a multitude of enzyme catalyzed biochemical reactions within the cell. Furthermore, the activated sluge process generally involves multiple substrates and a variety of microorganisms functioning simultaneously. The task of developing a comprehensive kinetic model for the substrate transformation process based on sound biochemical principles has therefore been complex. Over the years, however, several relatively simple kinetic formulations have been developed empirically for practical purposes from experimental observations (Eckenfelder, 1959; Eckenfelder and Ford, 1968 McKinney and Ooten, 1969; Lawrence and McCarty, 1970). Most of the these models slightly differ from each other and have evolved from the work of Monod (1949) who found a correlation between limiting nutrients concentration and specific growth rate of microorganisms of the form:

$$\mu = \mu_m \frac{C}{K_s + C} \qquad (6-1)$$

where, μ is the specific growth rate of microorganisms; μ_m is the maximum specific growth rate; C is the limiting nutrient concentration; K_s is the nutrient concentration at which specific growth rate is one-half the maximum value.

In this connection Van Uden (1967) contended that specific substrate removal rate rather than specific organism growth rate is the quantity directly affected by substrate concentration in reactor. Accordingly, Lawrence and McCarty expressed the relationship between substrate removal rate and reactor substrate concentration as:

$$\frac{dC}{dt} = \frac{-k\,X_a C}{K_s + C} \qquad (6-2)$$

where k is the maximum rate of substrate utilization per unit weight of microorganism; X_a is the concentration of microorganisms; K_s is a constant similar to the one defined above.

It is easily seen that if $C \ll K_s$, Eq. 6-2 may be approximated as:

$$\frac{dC}{dt} = -kX_a \qquad (6-3)$$

Again, if $K_s \ll C$, the same relationships may be written as:

$$\frac{dC}{dt} = -k'CX_a \qquad \text{where } k' = \frac{k}{K_s} \qquad (6-4)$$

These are in fact, the relationships given by Eckenfelder in his kinetic model. McKinney's formulation in addition assume that in activated sludge process, substrate removal rate is independent of microorganism concentration. Since the models of Eckenfelder and McKinney are but special cases of the Lawrence and McCarty development, the latter formulation will mostly be used in subsequent discussion.

Current researches by Eckenfelder (1976) uses $K_s + C_0$ as the denominator of Eq. 6-2. The first order reaction is obtained directly since K_s and C_0 remain constant during the reaction. However, the instantaneous reaction rate is dependent on the initial BOD concentration defined by C_0.

The net rate of microorganism growth is commonly defined in waste treatment literature as a function of substrate removal rate as:

$$\frac{dX_a}{dt} = Y\frac{dC}{dt} - k_b X_a \qquad (6-5)$$

where Y is the growth yield coefficient, mass of microorganisms produced/ mass of substrate utilized and k_b is the microorganism decay coefficient.

Eq. 6-2 and 6-5 have been applied in defining the kinetics not only of organic removal but also of nitrification (Downing et al 1964).

The oxidation-reduction state of the carbon source and nutrient elements, degree of polymerization of the substrate and pathways of metabolism are all important factors affecting the value of the growth yield coefficient (Ribbons, 1970). In practice, the yield coefficient is usually assumed to be constant for a given waste although in systems employing mixed cultures of organisms, it has been found to vary significantly (Gaudy and Ramanahtan, 1971).

The MLSS (mixed liquor suspended solids) or MLVSS (mixed liquor volatile suspended solids) concentration is most frequently used as a measure of microorganism concentration in the reactor. Recent research has, however, shown that active microorganism concentration does not bear a constant proportional relationship with either MLSS or MLVSS (Weddle and Jenkins, 1971). It has been found that the longer the sludge is retained in the system the lower the fraction of active microorganisms is in the sludge.

In addition, it has been observed that sludge decay rate coefficient, k_b, is lower at longer sludge *retention* time (McKinney and Ooten, 1969). Several attempts have been made to quantify these effects and the interested reader is referred to the published literature (Barnard et al. 1972, Goodman and Englande, 1974; Grady and Roper, 1974).

Perhaps, one of the most significant limitations of the kinetic models considered is their inability to depict observed process behavior under transient operating conditions. The equations imply that both the specific susbtrate removal rate and the specific growth rate of microorganisms at a given instant are uniquely determined by the substrate concentration at that instant. Experiments have, on the other hand, shown that the adjustment in specific growth rate to a change in substrate concentration involves a significant time lag (Storer and Gaudy, 1969). Others have observed that following the introduction of a shock loading condition, microorganisms exhibit an apparent ability for much faster rate of substrate removal than is to be expected according to the kinetic equations (McLellan and Busch, 1969). Recently some workers have attempted to remedy these shortcomions (Busby and Andrews, 1973; Venkitachalam, 1977), but the increased complexity of the resultant models has made the task of obtaining estimates of model parameters complicated.

Notwithstanding their limitations, the kinetic models considered above have been widely used in process design and operation due to the fact that they are the most rational approach available at present. In a subsequent section the application of these kinetic equations to reactor performance analysis will be illustrated.

Design Parameters

The design parameters for the activated sludge process define the interrelationship of the different process variables at steady state. The design parameters are distinctly different from the control parameters which define the interrelationship of the process variables under dynamic or unsteady state condition. The major defect in current research on the dynamic behaviour of the activated sludge process is the tendency for researchers to use the design parameters as control parameters. The interrelationship among the process variables under dynamic condition does not hold true at steady state and vice versa.

Since the mass balance and biochemical reaction in a completely mixed reactor are easier to establish, the design parameters are developed for the completely mixed reactor. For mixed flow reactors, the completely mixed model is extended to cases wherein it is divided into two or more reactors operated in series. As the number of small reactors operated in series approaches infinity, the whole system operates as a plug flow reactor. The detailed principles and derivation are discussed in Chapter III.

Food to Mass Ratio (F/M) – The mass balance of the incoming, outgoing, accumulation and biochemical degradation in a completely mixed reaction shown in Fig. 6-1, yields:

$$qC_o = qC + kX_aCV + VdC/dt \qquad (6-6)$$

where q is the wastewater flow rate, m^3/day; C is the BOD concentration in the reactor and at the effluent, mg/1; C_o is the influent BOD concentration in mg/1; k is the degradation constant, 1/day-mg; V is the volume of the reactor in m^3; X_a is the microbial mass concentration in the aeration tank, mg/1 and t is the time in days. At steady state dC/dt is equal to zero. Hence, Eq. 6-6 reduces to:

$$C = C_o/(1 + kX_a V/q) \qquad (6-7)$$

If the hydraulic detention time Θ is equal to V/q, then Eq. 6-7 simplifies to:

$$C = C_o/(1 + kX_a\Theta) \qquad (6-8)$$

The right hand side of Eq.6-8 could be divided and multiplied by $1/C_o$ which yields:

$$C = \frac{C_o \dfrac{1}{C_o}}{(1 + kX_a\Theta)\dfrac{1}{C_o}} = \frac{1}{\dfrac{1}{C_o} + \dfrac{kX_a\Theta}{C_o}} \qquad (6-9)$$

Since the influent BOD is often very much higher than one, then $1/C_o \rightarrow 0$ and Eq 6-9 simplifies to:

$$C = \frac{1}{k}\,\frac{C_o}{X_a\Theta} = \frac{C_oq}{k\ X_aV} = \frac{1}{k}\,\frac{F}{M} \qquad (6-10)$$

The food to mass ratio F/M is defined as the weight of BOD applied per unit of microbial mass per day. From Eq. 6-10:

$$F/M = \frac{C_o}{X_a \Theta} = \frac{C_o q}{X_a V} \qquad (6-11)$$

At steady state, the effluent quality from a completely mixed activated sludge is directly proportional to the food to mass ratio as defined by Eq.6-11. The F/M ratio defines the interaction of the microbial mass concentration X_a, the wastewater flow rate, q, the reactor volume, V and the influent BOD concentration C_o. The F/M ratio has similar function as the dimensionless groups discussed in Chapter I. In some textbooks food to mass ratio is also known as the sludge loading rate. (SLR)

Example 6-1. The pilot study shows that at F/M ratio of 0.1 kg of BOD/kg of MLVSS-day, the effluent quality is 25 mg/1 for an industrial waste with an initial BOD of 250 mg/1, a) Determine the k value. b) If the wastewater has a flow rate of 5,000 m^3/day determine the microbial mass concentration and volume of the aeration tank. c) Two years after the wastewater treatment plant has been constructed the wastewater flow rate has increased to 7,000 m^3/day. How would you accommodate the increased flow without incurring expensive reconstruction cost? d) what is the required F/M if the effluent quality is 20 mg/1.

Solution: a) From Eq. 6-10

$$k = F/M \; \frac{1}{C}$$

$$= 0.1/25 = .004 \text{ 1/mg-day}$$

b) From Eq. 6-11

$$F/M = \frac{C_o q}{X_a V} = .1$$

Hence,

$$X_a V = C_o\, q/(F/M)$$
$$= 12{,}500{,}000 \text{ mg-m}^3/1$$

The value of X_a and V could be adjusted over a wide range of values as long as $X_a V$ is equal to 12,500,000 mg-m^3/1.

It would be good practice to use the minimum possible microbial mass concentration to provide future flexibility in operation. Use X_a = 1,500 mg/1, hence

$V = 12{,}500{,}000 / (1{,}500) = 8333\ m^3$

c) When the flow rate is increased to 7000 m^3, the same effluent quality could be attained by increasing the microbial mass concentration in the aeration tank without increasing the aeration tank volume. From Eq. 6–11,

$$F/M = \frac{C_o q}{X_a V} = .1$$

For q of 7,000 m^3/day, V = 8,333 m^3 and C_o = 250 mg/1,

$X_a = C_o q / (.1V)$

$= 250 \times 7000 / (.1 \times 8333)$

$= 2100$ mg/1

Bench scale or pilot plant studies are often carried out under ideal operating conditions. The prototype operating results are expected to be poorer than the results from pilot plant studies. It is a good practice to initially design the activated sludge at the lowest possible MLVSS level. The performance of the process could be easily upgraded by increasing the microbial mass concentration in the aeration tank. If the microbial mass concentration is already at its maximum value, the desired effluent quality could only be attained by increasing the aeration tank volume which would incur expensive construction cost not to mention that in most instances, the space for the wastewater treatment plant in the factory site is very limited.

d) From Eq. 6-11

$C = (F/M)\ (1/k)$

k = .004/day and C = 20 mg/1 then,

$F/M = kC$

$= .004 \times 20 = .08$/day

Sludge Utilization Rate (SUR) – In some instances $kX_a\Theta/C_o$ is very close to $1/C_o$ in which case the simplication of Eq. 6-9 to 6-10 is not warranted. At steady state Eq. 6-6 could be simplified to:

$$C = \frac{q\,C_0 - q\,C}{kX_a\,V} = \frac{1}{k}\,\frac{q\,(C_0 - C)}{VX_a} = \frac{SUR}{k} \qquad (6-12)$$

When the effluent BOD is very low compared to the influent BOD, the sludge utilization rate is approximately equal to the food to mass ratio. In most instances, C varies from $0.05C_0$ to $.1C_0$. Hence, the sludge utilization rate is approximately 0.95 F/M to 0.9 F/M. The relationship between the F/M and sludge utilization rate is very similar to the areal loading rates and areal removal rates discussed in Chapter V.

If in the activated sludge process oxygen is supplied by mechanical aerators or diffusers, the dissolved oxygen concentration is designed to be non-rate limiting. The major constraint in the process is to provide sufficient residence time for the reaction to take place and microbial biomass to degrade the wastes. The sludge utilization rate defines the quantity of organic wastes that could be removed per unit weight of microbial mass. As the waste treatment efficiency increases, more microbial mass must be introduced into the reactor to remove the organic wastes.

The sludge utilization rate is more flexible in simulating the behavior of a number of completely mixed activated sludge operated in series than the food to mass ratio. However, the food to mass ratio is often simpler to apply and is more commonly used in practice.

Example 6-2. Determine the microbial mass concentration and volume for a two-stage activated sludge treating domestic sewage with an initial BOD of 250 mg/1, and flow rate of 3,000 m^3/day if the required effluent concentration is 25 mg/1. The k value of the waste is 0.01 1/mg-day. Compare the results obtained when using F/M and SUR as design parameters.

Solution: a) The required F/M ratio from Eq. 6-10 is:

$$F/M = k\;C$$

$$= \left(\frac{.0101}{\text{mg-day}}\right)\left(\frac{25\text{ mg}}{1}\right) = 0.25/\text{day}$$

Hence,

$$\frac{q\;C_0}{X_a\,V} = \frac{F}{M} = .25/\text{day}$$

$$X_a\,V = \frac{3{,}000 \times 250}{0.25} = 3{,}000{,}000\;\frac{\text{mg}-\text{m}^3}{1}$$

Using X_a = 1,500 mg/1

then, the volume V is equal to:

$$V = \frac{3,000,000}{1,500} = 2,000\ m^3$$

b) To optimize the aeration tank utilization, the BOD removal per stage should be proportional. Hence, as a trial

$$\frac{C_1}{C_o} = \frac{C}{C_1} \text{ or } \frac{C_1}{250} = \frac{25}{C_1}$$

or

$$C_1 = \sqrt{25 \times 250} = 79\ mg/1$$

For the first stage

$$79 = \frac{SUR}{0.01}$$

or

$$SUR = 0.79 = \frac{q\ (C_o - C)}{VX_a}$$

Then

$$V_1 X_a = 3,000\ (250 - 79)/0.79$$

$$= 649,367 \quad mg - m^3/1$$

Using $X_a = 1,500$

$$V_1 = 433\ m^3$$

For the second stage, the SUR could be calculated using Eq. 6-12, which yields:

$$25 = \frac{SUR}{0.01}$$

or SUR = 0.25

The value of X_aV is calculated from the definition of SUR which results in:

$$\frac{q\ (C_1 - C)}{VX_a} = SUR = .25$$

or

$$VX_a = 3500\,(79\text{-}25)\,/\,(.25)$$

$$= 648{,}000\ \text{mg}\text{-}\text{m}^3/\text{l}$$

To facilitate the construction, operation and maintenance of the units, the second unit should be of similar volume, hence, the microbial mass concentration in the second unit is:

$$X_a = 648{,}000/433 = 1497'\ \text{mg/l}$$

Actual X_a in the second tank is

$$X_a = X_{a,1} + Y\,(C_1 - C)$$

$$= 1500 + .5\ (79\text{-}25) = 1527\ \text{mg/l}$$

The required volume is only

$$V_2 = 648{,}000/1527 = 424\ \text{m}^3$$

Hence, the use of 433 m^3 aeration tank is safe. The total aeration tank volume is then equal to:

$$V = V_1 + V_2 = 433 + 433 = 866\ \text{m}^3$$

c) The volume required for a two-stage activated sludge is only 43% of the volume required using the F/M ratio due to the inability of the F/M to quantify the effects of reactor staging. The analysis using SUR as a design parameter could be easily extended to other variations of the activated sludge process such as step feed aeration, plug flow, tapered aeration in addition to the completely mixed activated sludge processes.

Sludge Retention Time(SRT) – In the derivation of the food to mass ratio and sludge utilization rate, X_a is assumed to be an independent process variable. However, X_a is also a by-product of the biochemical reaction, and it undergoes degradation in the aeration tank as well. Although the microbial mass concentration in the system could be very much higher than the microbial mass produced during the biochemical reaction, the error introduced in the design of the activated sludge process could be significant.

The mass balance boundary extended to cover the secondary clarifier in addition to the aeration tank is shown in Fig. 6-2. The mass balance of the microbial cell yields:

$$qX_o = q_w X_r + (q - q_w)\,X_e - qY\,(C_o - C) + k_b X_a V + V\frac{dX_a}{dt} \qquad (6-13)$$

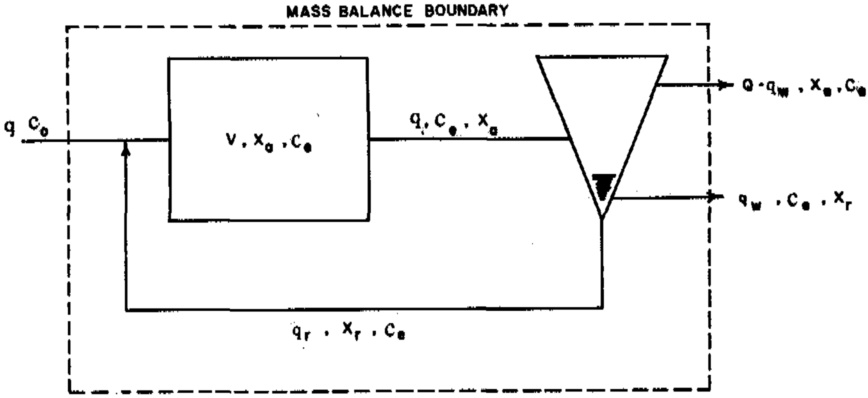


FIG. 6-2 Mass balance of an activated sludge process.

where X_0 is the microbial mass concentration in the influent, mg/l of MLVSS; X_r is the settled microbial mass concentration in the clarifier, mg/l of MLVSS; X_a is the microbial mass concentration in the aeration tank, mg/l of MLVSS; q_w is the desludging rate, m^3; k_b is the microbial respiration rate,/day, Y is the yield coefficient, mg of MLVSS produced/mg of BOD removed; C_0 is the influent BOD concentration, mg/1; and C is the BOD concentration in the aeration tank. At steady state, VdX_a/dt is equal to zero, hence, Eq 6-13 reduces to:

$$q_w X_r + (q - q_w) X_e + k_b X_a V = q Y (C_o - C) + q X_o \qquad (6\text{-}14)$$

If X_0 is very low, then Eq. 6-14 simplifies to:

$$q Y (C_o - C) - k_b X_a V = q_w X_r + (q - q_w) X_e \qquad (6 - 15)$$

Eq. 6-15 is nothing more than the restatement of the equilibrium or steady state condition. The net rate of microbial mass production in the system is equal to the rate of microbial mass removal out of the systems, otherwise, the microbial mass inventory will continuously decrease or increase. Both sides of Eq. 6-15 could be divided by X_aV, provided X_aV is not equal to zero to yield:

$$\frac{1}{SRT} = \frac{q Y (C_o - C) - k_b X_a V}{X_a V} = \frac{q_w X_r + (q - q_w) X_e}{X_a V} \qquad (6 - 16)$$

By definition, sludge retention time is equal to the microbial mass in the aeration tank divided by the rate of microbial mass removal out of the aeration tank and the clarifier *taken as one system*. Since the rate of microbial mass formation is equal to the rate of sludge withdrawal at steady state then sludge retention time is also equal to the microbial mass in aeration tank divided by the net rate of microbial mass formation as shown in Eq. 6-16. The left hand side of Eq 6-16 could be rearranged as:

$$\frac{1}{SRT} = \frac{Y\, q\,(C_o - C)}{X_a V} - k_b \qquad (6-17)$$

Substituting the definition of SUR in Eq. 6-12 into Eq. 6-17 yields:

$$\frac{1}{SRT} = Y\,(SUR) - k_b \qquad (6-18)$$

Since the effluent quality is defined by the sludge utilization rate as shown in Eq. 6-12, then,

$$\frac{1}{SRT} = YCk - k_b \qquad (6-19)$$

or

$$C = \frac{1}{Yk}\left\{\frac{1}{SRT} + k_b\right\}$$

Eq. 6-19 is more comprehensive than the F/M or the SUR as a design parameter since it includes the microbial mass kinetic parameters in defining the effluent BOD quality. Like the F/M and SUR, the design parameter SRT defines the interaction of the microbial concentration in the aeration tank, the aeration tank volume, the wastewater flow rate, the influent and effluent BOD concentration and the microbial kinetic constant. As a design parameter, SRT has the same function as the dimensionless groups described in Chapter I. It is erroneous to conclude from Eq. 6-19 that the effluent BOD of the completely mixed activated sludge depends only on the sludge retention time and is independent of the hydraulic retention time, microbial mass concentration in the aeration tank and influent BOD concentration as Marais (1973) did. In fact, SRT is a function of the hydraulic retention time, the influent BOD concentration and the microbial mass concentration in the aeration tank. Marais' conclusion is just like saying that friction losses in pipes depend on the Reynolds number and independent of the viscosity, velocity and density of the fluid.

Example 6-3. Determine the volume and microbial mass concentration required to treat the industrial wastewater whose characteristics are given in Example 6-1. The k value is .004 1/mg-day, Y is 0.5 and k_b is 0.01/day.

Solution: From Eq. 6-19.

$$\frac{1}{SRT} = Y C k - k_b$$

$$= .5 \times 25 \times .004 - .01$$

$$= .04$$

or

$$SRT = 25 \text{ days}$$

From Eq. 6-16, the required X_aV is calculated which yields:

$$X_aV = \frac{SRT \quad 3{,}000 \times .5 \; (250 - 25)}{(1 + .01)}$$

$$= 8{,}353{,}960 \text{ mg} - \text{m}^3/1$$

For X_a of 1500 mg/1,

$$V = 8{,}353{,}960/\ (1500) = 5569 \text{ m}^3$$

SRT Through MLVSS Desludging – If the sludge withdrawal is carried out at the aeration tank directly instead of the clarifier the mass balance of the microbial mass yields:

$$V \frac{dX_a}{dt} = q Y (C_o - C) - k_b X_a V - q_w X_a - (q - q_w) X_e \qquad (6-20)$$

where the terms have been defined before. At steady state, dX_a/dt is zero, hence Eq.6-20 simplifies to:

$$qY (C_o - C) - k_b X_a V = q_w X_a - (q - q_w) X_e \qquad (6-21)$$

Eq. 6-21 is a restatement of the steady state condition of the system which shows that the net rate of the microbial mass withdrawal is equal to the net rate of microbial mass production. If both sides of Eq. 6-21 are divided by X_aV, in view of the definition of SRT, then,

$$\frac{1}{SRT} = \frac{qY(C_o - C) - k_bX_aV}{X_aV} = \frac{q_wX_a - (q - q_w)X_e}{X_aV} \qquad (6-22)$$

Since, X_e is often very low with values less than 20 mg/1, compared to X_a which is in the range of 1,500 – 5,000 mg/1 then X_e could be neglected. Hence,

$$SRT = \frac{X_aV}{q_wX_a} = \frac{V}{q_w} \qquad (6-23)$$

The microbial mass concentration in the aeration tank depends on the sludge recirculation rate and the net microbial mass production rate. Some experts such as Garett and Sawyer (1959) proposed the use of MLVSS desludging to control the sludge retention time. Two clarifiers are provided per aeration tank where the first is used to concentrate the solids for recirculation to the aeration tank and the second, to concentrate the wasted sludge.

The proponents of MLVSS desludging to control the sludge retention time claim that the cost of additional clarifier is justified since the desired SRT levels are attained without further adjustment on the sludge recirculation rate, since SRT has become independent of microbial mass concentration X_a. The conclusion is erroneous in two major aspects. Although X_e is very low compared to X_a, the flow rate q-q_w could be a hundred times higher than q_w. The value of X_e $(q - q_w)$ could vary from 10 to 40% of X_aq_w depending on the type of activated sludge process. Hence, the assumption disregarding the X_e $(q$-$q_w)$ introduces a large error in the derivation of Eq. 6-23.

The mass balance boundary in deriving the SRT formula is based on the aeration tank and clarifier as one system.

To illustrate the failure of Eq. 6-23 to quantify the BOD removal in the system, consider the microbial mass concentration and BOD mass balance around the aeration tank and clarifier separately. The BOD balance around the aeration tank at steady state yields:

$$(q + q_r - q_w)C + q_wC + kX_eCV = qC_o + q_rC \qquad (6-24)$$

or

$$C = C_o / (1 + kX_aV/q)$$

Eq. 6-24 is equal to Eq. 6-6, which shows that the effluent BOD is unaffected by the MLVSS desludging rate q_w directly. The MLVSS desludging rate q_w could only affect quality C if and only if it influences the microbial mass concentration X_a as shown in Eq. 6-24. The other parameter V, q, and k in Eq. 6-24 are constants.

A mass balance of the microbial concentration X_a in the aeration tank at steady state yields:

$$qX_o + q_rX_r + Yq(C_o - C) - k_bX_aV = q_wX_a + (q - q_w)X_a \qquad (6-25)$$

When X_o is very low, then solving for X_a results in:

$$X_a = \frac{q_rX_r + Yq(C_o - C)}{q + k_bV} \qquad (6-26)$$

From Eq. 6-26, X_a is independent of q_w. Hence, the effluent quality C is unaffected by q_w. This conclusion is a contradiction to Eq. 6-19 if SRT is defined by Eq. 6-23.

To resolve the contradictions consider the mass balance of the microbial mass in the clarifier as equal to:

$$(q - q_w)X_a - q_rX_r - (q - q_w - q_r)X_e = \frac{d(X_rV_r)}{dt} \qquad (6-27)$$

If q_w increase such that $(q - q_w)X_a$ is lower than the sum of q_rX_r and $(q - q_w - q_r)X_e$, the residual solids in the clarifier defined by (X_rV_r) are depleted. Conversely, if $(q - q_w)X_e$ is higher than the sum of q_rX_r and $(q - q_w - q_r)X_e$, then the microbial solids in the clarifier will continuously pile up until it overflows. For any change in q_w, the clarifier could exist in steady state if q_r is simultaneously adjusted such that:

$$q_rX_r - (q - q_w - q_r)X_e = (q - q_w)X_a \qquad (6\text{-}28)$$

or

$$q_r = \frac{(q - q_w)(X_a - X_e)}{X_r - X_e}$$

Since, $X_a >> X_e$ and $X_r >> X_e$ then Eq. 6-28 simplifies to:

$$q_r = \frac{(q - q_w)X_a}{X_r} \qquad (6-29)$$

If q_r is a function of q_w, then substituting Eq. 6-29 into Eq. 6-26 yields:

$$(q + k_bV)X_a = (q - q_w)X_a + Y(C_o - C)q \qquad (6-30)$$

Dividing both sides of Eq. 6-30 by X_aV, yields

$$\frac{q}{V} + k_b = \frac{q}{V} - \frac{q_w}{V} + \frac{Y(C_o - C)q}{VX_a} \qquad (6-31)$$

or

$$k_b = \frac{-1}{SRT} + Y\,q \left[\frac{(C_o - C)}{VX_a}\right]$$

Substituting Eq. 6-12 into Eq. 6-31 yields:

$$\frac{q_w}{V} = \frac{1}{SRT} = YkC - k_b \qquad (6-32)$$

Hence, SRT level obtained through MLVSS desludging is a valid design parameter only if the sludge recirculation from the clarifier to the aeration tank is adjusted as defined in Eq. 6-29 to correspond with any change in q_w.

If the sludge recirculation rate from the clarifier to the aeration tank is held constant, the clarifier sludge will vary rather than the aeration tank MLVSS as intended. As very little biochemical activity takes place in the clarifier, the fluctuations of the clarifier solids will have no significant effect on the process performance.

The activated sludge process which controls the SRT levels by desludging the aeration tank MLVSS only complicates the process operation and increases the cost of the whole system without any benefits on the process performance. MLVSS desludging is only applicable in bench scale activated sludge units wherein the settled sludge is automatically recycled into the aeration tank since the clarifier is very shallow compared to the aeration tank. Automatic change of the recirculation rate takes place with variation of q_w in bench scale units specifically designed as shown in Fig. 6-3.

Process Parameters – The microbial mass balance in the aeration tank at steady state when desludging is made at the clarifier is:

$$q_r X_r + q\,Y(C_o - C) - k_b\,X_a\,V = (q_r + q)\,X_a \qquad (6-33)$$

Rearranging Eq. 6-33 yields:

$$1 = \frac{q_r}{q}\,\frac{X_r}{X_a} + \left(\frac{C}{C_o} - 1\right)\frac{C_oY}{X_a} \qquad (6-34)$$

From Eq. 6-34, the BOD removal efficiency could be expressed as:

$$\left(1 - \frac{C}{C_o}\right) = \frac{1 + k_b\,V/q + q_r/q - (q_r/q)\,(X_r/X_a)}{Y\,C_o/X_a} \qquad (6-35)$$

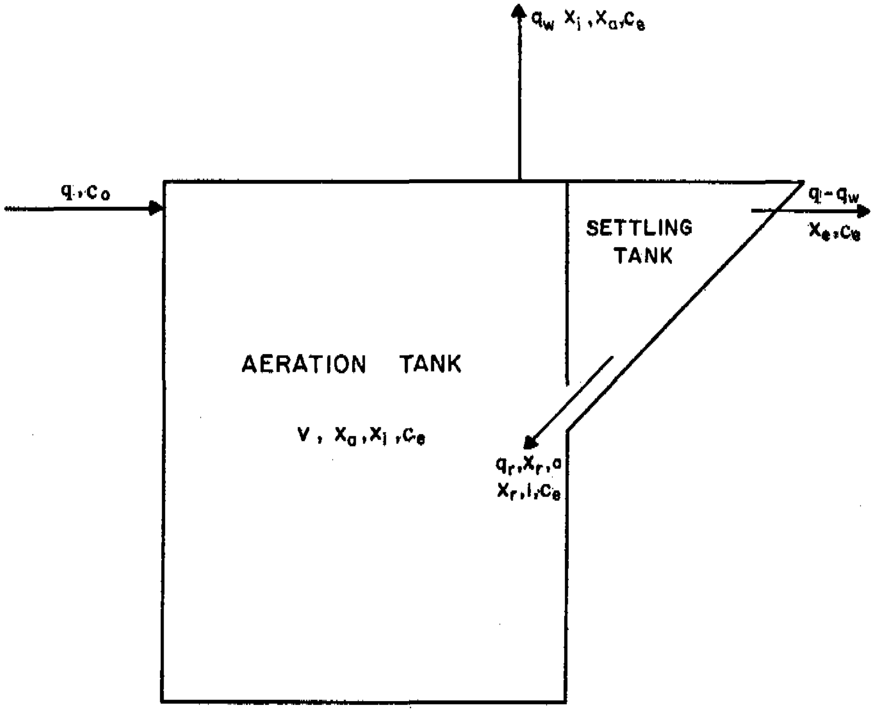


FIG. 6-3 Schematic diagram of bench scale activated sludge unit showing automatic q_r adjustment.

If desludging is carried out in the aeration tank the microbial mass concentration in the aeration tank could be expressed as:

$$X_a = \frac{q_r X_r + q\ Y\ (C_o - C)}{q_r + q - q_w + k_b V} \qquad (6-36)$$

The BOD removal efficiency could be derived from Eq. 6-36 as:

$$\frac{\left(C_o\ C\right)}{C_o} = \frac{\frac{q_r}{q} + \left(1 - \frac{q_w}{q}\right) + k_b\ \frac{V}{q} - (q_r/q)\left(\frac{X_r}{X_a}\right)}{Y\left(\frac{C_o}{X_a}\right)} \qquad (6-37)$$

or

$$\frac{C_o-C}{C_o} = f\,(q_r/q,\, q_w/q,\, k_b\ V/q,\, X_r/X_a,\, C_o/X_a,\, Y)$$

To maintain similitude between the prototype and the bench scale unit, the dimensionless groups defined by Eq. 6–35 and 6–37 must be conserved. Desludging the aeration tank MLVSS complicates the similitude requirements since q_w / q has to be conserved in addition to the other dimensionless variables.

If $k\ Y\ X_a C$ is used instead of $qY\,(C_0\text{-}C)$ to define the microbial mass produced in the aeration tank, it could be shown that the BOD removal efficiency could be expressed as:

$$\frac{C_o\text{-}C}{C_o} = f\left(q_r/q,\, k_b\ V/q,\, \frac{X_r}{X_a},\, \frac{1}{kYC_o}\right) \qquad (6-38)$$

If the wastewater used in the pilot plant is the same as that in the prototype, then k_b, K, Y, and C_o are inherent properties of the wastewater. Eq. 6-38 could be simplified as:

$$\frac{C_o - C}{C_o} = f\left(q_r/q,\, V/q,\, \frac{X_r}{X_a},\, 1/(kYC_o)\right) \qquad (6-39)$$

Similarly Eq. 6-35 could be expressed as:

$$\frac{C_o - C}{C_o} = f\left(q_r/q,\, V/q,\, \frac{X_r}{X_a},\, \frac{C_o}{X_a}\right) \qquad (6-40)$$

Eq. 6-39 and 6-40 define the process variables which have to be conserved for similitude to exist between the prototype and the pilot plant. The design parameters given in the previous section define the interaction between X_a and the detention time required for the same effluent quality. The detention time V/q is fixed by the aeration tank volume and the influent pumping rate q. The desired microbial mass concentration X_a is attained by manipulating the recirculation rate q_r, the MLVSS concentration in the clarifier and the influent BOD concentration. The term $q_r/\ q$ is known as recirculation ratio. The process variables define the conditions which must be maintained in order to attain the required MLVSS level in the aeration tank.

Sludge Production

The quantity of sludge wasted from the aeration tank is defined by Eq. 6-15. The weight of sludge wasted per day at equilibrium is equal to the net mass of sludge produced in the aeration tank. Hence,

$$M_s = (q\,Y\,(C_o - C) - k_b\,X_a V - (q - q_w)\,X_e\,)/e \qquad (6-41)$$

In an average clarifier the settled sludge concentration could vary from 8,000 to 15,000 mg/1. The desludging rate q_w is equal to:

$$q_w = \frac{M_s}{X_r} = \left\{ \frac{q\;Y\;(C_o - C)}{X_r} - \frac{k_b X_a\;V}{X_r} - \frac{qX_e}{X_r} + \frac{X_e}{X_r}\,q_w \right\} \frac{1}{e}$$

or

$$q_w = \frac{q\,Y(C_o - C) - k_b X_a\;V - q\;X_e}{(\;X_r - X_e)\;e} \qquad (6-42)$$

Dividing both sides of Eq. 6-42 by q, yields,

$$\frac{q_w}{q} = \frac{Y(C_o\; - C) - k_b X_a\;V/q - \;X_e}{(X_r - X_e)\,e} \qquad (6-43)$$

In a very efficient clarifier, X_e could vary from 5 to 30 mg/1 and would be small compared to X_r The value of e was estimated by Marais (1973) as:

$$e = 0.85/\;(SRT)^{\;0.1} \qquad (6-44)$$

Example 6-4. a) Calculate the mass of the sludge removed per day from an activated sludge process used in treating domestic sewage with an initial BOD of 250 mg/1. The sewage flow rate is 10,000 m^3/day, and the required effluent BOD is 20 mg/1. The Y and k_b values are 0.5 and 0.1/day respectively. The detention time in the aeration tank is 0.25 day. The MLVSS in the aeration tank is 2,000mg/1. b) If the MLVSS concentration in the clarifier is 10,000 mg/1, calculate the desludging rate. c) If the effluent MLVSS is 30 mg/1, calculate the solids outflow going with the treated effluent.

Solution: a) Assume e is close to one. From Eq. 6-43,

$$\frac{q_w}{q} = \frac{0.5\,(250\text{-}20)\; - 0.1 \times 2000 \times 0.25 - 30}{10{,}000 - 30}$$

$$= \frac{115 - 50 - 30}{10{,}000 - 30} = .0035$$

$$q_w = .0035 \times 10{,}000 = 35\ m^3\ /\ day$$

The mass of sludge wasted is equal to:

$$M_s = q_w X_r = 35 \frac{m^3}{day} \times 10{,}000 \frac{mg}{1} \times 10^{-3} \left(\frac{kg\text{-}1}{mg\text{-}m^3}\right)$$

$$= 350\ kg/day$$

b) q_w is 35 m^3/day from step a

c) The mass of solids in the overflow is equal to:

$$M = (q - q_w)\ (30)\ (10^{-3})$$

$$= (10{,}000 - 35)\ (30)\ (.001)$$

$$= 299\ kg/day$$

Although the solid concentration in the clarifier overflow is very low, the flow rate is very high compared to the desludging rate. About 20-50% of the total solids produced are carried out with treated effluent. For this reason the microbial mass removed from the activated sludge system through the clarifier effluent must be considered in the calculation of the sludge retention time.

Oxygen Requirement

Part of the BOD removed from the wastewater is converted to microbial cells or sludge. The five day BOD equivalent per gm of solids is approximately equal to 0.95 gm while the ultimate BOD is approximately 1.6 times the five day BOD. Hence, the total oxygen required to reduce the influent BOD from C_0 to C could be expressed as:

$$O_r = \left\{ - q\,Y\,(C_0 - C)\,(.95) + .95\,X_a V k_b + (C_0 - C)\,q \right\} 1.6$$

$$= 1.6\,(C_0 - C)(1 - .95\,Y)\,q + 1.52\,X_a V k_b \qquad (6-45)$$

When the sludge retention time exceeds the value,

$$SRT = 3.05\ (1.127)^{20-T} \qquad (6-46)$$

the nitrogen in the wastewater is oxidized. T is temperature in °C and SRT is in days. The nitrification reaction could be expressed as:

$$NH_4^+ + 4O_2 \longrightarrow 2\ NO_3^- + 2H_2O + 4\ H^+ \qquad (6-47)$$

Hence, for every mg of ammonia nitrogen oxidized 4.6 mg of oxygen is required. When the sludge retention time is lower than the limit defined by Eq. 6-46 the nitrifying bacteria are washed out and little nitrification is expected. Eq. 6-46 was experimentally determined by Downing, Painter and Knowles (1964).

Part of the nitrogen in the wastewater is converted to cellular protein by the bacteria. The nitrogen content of bacterial cells could vary from 7–10% by weight. The mass balance of the different nitrogen mass flow in the aeration tank yields:

$$q\,N_o = q\,N_e + N_x q + f\,(q\,Y\,(C_o - C) - k_b\,X_a V) \qquad (6\text{–}48)$$

where N_o is the influent ammonia nitrogen concentration, mg/1; N_e is the effluent ammonia nitrogen concentration, mg/1; N_x is the oxidized or nitrate nitrogen concentration, mg/1; f is the weight fraction of nitrogen in the sludge produced and the other terms have been defined before. When the bacterial cells die, the nutrients are released back to the wastewater in a process known as lysis. However, part of the nutrients are retained in the dead cells. Eq. 6-48 would result in a conservative estimate of the oxygen required for nitrification assuming that all the nitrogen are released during lysis and are subsequently oxidized to nitrate.

Rearranging Eq. 6-48 yields:

$$qN_x = q\,(N_o - N_e) - f\,(q\,Y\,(C_o - C) - k_b X_a V\,) \qquad (6-49)$$

Multiplying Eq. 6-49 by 4.6 gives the oxygen required for nitrification. Hence,

$$O_n = 4.6\,(q\ \ (N_o\text{-}N_e) - f(q\ \ Y\,(C_o\text{-}C)\ - k_b X_a V)\,) \qquad (6-50)$$

At steady state the net rate of sludge production is equal to the rate of sludge withdrawal. Hence, introducing the definition of SRT into Eq. 6-50, yields:

$$O_n = 4.6 \left\{ q\,(N_o\text{-}N_e) - f \left(\frac{X_a V}{SRT} \right) \right\} \qquad (6-51)$$

The effluent nitrogen concentration is often very low at 1-2 mg/1 even if SRT is just a day or two higher than the minimum requirement. The concentration of the nitrifying bacteria seems to be the rate limiting condition rather than the ammonia nitrogen. Exceptions may occur when treating industrial wastes with very high ammonia nitrogen.

Example 6-5. Calculate the oxygen required to treat 10,000 m^3/day of domestic sewage with 40 mg/l of ammonia nitrogen, and 250 mg/l of influent BOD. The value of Y is 0.4: k, 0.03/day, and k_b, 0.1/day. The required effluent BOD is 20 mg/l. Operating temperature is 30°C.

Solution: i) From Eq. 6-19 which is,

$$C = \frac{1}{Yk}\left\{\frac{1}{SRT} + k_b\right\}$$

SRT is calculated as:

$$20 = \frac{1}{.4 \text{x} .03}\left\{\frac{1}{SRT} + .1\right\}$$

SRT = 7 days

ii) The X_aV value could be calculated from the SUR given in Eq. 6 –12

$$20 = \frac{(C_o - C)\ q}{k\ X_a\ V} \simeq \frac{(250-20)q}{.03\ (X_aV)}$$

$$X_aV = 383\ q\ \frac{mg \cdot m^3}{l}$$

iii) The minimum SRT from Eq. 6-46 is

$SRT = 3.05\ (1.124)^{20-30} = .92$ days < 7 days. Hence, nitrification is expected.

From Eq. 6-50,

$$O_n = 4.6 \text{ x } .001\left\{40 - \frac{.1 \text{ x } 383}{7}\right\} 10{,}000$$

$$= 1588 \text{ kg/day}$$

iv) The oxygen required for BOD removal is calculated from Eq. 6-45 which yields:

$$O_r = 10{,}000 \text{ x } .001\left\{(250\text{-}20)\ (1.6)\ (1 - .95 \text{ x } .4) + 1.52\,(.1)(383)\right\}$$

$$= 2683 \text{ kg/day}$$

Hence, total oxygen required is:

$$O_2 = 2683 + 1588 = 4451 \text{ kg/day}$$

For nitrification, the minimum detention time is dependent on the dissolved oxygen concentration which should not be lower than 1 mg/1. The detention time for nitrification could be calculated as:

$$\Theta_n = \frac{SRT\ Y\ (C_o - C)}{X_a\ (1 + k_b\ SRT)}$$

Normally Θ_n is smaller than Θ required for BOD degradation unless the effluent BOD quality is very high compared to that of influent BOD. From the above example:

$$\Theta_n = \frac{7 \times .4\ (250\text{-}20)}{(1 + .1\ \times 7)X_a} = \frac{378}{X_a}$$

If the detention time for BOD degradation is set at 6 hours or 0.25 day, then from step ii,

$$X_a = 383\frac{q}{V} = \frac{383}{.25} = 1532 \text{ mg/1}$$

Hence,

$$\Theta_n = \frac{378}{1532} = .246 \text{ days} < .25 \text{ days}$$

The oxidation equipment sizes could be calculated in the same manner as in the aerated lagoon. Refer to Chapter V for aerator and diffuser sizing.

Parameter Estimation

To determine the value of Y, k, and k_b a completely mixed activated sludge model is used. The aeration tank is often very deep compared to the clarifier tank. The clarifier bottom unit slopes steeply towards the aeration tank so that settled solids are automatically recycled to the aeration tank. In cases where mechanical aerators are used, the mixing pattern is adjusted to facilitate the recirculation of the settled sludge. For activated sludge unit of this type the q_r/q value is adjusted automatically. The MLVSS concentration in the aeration tank is controlled by varying the desludging rate. Since, the sludge recirculation rate, q_r, and X_r are invariant in this process, SRT is favorably controlled by desludging the aeration tank MLVSS. Fig. 6-3 is a typical schematic diagram of bench scale activated sludge process.

The k value is the average of the calculated values using the sludge utilization rate relation given by Eq. 6-12. Hence,

$$k = \frac{1}{n} \sum_{i=1}^{n} \frac{q\ (C_o - C_i)}{C_i\ V\ X_a} = \frac{1}{n} \sum_{i=1}^{n} \frac{SUR}{C_i} \qquad (6-52)$$

The relationship between sludge utilization rate (SUR) and the sludge retention time (SRT) given in Eq. 6-18 is a restatement of the relationship between the microbial mass production and the BOD removal rate, defined as:

$$\frac{dX_a}{dt} = -Y \frac{dS}{dt} - k_b X_a \qquad (6-53)$$

As discussed in Chapter III, dividing Eq. 6-53 by X_a, yields

$$\frac{1}{X_a} \frac{dX_a}{dt} = \frac{-Y}{X_a} \frac{dS}{dt} - k_b \qquad (6\text{-}54)$$

Eq. 6-54 could be simplified further as:

$$\frac{1}{SRT} = Y\ SUR - k_b \quad \text{if } SUR = -(dC/dt)/X_a \qquad (6-55)$$

From Eq. 6-55 or 6-19, the values of Y and k_b are determined by the least squares method.

BOD/COD Ratio – The cost of pilot plant testing and evaluation could vary from 20 to 50% of the total capital cost of small activated sludge units treating industrial wastewater so that designers are often forced to make rough estimates of the design parameters intuitively from past experience. The problem is very critical in developing countries where more than 90% of the industries are capitalized at less than US$50,000 and the designers of waste treatment plants have limited experience.

The BOD/COD ratio has been used for a long time to describe qualitatively the suitability of biological treatment processes to stabilize a particular type of industrial wastewater. Wastewaters with a BOD/COD greater than 0.4 are often considered biodegrable and amenable to biological waste treatment processes.

Domestic sewage has a BOD/COD ratio varying from 0.4 to 0.6 depending on the proportion of industrial wastewater discharges to the domestic sewers. However, the quantitative relationship between the BOD/COD ratio and the process design parameters has not been established.

This section reviews the operating performance of completely mixed and extended aeration processes as reported in literature, after which the design parameters are quantitatively correlated with the BOD/COD ratio using regression analysis.

The BOD test measures the quantity of oxygen used in 5 days at 20°C by very dilute concentration of microbial mass with sufficient quantities of trace elements and nutrients essential for microbial growth feeding on wastewaters. The 5 day BOD (BOD_5) is a function of the ultimate carbonaceous (BOD_u) which is defined as:

$$BOD_5 = BOD_u \; (1 - \exp(-5\,K)) \qquad (6-56)$$

where K is the rate of biological degradation of the wastewater under condition of the BOD test described above, day $^{-1}$.

With the proper catalyst, the COD value is a close approximation of BOD_u. Hence, from Eq. 6-56 the degradation rate constant K, under the conditions of the BOD test could be expressed as:

$$K = \frac{1}{5} \ln \; (1 - BOD/COD) = f\,(BOD/COD) \qquad (6-57)$$

Similarly, the activated sludge process consists of a biological reactor with the reaction carried out at high microbial mass concentration, sufficient oxygen, essential nutrients and trace elements. A mass balance around the reactor shown in Fig. 6-2 yields;

$$V \frac{dC}{dt} = qC_o + q_rC - (q + q_r)\; C - kCX_aV/\,(K_m + C_o) \qquad (6-58)$$

where V is the volume of the reactor, m^3, C is the effluent substrate concentration expressed as mg/1 of BOD or COD; q is the influent flow rate, m^3/hr; q_r is the recirculation rate, m^3/hr; C_o is the initial substrate concentration, mg/1 of BOD or COD; k is the treatability constant, hr^{-1}; X_a is the microbial mass concentration, mg/1 or MLVSS; and K_m is the Monod constant, mg/1.

A number of variations to the inhibitory term other than $K_m + C_0$ have been proposed. Current studies by Adams (1974); Roth, Eckenfelder, and Oleszkiewics (1975); Eckenfelder, Roth, and McMullen (1975); and Eckenfelder (1976) show that ($K_m + C_o$) is the best quantification of the inhibitory term for industrial wastes.

At steady state, Eq. 6-58 simplifies to:

$$k = \frac{q\,(C_o - C)\;(K_m + C_o)}{VX_a \quad C} \qquad (6-59)$$

When $C_0 \gg K_m$ and using the definition of the food-to-mass ratio ($F/M = qC_0/(VXa)$), Eq. 6-59 could be expressed as:

$$k = \frac{F\,(C_0 - C)}{M\,C} \qquad (6-60)$$

Since C is defined by the effluent standard and C_0 by the waste characteristics of the industrial processes, then, if k is known, the food-to-mass ratio required in the treatment process could be calculated from Eq.6-60. If the designer likes to use the sludge retention time (SRT) he could easily calculate the desired SRT values from the F/M once the yield coefficient and the endogeneous respiration rate is known.

Since the BOD test and the activated process are both aerobic processes taking place with sufficient nutrient and oxygen, a waste with high k value is expected to have proportionately high K value. Hence, the waste treatability constant k, could be expressed as:

$$k = g\,(K) = g\,(f\,(BOD/COD)) = h\,(BOD/COD) \qquad (6-61)$$

For a particular process at steady state, as long as the F/M ratio is maintained constant, the effluent quality C will vary linearly with C_0 as given in Eq. 6-11. The designer could maintain a constant F/M ratio by varying the microbial mass concentration inversely with the detention time, V/q as shown in Eq. 6-11. Since the microbial mass concentration is easier to control than the detention time once the treatment plant has been commissioned, most designers tend to be conservative in their choice of the required detention time. As a rule-of-thumb, most designers prefer long retention time for stronger waste. Hence, a relationship between C_0 and the hydraulic detention time is expected.

The microbial mass concentration could be increased or decreased by changing the recirculation rate whereas to change the detention time for a given flow rate, q, would mean expanding the volume of the reactor. The latter alternative would involve additional construction cost.

The treatability constant, k at various temperature could be estimated using the formula given by Eckenfelder, (1975) which is:

$$k_T = K_{20}\ (1.024)^{T-20} \qquad (6-62)$$

where k_T is the corrected treatability constant at a given temperature T, and k_{20}o is the treatability constant at a base temperature of 20°C.

Table 6-1 summarizes the operating characteristics of twenty nine activated sludge units reported in literature giving all the pertinent data required. Most operating data before 1962 reports the BOD values only and later on, the COD or TOC values. The BOD and COD values are seldom reported simultaneously.

TABLE 6-1. Summary of Operational Data from Literature

PLANT No.	TYPES OF WASTE	GIVEN DATE								TREATMENT PLANT SIZE
		INFLUENT		EFFLUENT						
		BOD_5	COD	BOD_5	COD	MLVSS	DET. TIME	F/M based on BOD	TEMP.	
		mg/1	mg/1	mg/1	mg/1	mg/1	hrs.	day^{-1}	°C	
1	High Phen. & Ammonia	6170	7976	26	87	1776	480	–	23	bench scale
	High Phen. Ammonia	6666	8341	64	468	2811	199.2	–	23	bench scale
	High Phen. & Ammonia	6666	8341	201	553	2785	129.6	–	23	bench scale
2	Formaldehyde & Alco.	5040	7400	47	296	3200	271	–	29	full scale
3	Ice Cream Waste	3250	4072	–	244	1038	240	–	20	bench scale
4	Hardboard Waste	4012	6120	321	–	3641	–	0,20	21	full scale
5	Diver. Org. Waste	3070	4760	77	723	2620	127,2	–	17	pilot scale
6	Batch. Org. Molecule	2270	3860	53	603	2240	98	–	17	full scale
7	Distellery Waste ***	1992	2723	27	113	3649	75,1	–	21.3	full scale
8	Coke Plant Waste 1	4140	6088	146	1491	2502	158,4	–	20	bench scale
	Coke Plant Waste 2	4140	6088	107	1364	2592	62,4	–	20	bench scale
	Coke Plant Waste 3	4100	6088	499	1368	2398	43,2	–	20	bench scale
9	Pol. Fib. & Fil. Staple	4400	6100	40	260	3000	52	–	25	full scale
10	Nylon 66	1300	2100	150	260	–	33	0,95	25	full scale
11	Potato Waste	1680	3050	85	380	–	26,16	0.30	16.4	full scale
12	Pthalic Anhydride	1600	4200	250	800	2700	24	–	21	full scale
13	Polyalcohols	630	1865	35	284	4320	34	–	28	full scale
14	Textile Waste	457	1655	132	1056	–	23.6	0,31	24	full scale
15	Dyeing Waste	818	2852	123	1711	–	18	0,5	20***	bench scale
16	Sewage	268	463	10	39	–	8	0,2	18	full scale
17	NA	200	350	7	25	2000	8	–	30	pilot scale
18	Sew. & High Ind. Waste	131	558	29	131	–	7	0.15**	18,4	full scale
19	Brewery Waste	650	1000	–	41	1321	6	1.968	30	bench scale
20	Rayon Staples	125	316	30	102	3153	6	–	24	full scale
21	Sew. & High Ind. Waste	256	661	52	236	–	3,9	0.4**	20	pilot scale
22	Weak Sew. & Com. Waste	122	208	15	60	2560	2,2	–	14	full scale
23	Cellophane	61	186	19	115	3380	0.75	–	38	full scale
24	Tetra Ethyl Lead	62	96	–	29	–	0.72	0,375	20	full scale
25	Nylon Polyester	323	604	22	83	–	–	0.78	26	full scale

* Temperature Correction Coefficient = 1.024

** Based on Mass Liquor Suspended Solids

*** Assumed Value

**** Data Used in sample problem

Table 6-1 Cont'd.

COMPUTED DATA									REFERENCES
BOD/COD Ratio	MLVSS	Based on BOD_5		Based on COD		% BOD Removal	% COD Removal		
		F/M	$k_{30^\circ C}$*	F/M	$k_{30^\circ C}$*				
	mg/1	day^{-1}	day^{-1}	day^{-1}	day^{-1}				
0.774	–	0.174	48.461	0.225	24.039	99.58	96.91		Adams (1976)
0.799	–	0.266	34.795	0.358	7.100	99.04	94.39		Adams (1976)
0.799	–	0.443	16.831	0.555	9.221	96.96	93.37		Adams (1976)
0.681	–	0.139	15.174	0.206	5.033	99.07	96.00		Eckenfelder et al (1975)
0.798	–	0.313	–	0.392	7.802	–	94.01		Dul (1972)
0.656	–	–	2.874	0.305	–	92.00	–		Roth et al (1975)
0.645	–	0.221	11.697	0.343	2.605	97.49	84.81		Koon et al (1950)
0.588	–	0.248	14.130	0.422	3.103	97.67	84.32		Eckenfelder et al (1975)
0.732	–	0.174	16.606	0.238	6.770	98.64	95.85		Thomas et al (1974)
0.680	–	0.251	8.694	0.369	1.441	96.47	75.51		Adams et al (1974)
0.680	–	0.614	29.352	0.903	3.966	97.42	77.60		Adams et al (1974)
0.680	–	0.959	8.872	1.410	6.228	87.95	77.69		Adams et al (1974)
0.721	–	0.677	83.074	0.938	23.733	99.09	95.74		Eckenfelder (1976)
0.619	995	–	8.200	1.530	12.191	88.46	87.62		Eckenfelder (1976)
0.551	5138	–	7.772	0.550	5.335	94.94	87.54		Ritchter et al (1971)
0.429	–	0.667	5.117	1.556	8.184	86.11	80.95		Eckenfelder et al (1975)
0.338	–	0.103	1.835	0.305	1.779	94.94	84.77		Eckenfelder et al (1975)
0.294	1506	–	0.880	1.050	0.572	71.12	32.09		Rinker et al (1974)
0.287	2181	–	3.581	1.740	1.471	84.96	40.01		Machlock et al (1974)
0.579	4594	–	6.859	0.340	4.913	96.27	91.58		Spetzierer et al (1974)
0.571	–	0.300	8.271	0.525	6.825	96.50	92.86		Eckenfelder et al (1975)
0.235	2042	0.220	1.019	0.936	4.017	77.86	76.52		Banks (1976)
0.650	–	1.968	–	3.028	70.826	–	96.90		Tanong Tantideeravit (1975)
0.396	–	0.159	0.579	0.401	0.970	76.00	67.72		Eckenfelder et al (1975)
0.387	2995	0.526	2.616	1.360	3.105	79.69	64.30		Banks (1976)
0.592	–	0.520	5.420	0.878	3.122	87.70	70.87		Toerber (1972)
0.328	–	0.578	1.056	1.761	0.899	68.85	38.17		Eckenfelder et al (1975)
0.646	5511	–	–	0.581	1.702	–	69.79		Eckenfelder et al (1975)
0.535	–	0.780	12.016	1.460	10.318	93.19	86.26		Eckenfelder et al (1975)

Regression analysis between the log BOD/COD and the log (k) value shows a very strong correlation betwen the two variables as shown in Fig. 6-4. The relationship between the two variables could be expressed as:

$$\log(\mathrm{BOD/COD}) = 0.228 \log(k) - 0.461 \qquad (6-63)$$

with a correlation coeffficient of 0.825. Similarly regression analysis was carried out between log C_o and log Θ which shows a very strong correlation as shown in Fig. 6-5. The relationship between the two variables could be expressed as:

$$\log C_o = 0.817 \log \Theta + 1.837 \qquad (6-64)$$

with a correlation coefficient of 0.9326.

As discussed before the designer has two interchangeable variables to manipulate in order to satisfy the required F/M or SRT. For a highly degradable waste, with BOD/COD greater than 0.5 the results from Fig. 6-5 would result in a very low MLVSS concentration. Under this condition it would be worthwhile to overdesign the recirculation pump to provide greater flexibility in operation.

Example 6-7. Design a treatment plant for Pepsi-Cola waste at Sermsuk Co. Ltd. in Bangkok with the actual wastewater characteristics as tabulated in pp.235. The average influent flow rate, $q = 52\ m^3/hr$.

Solution:

The required BOD effluent from the waste treatment plant is 30 mg/1.

i) From Fig. 6-5, the estimated detention time V/q at maximum C_o of 890 mg/1 is 21 hrs.

ii) The minimum BOD_5/COD ratio of the influent is 0.553. From Fig. 6-4, k is equal to 8.0 at 30 °C.

iii) The lowest temperature is 26 °C, using Eq. 6-62;

$$k_{26^o} = k_{30^o} \ (1.024)^{26-30} = 7/\mathrm{day}$$

iv) The required F/M from Eq. 6-60 is:

$$F/M = \frac{C\ k}{C_o - C} = \frac{30 \times 7}{890 - 30} = 0.244/\mathrm{day}$$

v) The required microbial mass concentration in the reactor could be determined using the definition of the F/M ratio.

$$F/M = \frac{qC_o}{V X_a}$$

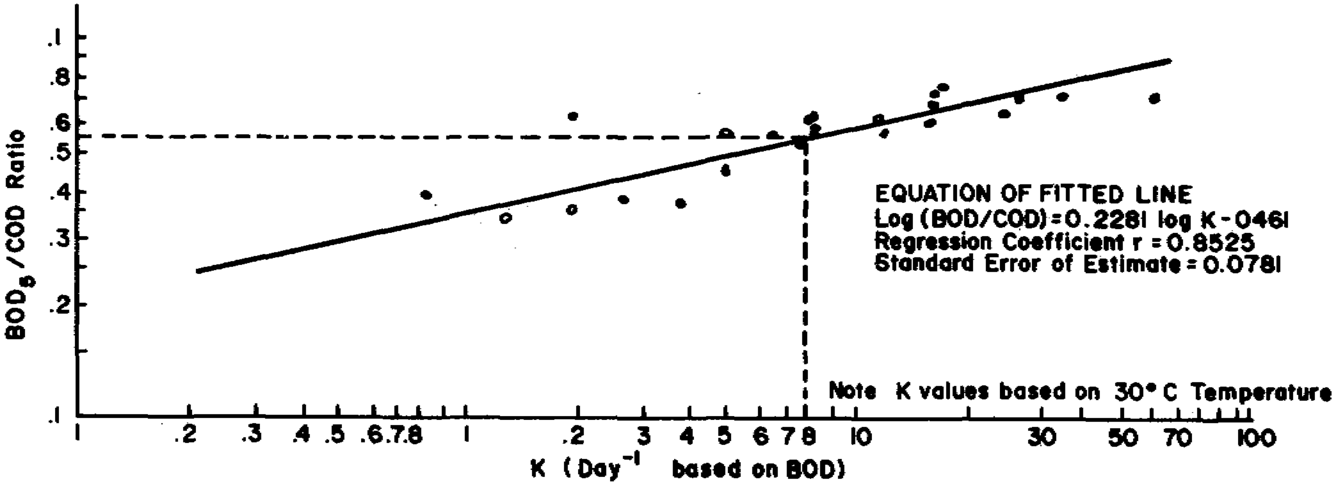
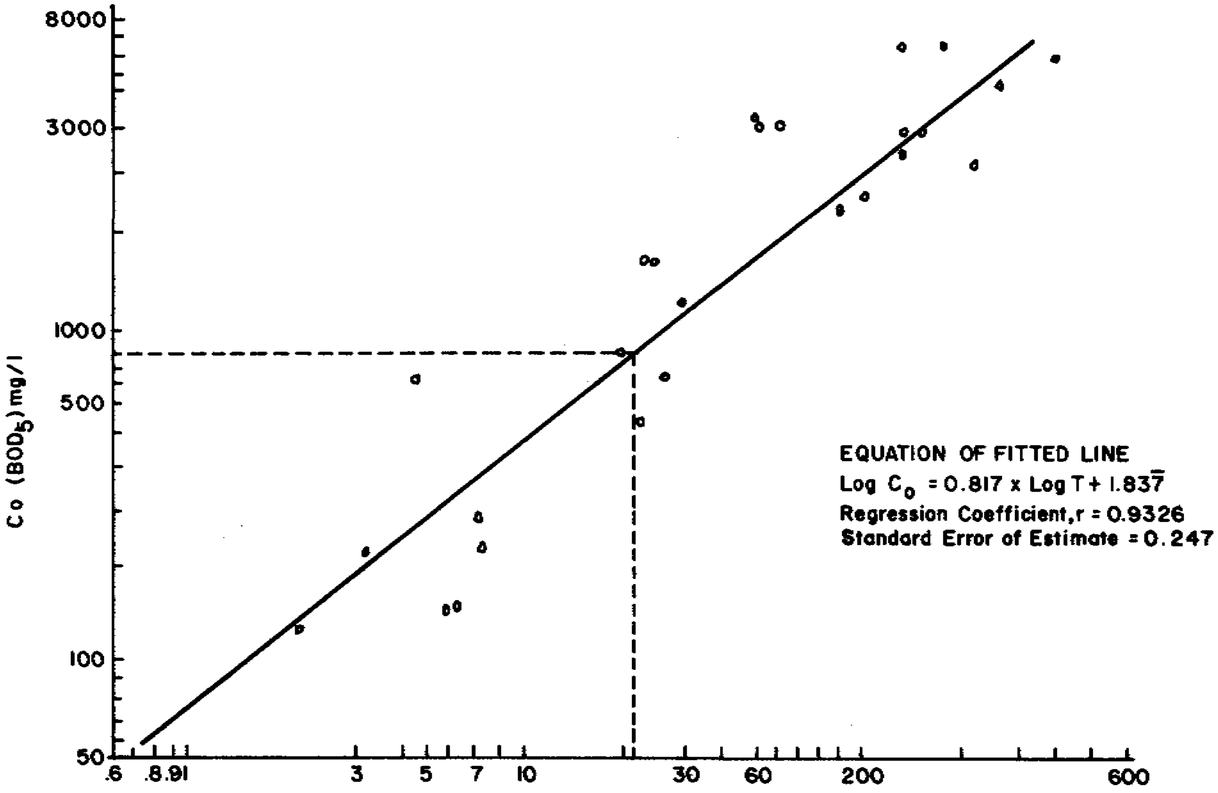


FIG. 6-4 Relationship of BOD_5/COD ration vs. substrate removal rate coefficient, K.



EQUATION OF FITTED LINE

Log C_0 = 0.817 x Log T + $1.83\overline{7}$

Regression Coefficient, r = 0.9326

Standard Error of Estimate = 0.247

FIG. 6-5 Aeration tank influent concentration, C_0 (BOD) vs. hydraulic detention time.

DATE OF SAMPLING	BOD_5 mg/l	COD mg/l	COMPUTED BOD_5/COD	TEMP °C
Jan. 3, 1973	610	–	–	26
	890	–	–	23
	790	–	–	29
	780	–	–	26
Jan. 8, 1973	780	1340	0.582	27
	780	1390	0.560	31
	670	1210	0.553	29
	770	1310	0.588	26
Jan. 10, 1973	760	960	0.79	27
	710	1160	0.610	32
	770	1150	0.67	30
	850	1340	0.752	26
Jan. 12, 1973	790	1130	0.670	–
	750	1000	0.75	–
	800	–	–	–

$$\frac{C_o}{(V/q)(F/M)} = \frac{890 \text{ mg/l} \times 24 \text{ hr/day}}{0.224/\text{day} \times 21 \text{ hr.}} = 4169 \text{ mg/l}$$

vi) At this level the settled solids concentration X_r, could vary from 10,000 to 20,000 mg/l. To size up the recirculation pump, the recirculation ratio is calculated from Eq.6-26,

$$q_r = \frac{qX_a - Y\,q\,(C_o - C) - k_b X_a V}{X_r - X_a}$$

From a previous study, Y has a value of 0.5 to 0.6 and k_b of 0.04. For engineering purposes, the effect of the two parameters can be neglected which is equivalent to a factor of safety of 90-150%, hence,

$$\frac{q_r}{q} = \frac{X_a - Y(C_o - C) - bX_a\Theta}{X_r - X_a}$$

$$= \frac{4169}{10{,}000 - 4169} = 0.715$$

vii) The oxygen requirement could be determined from Eq. 6-45. Again neglecting Y and the endogeneous respiration for sufficient factor of safety which could be from 30 to 70% of the actual oxygen requirement, then,

$$O_r = (890\text{-}30)\,10^{-3} \times 50 \times 24$$

$$= 1073 \text{ kg/day at } 30^{o}\text{C.}$$

The oxygen required at 20°C could be calculated using Eq. 5-4

$$N_{30} = \frac{N_o\ (9.17)}{(O_s - O)(1.024)^{T-20}}$$

where N_{30} is the oxygen required at 30°C, T is the temperature in °C, O is the required dissolved oxygen concentration in the tank, mg/l and O_s is the dissolved oxygen concentration at saturation, mg/l. If the dissolved oxygen concentration in the tank is set at 2 mg/l, then using the above mentioned formula, the oxygen required at 20°C, is 1292 kg of oxygen/day. Since the high speed aerators have an oxygen transfer capacity of 0.5 kg of oxygen/hp at 20°C, the required aerator horsepower is 110 hp. The characteristics of the water treatment plants as operated and designed are shown below.

Parameters	Actual Operation	Design Based on this Method
F/M, day $^{-1}$	0.2 based on MLVSS	0.244 based on MLVSS
Microbial mass, mg/l	5000-7000 (MLSS)	4169 (MLVSS)
Detention time, hr.	17	21
Recycle ratio	0.7	0.715
Aerator Horse power	120 Hp	110 Hp

Process Variation

The initial reaction rate is slow with the microbial mass as the rate limiting constituent. As more microbial mass is produced the reaction rate reaches a peak when the substrate and microbial mass are at their optimal proportion after which the reaction rate again slows down as the substrate is used up and becomes rate limiting. At the later stage the microbial mass enters the endogeneous phase. As the objective of most design is to minimize the size of the reactor in order to minimize the construction cost, a number of reactor types have been proposed to operate at maximum rate of substrate degradations.

Conventional Activated Sludge — In the original design development of the activated sludge process, the autocatalytic nature of the biochemical reaction was not recognized. The units were designed to conform as much as possible to plug flow conditions such that the tanks were designed with very large length to width ratio and baffles were added perpendicular to the direction of flow. This process is also known as the plug flow activated sludge process.

Step-Feed System — The step-feed system is a modification of the plug flow system wherein the substrate is fed at a number of points along the reactor to maintain the ideal food to mass ratio or optimal substrate degradation rate. In this type of reactor endogeneous metabolism of the bacteria is minimized and the number of sections where substrate is rate limiting is minimal. Hence, better process efficiency is expected per unit of aeration tank volume. Fig. 6-6c shows a schematic diagram of the process.

Completely Mixed Process — As the number of feeding points across the step feed system approaches infinity, and if the lateral and vertical dispersion is very high, then the process is almost similar to a completely mixed system. Hence, designers feel it is simpler to deal with a completely mixed system rather than a step feed system. The completely mixed system is easier to quantify and has better damping control on shock loads resulting in more stable operating condition. Fig. 6-6b shows the schematic diagram of the process.

Tapered Aeration Units — The aeration tanks or reactors are often constructed of reinforced concrete for a design period of 15 to 20 years and most often they last for 40 years. As in most designs, a considerable factor of safety and allowance is provided to accommodate future increases in wastewater quality and quantity, as well as for diurnal and seasonal fluctuations. Any savings in minimizing the construction cost is minimal compared to savings in the operating cost. Hence, instead of modifying the substrate feeding configuration and mixing patterns, the oxygen supply is reduced in the conventional activated sludge to conform with the oxygen utilization rate, thereby minimizing the power requirements which is the major operating cost. Fig. 6-6d shows a schematic diagram of the process.

Extended Aeration Unit — In Eq.6-5, Y has a value in the range of 0.3 to 0.9 which means that approximately 30% to 90% of the COD or substrate removed from the wastewater is converted to solid cellular mass. In small treatment plants it would be very expensive to install complete sludge handling process to treat small volume of solids. Hence, designers would prefer to degrade the sludge aerobically, i.e., extending the region of the endogeneous

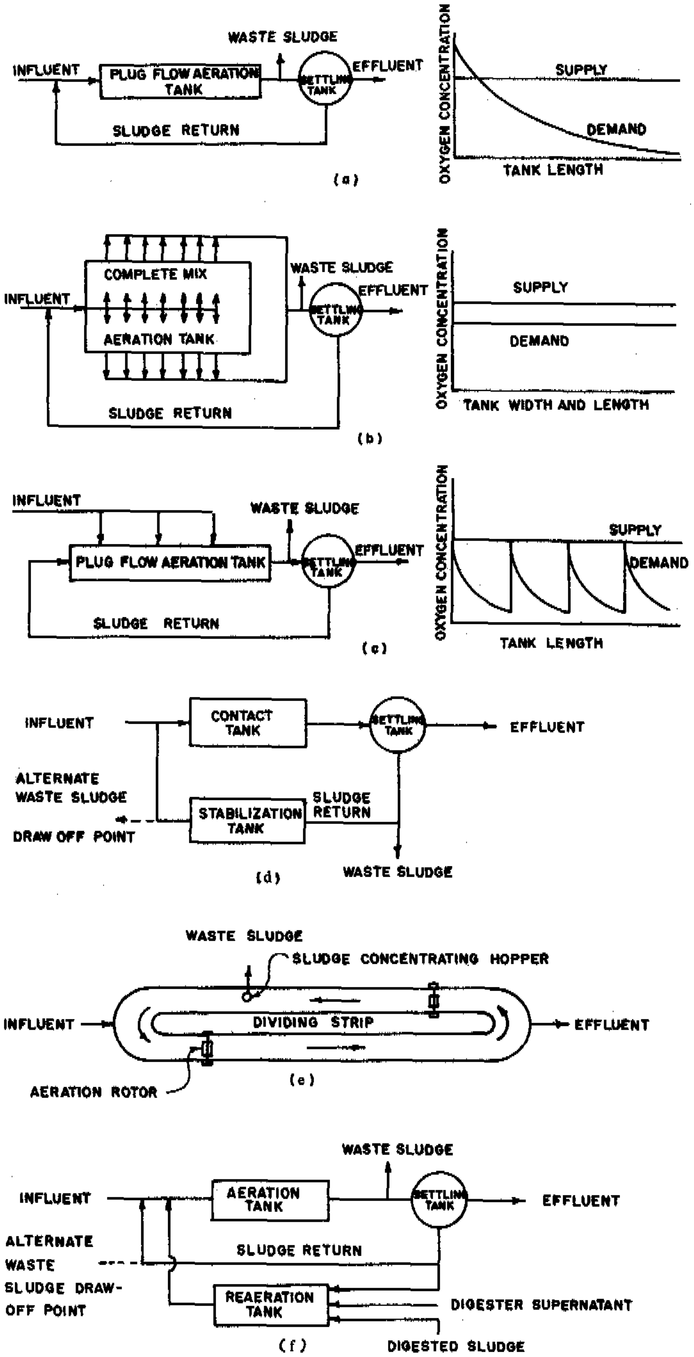


FIG. 6-6 Flow sheet and plot of oxygen demand and oxygen supply vs. tank length

phase. To minimize further the need for a separate digestion unit, the aeration unit is utilized simultaneously for BOD degradation and sludge stabilization. This would involve long sludge retention time, high sludge recirculation ratio and high mixed suspended solids concentration in the reactor. The oxygen requirement in the aeration system is approximately twice the normal level. Fig. 6-6e shows the schematic diagram of the process.

Kraus Process – A number of organic industrial wastes such as soft drinks bottling, tapioca processing and sugar refining are nutrient deficient. The cost of nutrient fortification could be very expensive. On the other hand, the need for nutrients is transitory in nature. The microorganism needs the nutrients during the growth phase and when it undergoes endogeneous metabolism it releases the nutrients back into the solution. The Kraus process is a modification of the activated sludge which recycles the waste from anaerobic digestion into the aeration unit. Fig. 6-6f shows the schematic diagram of the process.

Reactor Type Selection

Current trends have been to design completely mixed activated sludge process except in certain instances where a conventional activated sludge process is to be modified. The tapered aeration type is commonly used if the existing unit has large extra capacity wherein the cost of supplying oxygen is the major consideration. Otherwise, if the objective is to increase the capacity of the existing process, the step-feed system is used.

Extended aeration type is commonly used in package treatment works or in the treatment of nutrient deficient wastes where the nutrients from bacteria in the endogeneous phase could be reutilized. The Kraus process is used for nutrient deficient wastes where an anaerobic digester for sludge handling is available. Table 6-2 and 6-3 summarize the design and operating values of the different processes.

Dispersed Flow Models

In the previous section, the design parameters, process parameters, oxygen requirements and sludge production were discussed. For dispersed flow reactors, the unit could be visualized as a number of smaller completely mixed systems operated in series as shown in Fig. 6-7. To facilitate the calculation and mass balance around each unit, the recycled flow and incoming flow are assumed to be completely mixed at a point just before the first completely mixed unit. Hence, the flow rate into the first tank, q', is equal to the sum of the incoming flow q and the recycled flow q_r or,

$$q' = q + q_r \qquad (6-65)$$

TABLE 6-2 Operational Chracteristics of Activated-Sludge Processes
(From Metcalf and Eddy, 1972)

Process modification	Flow Model	Aeration System	BOD removal efficiency, %	Application
Conventional	Plug-flow	Diffused-air mechanical aerators	85-95	Low strength domestic wastes, susceptible to shock load
Complete-mix	Complete-mix	Diffused-air mechanical aerators	85-95	General application, resistent to shock loads
Step - aeration	Plug-flow	Diffused-air	85-95	General application to wide range of wastes
Modified-aeration	Plug-flow	Diffused-air	60-75	Intermediate degree of treatment where cell tissue in the effluent is not objectionable
Contact-stabilization	Plug-flow	Diffused-air mechanical aerators	80-90	Expansion of existing systems, package plants flexible
Extended-aeration	Complete-mix	Diffused-air, mechanical aerators	75-95	Small communities, package plants, flexible, surface aerator
Kraus process	Plug-flow	Diffused-air	85-95	Low-nitrogen, high strength wastes
High-rate aeration	Complete-mix	Mechanical aerators	75-90	Use with turbine aerators to transfer oxygen and control the floc size, general application
Pure-oxygen systems	Complete-mix reactors in series	Mechanical aerators	85-95	General application, use where limited volume is available, use near economical source of oxygen, turbine or surface aerators

TABLE 6-3 DESIGN PARAMETERS FOR ACTIVATED-SLUDGE PROCESSES

(From Metcalf and Eddy, 1972)

Process modification	PARAMETERS					
	SRT - days	F/M, kg BOD_5/kg MLVSS – day	Volumetric loading kg BOD_5/62 m^3	MLSS, mg/liter	V/q, hr	q_r/q
Conventional	5–15	0.2–0.4	20–40	1,500–3,000	4–8	0.25–0.5
Complete–mix	5–15	0.2–0.6	50–120	3,000–6,000	3–5	0.25–1.0
Step-aeration	5–15	0.2–0.5	40-60	2,000–3,500	3–5	0.25–0.75
Modified-aeration	0.2–0.5	1.5–5.0	75–150	2000 - 5000	1.5–3	0.05–0.15
Contact-stabilization	5–15	0.2–0.6	60–75	(1,000–3,000)* (4,000–10,000)+	(.05-1.0)* (3–6)+	0.25–1.0
Extended-aeration	20–30	0.05–0.15	10–25	3,000–6,000	18–36	0.75–1.50
Kraus process	5–15	0.3–0.8	40–100	2,000–3,000	4–8	0.5–1.0
High-rate aeration	5–10	0.4–1.5	100–1,000	4,000–10,000	0.5–2	1.0–5.0
Pure-oxygen systems	8–20	0.25–1.0	100–250	6,000–8,000	1–3	0.25–0.5

* Contact unit.

\+ Solids stabilization unit.

Similarly, the influent BOD, C_o' is equal to

$$C_o' = (q C_o + q_r C_n)/q' \tag{6-66}$$

where C_n is the effluent BOD of the last completely mixed tank in the series and the other terms are as defined before. The influent microbial mass concentration, X_a' is equal to:

$$X_a' = (q X_o + q_r X_r)/q' \tag{6-67}$$

The mass balance of the BOD at each stage is equal to:

$$C_{m-1} q' = C_m q' + k \frac{X_{a,m} V}{n} C_m \tag{6-68}$$

where C_m is the effluent BOD and BOD concentration in the mth tank, mg/1; C_{m-1} is the effluent BOD from the (m–1) th tank and is equal to the influent BOD of the mth tank; $X_{a,m}$ is the microbial concentration in the mth tank, n is the number of tanks in series and the other terms are as defined before.

The mass balance of the microbial mass yields:

$$X_{a,m} q' = X_{a,m-1} q' + q' Y (C_{m-1} - C_m) - k_b X_{a,m} V \tag{6-69}$$

Eq. 6-68 could be rearranged as:

$$C_m = \frac{q'(C_{m-1} - C_m)}{k X_{a,m} \; V/m} = \frac{1}{k} SUR \tag{6-70}$$

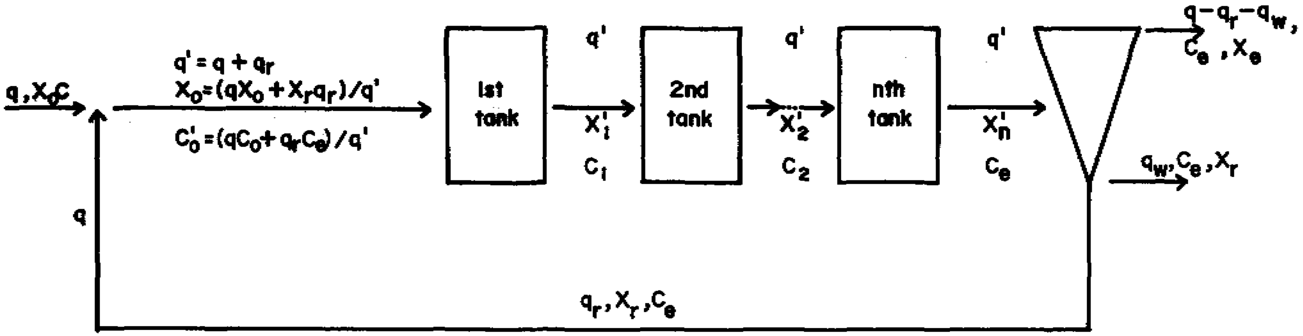
Similarly, Eq 6-69 could be rearranged as:

$$(X_{a,m} - X_{a,m-1}) q' = q' Y (C_{m-1} - C_m) - \frac{k_b X_{a,m} V}{n}$$

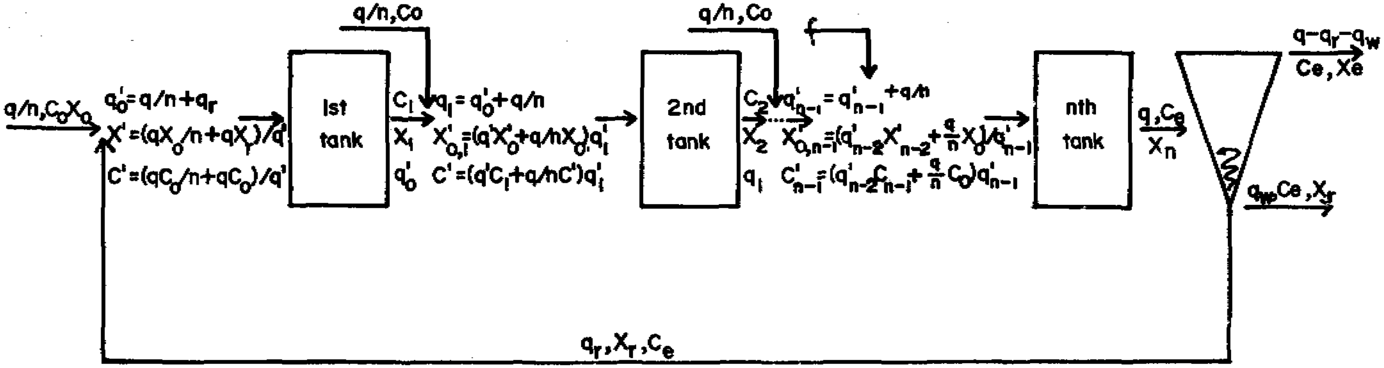
or

$$\frac{1}{SRT} = \frac{(X_{a,m} - X_{a,m-1}) q'}{X_{a,m} \; V/n} = \frac{q' Y (C_{m-1} - C_m)}{X_{a,m} \; V/n} - k_b \tag{6-71}$$

Introducing Eq. 6-70 to 6-71 yields:



Mass balance in a conventional and mixed flow activated sludge.



Mass balance in stepfeed activated sludge.

FIG. 6-7 Modified mass balance to simulate plug flow and dispersed flow activated sludge systems

$$\frac{1}{SRT} = YkC_m - k_b \qquad (6-72)$$

and

$$\frac{1}{SRT} = Y\ SUR - k_b \qquad (6-73)$$

Eq 6-72 and 6-73 are the same relationship as is process variables in a completely mixed system in eq. 6-17, and eq. 6-19. Normally $X_{a,1} = X_{a,3} = X_{a,n}$, since the recycled microbial mass is often much higher than the microbial mass produced in the system. As an example, for a system operated with SRT of 7 days, the microbial mass is often much higher than the microbial mass produced per day. Normally, the detention time at each stage could vary from 1-6 hours depending on the number of the stages. Hence, the microbial mass produced is only 5–10% of the total microbial mass in the aeration tank. The values of $C_1, C_2 \ldots C_{n-1}$ could be estimated using the following equation,

$$\frac{C_1}{C_o} = \frac{C_2}{C_1} = \frac{C_3}{C_3} = \ldots \frac{C_{n-2}}{C_{n-1}} = \frac{C_n}{C_{n-1}} \qquad (6-74)$$

The actual values of $C_1, C_2, C_3 \ldots C_{n-1}$ are calculated using Eq. 6-70 and 6-71 simultaneously.

The oxygen requirements could be calculated using Eq. 6-45 and 6-51. However, the oxygen requirements for nitrification in the first two stages could be very low.

The quantity of sludge wasted could be calculated from the mass balance of the microbial mass in the clarifier. This yields;

$$(q + q_r)X_{a,n} = (q - q_w - q_r)X_e + q_rX_r + q_wX_r \qquad (6-75)$$

or

$$q_w = \frac{(q + q_r)\ X_{a,n} - (q - q_r)\ X_e - q_rX_r}{X_r - X_e}$$

Example 6-8. The tapered aeration process is a variation of the activated sludge process wherein the oxygen supply is tapered from the inlet to the outlet section of the aeration tank to conform with the oxygen demand. Calculate the microbial mass concentration, the effluent BOD at each section if the whole unit is approximated as a series of three completely mixed tanks in series. The total aeration tank volume is 10,000 m³/day. The influent BOD is 300 mg/l and the required effluent BOD is 30 mg/l.

The wastewater flow rate is 40,000 m^3 /day. The value of k,Y, and k_b are 0.01 1/mg-day, 0.4 and 0.4 and 0.1/day respectively.

Solution: i) Estimate the effluent BOD at each stage

$$\frac{C_1}{C_o} = \frac{C_2}{C_1} = \frac{C_3}{C_2}$$

where $C_3 = 30, C_o = 300$

hence,

$$\frac{C_2}{C_1} = \frac{30}{C_2} \text{ and } \frac{C_1}{300} = \frac{C_2}{C_1}$$

$$C_1^{3/4} = 40.5$$

or

$$C_1 = 139 \text{ mg/1}$$

$$C_2 = 5.477 \quad \sqrt{139} = 64.6 \text{ mg/1}$$

ii) Neglect first the effect of recirculation rate, and calculate the X_a for the first stage from the SUR relationship.

$$\frac{q\ (C_o - C_1)}{X_a \quad V/n} = kC_1$$

$$\frac{40{,}000\ (300-139)}{10{,}000/3\ (X_a)} = .01\ (139)$$

$$X_a = \frac{3 \times 40{,}000\ (161)}{0.1\ (139) \times 10{,}000} = 1390 \text{ mg/1}$$

iii) Assume the volatile suspended solid concentration in the settled sludge is 7,000 mg/1. This would allow up to 30% solids in the sludge for a settled solids concentration of 10,000 mg/1. Estimate the recirculation rate by making a mass balance in the first tank;

$$q_r\,(7000 + 40{,}000\,(.40)(300-139)-$$

$$-.1\,\frac{(1390)(10{,}000)}{3}\;1390\;(40{,}000\;q_r)\;7000\;q_r +$$

$$2{,}576{,}000 - 463{,}333 = 55{,}600{,}000 + 1390\;q_r$$

$$q_r = \frac{53{,}487{,}333}{7000-1390} = 9534\ \text{m}^3/\text{day say } 10{,}000\ \text{m}^3/\text{day}$$

$q_r/q = 0.25$ which is within the normal range for tapered aeration process.

iv) Calculate $C_o', X_{a,o}'$ and q'

$$q' = 40{,}000 + 10{,}000 = 50{,}000\ \text{m}^3/\text{day}$$

$$X_{a,o}' = 7{,}000 \times 10{,}800 / (50{,}000) = 1{,}400\ \text{mg/1}$$

$$C_o' = \frac{10{,}000 \times 30 + 40{,}000 \times 3000}{50{,}000} = 246\ \text{mg/1}$$

v) Check the value of $X_{a,1}$ and C_1 using Eq. 6-68 and 6-71 which yields;

$$X_{a,1} = \frac{Y(C_o - C_1) + X_{a,o}}{1 + k_b\Theta}$$

where $\Theta = V/qn$. The BOD balance yields:

$$C_1 = \frac{C_o - C_1}{kX_{a,1}\ \Theta}$$

Then combining the values of $X_{a,1}$ into the BOD balance results to:

$$\frac{Y\Theta kC_1^2}{1 + k_b\Theta} - \left\{\frac{k\Theta}{1 + k_b\Theta}\ YC_o + X_{a,o} + 1\right\} C_1 + C_o = O$$

For the problem, $\Theta = 10{,}500/50{,}00 = .2$ day, hence,

$$\frac{(.4)(.2)(.01)}{1 + .1\ (.2)} C_1^2 - \left\{ \frac{(.01)(.2)}{1 + (.1)(.2)} .4 \times 246 + 1{,}400 + 1 \right\} C_1 + 246 = 0$$

$$.00078\ C_1^{\ 2} - 2.94\ C_1 + 246 = 0$$

$$C_1 = \frac{2.94 \pm \sqrt{(2.94)^2 - 4\ (.00078)\ (246)}}{2\ (.00078)} = 86 \text{ mg/l}$$

$$X_{a,1} = \frac{.4\,(246\text{-}86) + 1400}{1 + (.1)(.2)} = 1435 \text{ mg/l}$$

vi) The calculation is continued for the second stage using the same equation.

$$X_{a,2} = \frac{.4(86 - C_2) + 1469}{1 + (.1)\ (.2)}$$

$$.00078\ C_2^2 - \left\{ \frac{(.01)(.2)}{1 + (.1)(.2)} .4 \times 86 + 1435 + 1 \right\} C_2 + 86 = 0$$

$$.00078\ C_2^2 - 2.08\ C_2 + 86 = 0$$

$$C_2 = \frac{2.98 - \sqrt{(2.98)^2 - (4)\ (86)\ (.00078)}}{2\ (.00078)}$$

$$= 30.1 \text{ mg/l}$$

$$X_{a,2} = \frac{.4(86-30) + 1435}{1 + .2\ (.1)} = 1429 \text{ mg/l}$$

vii) The effluent BOD and X_a from the third stage is calculated as:

$$X_{a,3} = \frac{(.4)(30 - C_3) + 1429}{1 + .02}$$

$$.00078\ C_3^2 - \left\{ \frac{(.01)\ (.2)}{1.02}\ .4 \times 30 + 1429 + 1 \right\} C_3 + 30 = 0$$

$$.00078\ C_3^2 - 2.827\ C_3 + 30 = 0$$

$$C_3 = \frac{2.827 - \sqrt{(2.827)^2 - 4\,(30)\,(.00078)}}{2\,(.00078)} = 2\ \text{mg/l}$$

$$X_{a,3} = \frac{.4\left(30-2\right) + 1429}{1.02} = 1412\ \text{mg/l}$$

It should be noted that X_a values varied from 1400 mg/l in the influent to 1412 mg/l in the effluent. At each stage X_a varied by less than 5% from the influent X_a concentration. Hence, to simplify the analysis, X_a is often taken as a constant.

Process Control and Instrumentation

The influent wastewater quality and quantity to the activated sludge process changes with time as discussed in Chapter II. The wastewater quality and quantity fluctuations could disrupt the process performance due to wash-out of the microbial solids, ecological disruption of the microbial population resulting in the predominance of poor settling sludge, and depletion of the dissolved oxygen concentration. The effects of the wastewater quality and quantity fluctuations on the waste treatment performance could be minimized by adding an equalization system as discussed in Chapter II or by overdesigning the activated sludge process to accommodate the maximum wastewater quality and quantity. The overdesign of the waste treatment facility could be an expensive solution.

The inherent equalizing properties of the aeration tanks and clarifiers must be utilized to the maximun. Process stability is further enhanced by the feedback effect of the sludge recirculation. The sludge recirculation dilutes the incoming wastewater quality and build up of the microbial mass concentration in the aeration tank. Process control is the evaluation of techniques and procedures for maintaining the process stability to meet the desired effluent quality from the effects of time varying influent quality and quantity. The development of the process control parameters depends on the quantification of the dynamic behaviour of the wastewater treatment process.

The development of the appropriate process control parameter for the activated sludge process has been retarded due to the inability of past researchers to distinguish the differences between the design parameters and control parameters. The design parameters establish the relationship and effect of the different process variables in the performance of the wastewater treatment process under steady state condition and are meaningless in quantifying the behaviour of the same wastewater treatment system under dynamic conditions. The design parameter discussed in the previous section such as F/M ratio, SUR, and SRT are valid only at steady state and are useless at dynamic state.

The activated sludge is the only process whose dynamic and transient state is defined and controlled by a system of steady state equations or parameters derived from steady state. Roesler, Bishop and Ringelman (1977) reported serious limitations of SRT, F/M, X/X_r and SUR as control parameters for activated sludge system in a survey of the performance of 182 waste treatment plants. Ouano et al (1977) using a computer simulation model of the currently accepted kinetic models of the activated sludge, have demonstrated the inadequacy of the steady state equations or parameters at steady state for process control. Control parameters derived from a system at steady state are meaningless when applied to the same system under transient or dynamic state.

Although the dynamic equations of the activated sludge have been defined and its applicability as a control strategy have been discussed, researches have been concentrated on the steady state model due to its apparent simplicity without considering its effectivity. The substitution of steady state relationship into the dynamic equations linearizes two interrelated non-linear differential equations thereby simplifying the analysis.

Crittenden and Asano (1977) suggested Taylor series approximation to linearize the objective and dynamic equations of the activated sludge. If the microbial growth rate and BOD degradation could be expressed as:

$$f(C,X_a) = dC/dt = (C_o q - Cq - k \quad X_a CV) \; / V \qquad (6-76)$$

and

$$g(C,X_a) = d X_a/dt = q_r X_r + q\, Y(C_o - C) - k_b X_a V - \quad X_a \; (q+q_r) \,/\, V \qquad (6-77)$$

then, the Taylor series approximations of the above equations are:

$$f(C,X_a) = f(C_s, X_{a,s}) + \frac{\partial f}{\partial C}(C - C_s) + \frac{\partial f}{\partial X_a}(X_a - X_{a,s}) \qquad (6-78)$$

and

$$G(C,X_a) = g(C_s, X_{a,s}) + \frac{\partial g}{\partial C}(C - C_s) + \frac{\partial g}{\partial X_a}(X_a - X_{a,s}) \qquad (6-79)$$

The terms C_s, $X_{a,s}$, and C_s, and $X_{a,s}$ are the values at steady state. Since at steady state, dC/dt is equal to zero, then the variable $X_a' = X_a - X_s$ and $C' = C - C_s$ are defined as:

$$U = \begin{vmatrix} X' \\ \\ S' \end{vmatrix} \qquad (6-80)$$

The derivative of the vector U is defined as:

$$h(t) = \frac{dU}{dt} = \begin{vmatrix} \frac{\partial f}{\partial C} & \frac{\partial f}{\partial X_a} \\ \\ \frac{\partial g}{\partial C} & \frac{\partial g}{\partial X_a} \end{vmatrix} \qquad (6-81)$$

where h (t) is the forcing function causing the disturbance or deviation from steady state condition. The eigenvalues and eigenvectors of the Jacobian matrix given in Eq. 6-81comprise the complementary solution to the transient state. The stability of the solution to the differential equations defined by 6-81 is related to the real part of the eigenvalues. Steady state recovery will be achieved if the real parts of the eigenvalues are negative. The higher the absolute value of the eigenvalues, the faster the shockloads are dampened. Jones (1976) conducted similar analysis using the Lyapunov criteria. The activated sludge stability is best attained by maintaining the recirculation ratio greater than

$$q_r/q^* \geqslant \frac{C_o^* \,(Y - k_b)}{X_r/X_a - 1} \qquad (6-82)$$

where q^* is the maximum expected wastewater flow rate, m^3/hr, C_o^* is the maximum expected influent BOD concentration mg/l and the other terms are as defined before.

DESIGN OF AEROBIC SLUDGE DIGESTERS

Except for the extended aeration variation of the activated sludge process, the wasted sludge still contains large quantities of degradable organic matter which could attract flies and cause odor problems when sprayed or disposed on land. The organic wastes are stabilized either through aerobic or anaerobic biological activity. The anaerobic sludge digestion is very slow compared to aerobic digestion. Where land is very limited and power is available cheaply, aerobic sludge digestion is preferred.

The principle of aerobic sludge digestion is similar to the activated sludge process except that the microbial sludge undergoes endogeneous respiration only. The microbial mass balance around the aerobic sludge digester is similar to the activated sludge aeration tank without recirculation except that q Y (C_o -C) is equal to zero. Hence, the microbial mass balance is equal to:

$$V \frac{dX_a}{dt} = q_w X_r - k_b X_a V - q_w X_a \qquad (6-83)$$

At steady state, dX_a / dt is equal to zero, hence, Eq. 6-83 simplifies to:

$$X_a = \frac{X_r}{\left(1 + k_b V/q_w\right)} \qquad (6-84)$$

The term q_w is the desludging rate, m^3/day, X_r is the settled sludge concentration mg/l, V is the aerobic digester volume, m^3, and k_b is the endogeneous respiration rate. The inert solids are normally 10–20% of the volatile solids. Hence, a mass balance of the inert solids yields,

$$V \frac{dX_i}{dt} = q_w X_{r,i} + (.1)\, k_b X_a V - q_w X_i \qquad (6-85)$$

At steady state, Eq. 6-85 simplifies to:

$$X_i = \frac{q_w X_{r,i} + (.1)\, k_b X_a V}{q_w}$$

$$= X_{r,i} + .1\, k_b X_a V = X_{r,i} + \frac{.1\, k_b X_r (V/q_w)}{1 + k_b V/q_w}$$

Normally, from 70-80% of the volatile solids are stabilized in the aerobic digester. The remaining 20-30% may still undergo aerobic decomposition but those are primarily composed of very stable organic susbtances such as cellular walls, lignin, and cellulose. The total solids disposed is equal to the sum of the residual volatile solids and the inert solids. Hence, the stabilized mass is equal to:

$$M_s = q_w (X_i + X_a) \qquad (6-86)$$

The oxygen required to stabilize the microbial solids from X_r to X_a is equal to

$$O_r = q_w (X_r - X_a)(1.6)(.001) \qquad (6-87)$$

since 1.6 kg of oxygen is required per kg of microbial mass stabilized.

Example 6-9. The settled solids concentration is 10,000 mg/l from an activated sludge operated with an SRT of 20 days. The sludge is to be stabilized in an aerobic digester. The value of k_b is 0.1/day. The sludge flow rate is 1,000 m^3/day. a) Calculate the digester volume and the oxygen required b) Estimate the quality of the effluent sludge flow if 70% of the sludge volatile solids is removed.

Solution: The volatile suspended solids fraction from Eq. 6-44 is calculated as:

$$e = 0.85/ (SRT)^{.1} = .63$$

Hence, X_r = .63 x 10,000 = 6,300 mg/1. For 70% BOD removal V/q_w is calculated from Eq. 6–84,

$$.7 = \frac{1}{1 + .1 \ (V/q_w)}$$

$$V/q_w = 4.28 \text{ say } 4.5 \text{ day}$$

$$V = q_w \Theta = 4.5 \times 1{,}000 \text{ m}^3/\text{day} = 4{,}500 \text{ m}^3$$

b) $$O_r = q_w \ (X_r - X_a)$$

$$= 1{,}000 \text{ m}^3/\text{day} \ (6300 - .3 \ (6300) \ (1.6)(10^{-3})$$

$$= 7056 \text{ kg/day}$$

c) $$X_i = (3{,}700) + (.1) \ (6300)(7)$$

$$= 4141 \text{ mg/l}$$

$$X_a = .3 \ (6300) = 1890 \text{ mg/l}$$

$$M_s = q_w \ (1890 + 4141)$$

$$= 1000 \ (2220 + 5422) \ (10^{-3}) = 6031 \text{ kg/day}$$

The power required for mixing could be higher than the power required for oxygenation. Due to the relatively high suspended solids concentration in the aeration tank, the mixing power required is maintained at 0.01 to 0.05 Hp/m^3·(.007 to .04 kw/m^3).

HIGH RATE ANAEROBIC DIGESTERS

Anaerobic digesters are often used to stabilize the wasted sludge from the activated and trickling filters. Recently, anaerobic digesters have become popular in treating strong industrial and agricultural wastewaters due to the high power cost of activated sludge processes and other secondary waste treatment processes. Methane recovery from anaerobic digestion has been utilized for operating aerobic digestion units, lighting and other electrical uses within the sewage treatment works. The excess methane is often burned in flare burners. With the increasing cost of oil and other energy sources, methane from anaerobic digesters could be tapped to supplement gas and electricity supply.

In small rural areas, biogas digesters are utilized to convert farm and human wastes to methane or biogas. The nitrogen and essential elements of the agricultural wastes are conserved in the anaerobic digester as the carbonaceous matters are utilized. The effluent from the anaerobic digester is utilized to supplement the fertilizer requirements on the farm.

Anaerobic sludge is rich in Vitamin B.In large poultries and piggeries, the anaerobic sludge is sterilized and mixed with the animal feed to minimize the Vitamin B supplement required from chemical sources. The full potential of the anaerobic digestion process has not been thoroughly evaluated. Before 1972, anaerobic digestion was slowly replaced by the aerobic processes due to its slow reaction rate requiring very large tanks. The anaerobic effluent is highly colored and malodorous which would require further aerobic treatment before discharge to the water course. The rapid increase of power cost and even restriction of power supply after 1972 made the anaerobic process popular.

Process Variation – The conventional anaerobic digester merely consists of a tank where the wastes are continuously or intermittently fed and the effluent is correspondingly removed in consonance with the influent feeding rate. The solids in the wastes are bouyed up by the gases produced during anaerobic digestion to form a scum layer on the top of the wastewater surface. The scum layer may be broken by intermittent or continuous stirring.

The heavy solids and the stabilized solids settle at the bottom of the digester. The effluent are tapped at the clear zone between the scum and the settled solids layer. The settled solids are removed for drying or sludge treatment. The scum layer reduces the rate of methane gas and carbon dioxide evolution. Furthermore, the scum layer could form very thick and rigid barriers on the surtace of the wastewater. For proper operation of the anaerobic digester, the scum layers should be destroyed.

Fig. 6-8 shows a schematic diagram of a conventional anaerobic digester.

The anaerobic digester is basically a holding tank with provision for intermittent mixing of the content.

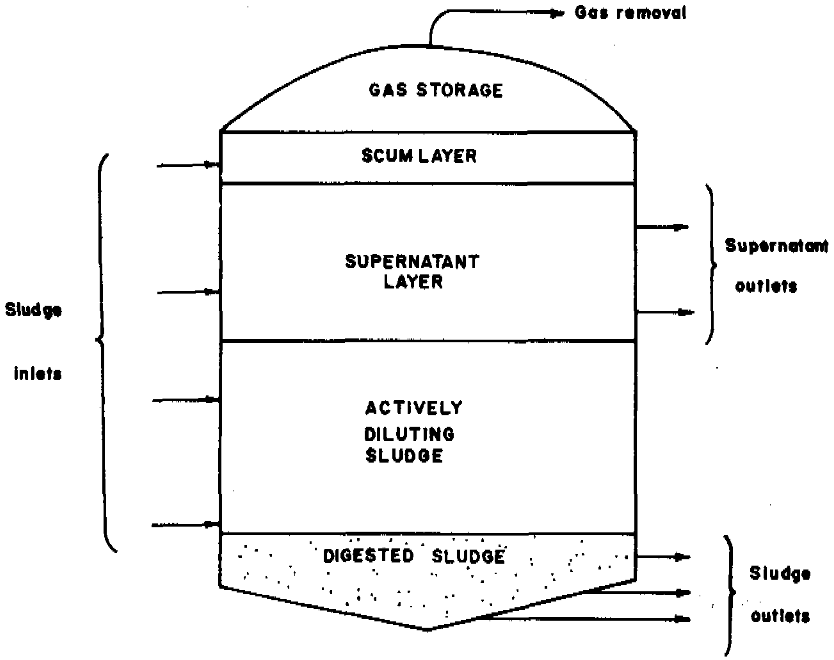


FIG. 6-8 **Schematic diagram of conventional digester used in the single stage**

If methane gas is burned and wasted, the digester cover is often fixed. The gas is bled off continuously to the flare burner. If the methane gas is to be recovered, the digester cover is designed to float and maintain the gas pressure for utilization in the boiler and gas engine. In some instances, where the digester is designed with fixed covers, the gases are bled off to another tank with floating top and filled with water to prevent gas backflow to the anaerobic digester.

High Rate Anaerobic Digester – A large portion of the time required for the wastewater or sludge stabilization in the conventional anaerobic digester is due to the very slow growth rate of methane forming bacteria. The acclimatization time could be reduced if the methane bacteria are concentrated and recycled into the digester. Unlike the activated sludge, the anaerobic solids, specially the portion with active microbial population, do not settle freely due to trapped gases.

The effluent is first passed to a tank operated with negative atmospheric pressures to remove the trapped gases. The solids are removed and concentrated in a clarifier thickener. The concentrated solids are recycled back to the reaction tank. Continuous stirring of the reaction tank is required to disperse the active microbial population uniformly. Mixing could be carried out mechanically or by means of diffusers operated with the recycled gaseous by-products. Methane and oxygen form an explosive mixture, hence, it is very important to seal the digester from possible infiltration of oxygen from the atmosphere. The anaerobic bacteria are also killed by the presence of oxygen.

The methane-forming bacteria are very sensitive to changes in the wastewater pH, temperature and shock loads. The acid forming bacteria are more tolerant and could operate at a wider range of pH, temperature and influent variations. Anaerobic digesters are often heated to maintain a constant operating temperature between 28-35°C. The digesters are often painted black to absorb the solar energy and aid in heating the digester.

The influent feeding is rigidly controlled to prevent digester failure. Addition of lime is often required to control the pH within the tolerable range for methane-forming bacteria. The current design of the anaerobic digester is in the form of an egg shell to facilitate the removal of settled solids and mixing of the tank content. Fig. 6-9 shows a schematic diagram of the high rate digester.

The anaerobic digester could be designed as a two-stage unit with the first stage specialized for the conversion of the raw wastes to volatile fatty acid and, the second stage, for the conversion of the raw wastes to volatile acid to methane gas. Although the process has been successful in the laboratory and pilot plant units, to date no prototype has been constructed. The two stage anaerobic digester will decrease the reactor volume by decreasing the acclimatization periods. The relation between the acid and methane formers is symbiotic in nature. Hence, the reaction proceeds to completion faster if mixed culture is utilized simultaneously. For example, the acid former could not complete the conversion of waste with BOD greater than 20,000 mg/1 without reducing the pH to less than 6.0 A two stage process will have to control the extent of fatty acid formation such that the second stage is not inhibited. It is the continuous removal of the volatile fatty acid by the methane former which improves the process stability and allows higher BOD removal efficiency.

Design Parameters — The same design parameters in the activated sludge could be used in the anaerobic digester. The derivations of the design parameters are exactly the same as discussed in the previous section.

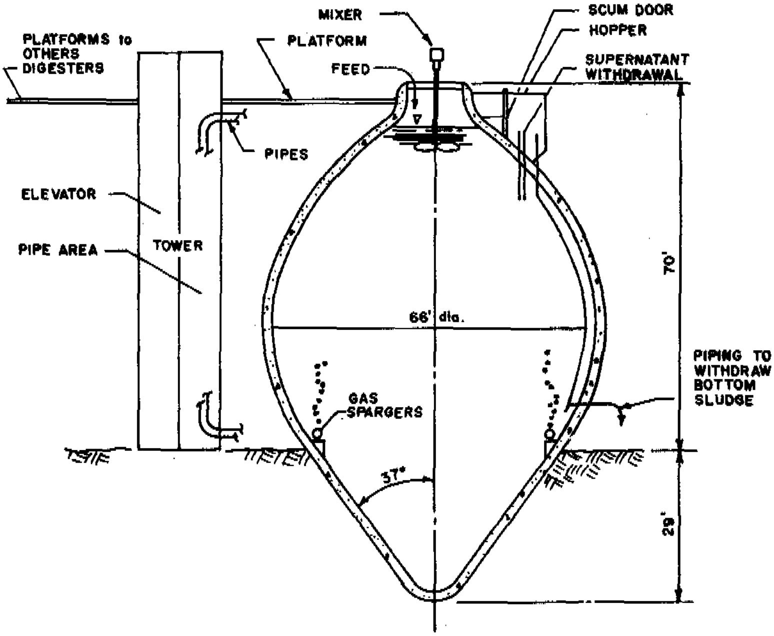


FIG. 6-9 Egg – shaped anaerobic digestion tank at terminal island treatment plant, Los Angeles.

The sludge retention time depends on the operating temperature. The minimum and recommended sludge retention time for various temperature are summarized by Metcalf and Eddy (1972) as shown below:

Temperature	Minimum SRT (days')	Recommended SRT (days')
18	11	28
24	8	20
30	6	14
35	4	10
41	4	10

Current researches have been utilizing the thermophilic anaerobic reaction. The thermophilic reaction is much faster than the mesophilic reaction and could be advantageous for treating hot wastewater such as alcohol slop and oil palm wastes. Thermophilic digestion is still in its developmental stage.

Methane Gas – The quantity of methane gas produced is proportional to the BOD removed less by the equivalent BOD converted to bacterial cells. Since, 0.95 gm of bacterial cells is equivalent to one gm of five day BOD, then the methane gas produced could be calculated as:

$$\frac{dCH_4}{dt} = qY_1 \ (C_o - C) - .95 \frac{dX_a}{dt} \qquad (6-88)$$

where Y_1 is the liters of methane gas at standard temperature and pressure (STP) produced per kg of five day BOD removed, and the other terms are as defined before. Since, the sludge retention time (SRT) is related to dX_a/dt by the following formula

$$SRT = \frac{1}{X_a} \frac{dX_a}{dt} = \frac{Yq\,(C_o - C)}{VX_a} - k_b$$

then

$$\frac{dX_a}{dt} = \frac{Y\,q\,(C_o - C)}{1 + k_b \ SRT} \qquad (6-89)$$

Substituting Eq. 6-89 into Eq. 6-88 yields:

$$\frac{dCH_a}{dt} = q\,Y_1 \ \frac{(C_o \ C)\ (1 - .95Y + k_b \ SRT)}{1 + k_b \ SRT} \qquad (6-90)$$

The value of Y_1 is approximately 150-250 liters of CH_4/kg of five-day BOD removed with an average of 220 liters of CH_4/kg of BOD removed. The values could be divided by 1.5 if the ultimate BOD is used in Eq. 6-90. The values of Y for anaerobic digestion is approximately 0.1. The value of Y varies from .082 for fatty acids to .36 for carbohydrates, while k_b is approximately 0.03/day.

The detention time in conventional anaerobic digester varies from 30 to 90 days while for high rate anaerobic digester it is from 10 to 20 days. Mixing should be sufficient to mix all the digester's content every 30 minutes. The mixing power requirement varies from .02 to .04 Hp per cubic meter of digester volume or (.015 to .03 kw/m^3).

CHAPTER VII

FIXED FILM REACTORS

In the previous chapter, it was stressed that the microbial concentration in the aeration tank of the activated sludge process is increased to match the incoming organic wastes concentration by recirculating the settled microbial mass in the secondary clarifier. Although the procedure is very effective in hastening the biochemical reaction to degrade the organic wastes, the process has unstable characteristics if the influent quality and quantity fluctuate drastically. The activated sludge tends to bulk due to nutrient deficiency, shock loads, changes in pH, and the operating environment. If the dissolved oxygen is insufficient, the sludge will turn septic and starts to rise due to denitrification and trapped gases in the sludge. Very long sludge aeration will result in the formation of very fine sludge particles.

Bulking, rising and fine sludge lower the secondary clarifier efficiency in concentrating the microbial sludge. The activated sludge process thus requires skilled operators for the process to function efficiently.

The microbial concentration in the reaction tank could also be increased by providing surface area inside the tank for the microbial organisms to cling and form slime layers. The microorganism forms a fixed film waiting for the organic wastes to trickle along its surfaces. Since the microbial population is already in excess inside the reaction tank waiting for the organic wastes, the process is more stable and does not require stringent process control to match the microbial population and the influent waste flow. The organic wastes are adsorbed and later absorbed by the microbial slimes. Dissolved oxygen and essential nutrients from the wastewater are adsorbed and absorbed simultaneously with the organic wastes.

As the slime layer matures, it becomes thicker until the cohesive forces between the contact surface and slime layer become insufficient to support the total weight of the slime layer.The slime layer then sloughs off and a new slime layer starts to form. The reaction at the top of the slime layer is aerobic while the main bulk of the microbial reaction is anaerobic. The anaerobic reaction involves degradation of the organic wastes and stabilization of the microbial sludge. The symbiotic reaction between the aerobic and anaerobic organisms recycles the limited nutrients in the wastewater. This improves the stability of the process to treat nutrient deficient wastes.

The anaerobic stabilization of the sludge lowers the biodegradable mass of the sludge. For this reason the secondary clarifiers and sludge digesters are smaller for fixed film reactors than in the activated sludge process.

Although the fixed film reactor was developed before the activated sludge process, the major contact media for slime formation was crushed rocks. Due to the weight of the rocks, massive structures were required for fixed film reactors. For this reason, the popularity of the process declined in the early 1900 in favor of the activated sludge. In the 1950's plastic sheets were substituted for crushed rocks as contact media for slime formation. The use of plastic media allowed the construction of very tall towers with very low structural cost. However, lately with the increasing oil prices, the cost of the plastic media has increased proportionally. Although the fixed film reactor operating cost is very low compared to the activated sludge, the initial capital cost is much higher.

Process Description

The original approach to wastewater purification was very similar to water purification for domestic water supply. The wastewater was filtered after removal of the coarse solids in primary clarifiers. The approach was not very successful since the biological slimes clog the filter media decreasing the waste treatment efficiency. To overcome the problem of clogged filters, the filter media size was increased from the normal sand to crushed rocks with 2.0 cm to 4 cm diameters.

When the filter media was changed from sand to crushed rocks, the purification mechanism shifted from the physical operation known as filtration to fixed film biochemical reaction. Due to its historical origin, fixed film biochemical reaction is commonly known today as biofiltration. Biofiltration is a misnomer as the organic wastes are stabilized due to biochemical reaction rather than by straining operation.

At present, the main wastewater treatment process applying the principle of biofiltration are the trickling filter, the rotating filter and the anaerobic filter.

Trickling fitler – The trickling filter is a fixed bed reactor filled with a solid media on which microbial slime grows and over which wastewater trickles. The biological reaction releases heat which maintains the unit at a temperature higher than ambient conditions. Aside from maintaining the stability of the filter to diurnal variations in temperature, the temperature difference induces air circulation, as shown in Fig. 7-1. When designing the underdrains, 50% of the conduit's area should be alloted to incoming air. In some installations an air blower may be used to increase the oxygen supply. The applicability of this method is discussed in later sections.

The most common type of trickling filter consists of a concrete tank, 2 to 3 m high and filled with crushed rocks whose diameter varies from 5 to 10 cm. If a circular tank is designed, a rotary distributor is commonly used to distribute the wastewater over the filter surface. The distributor arm rotates due to the reaction of the released water, although in some instances an electric motor may be used.

Most synthetic filter media are constructed in cubical sections. For small treatment works, it is inconvenient to cut the edges to suit circular tanks, so

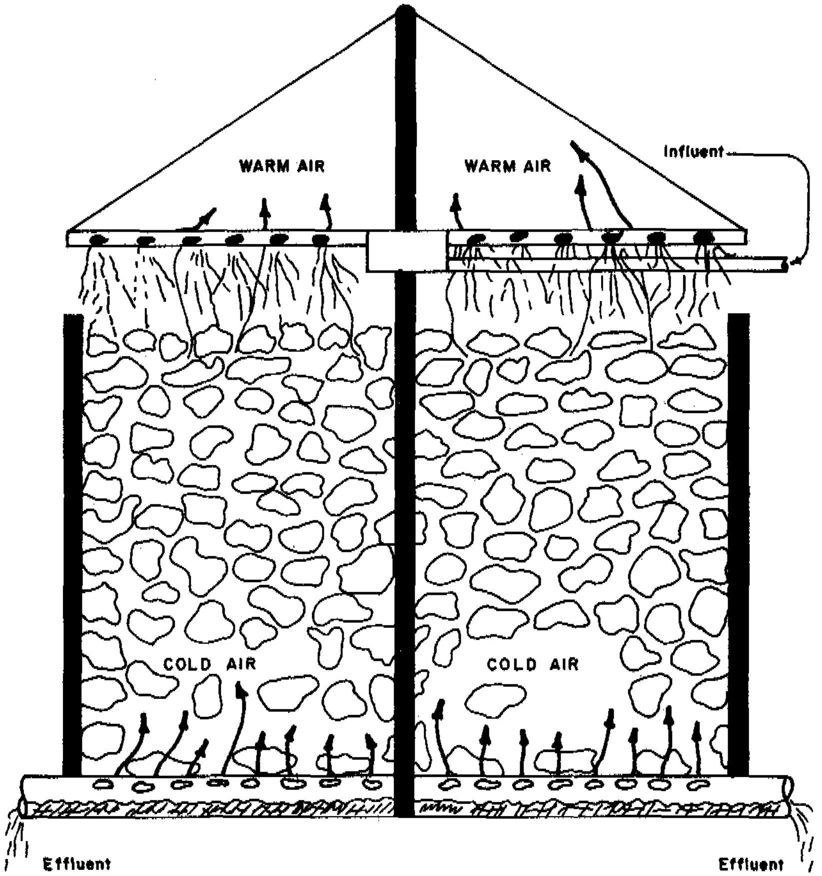


FIG. 7-1 Circulation in trickling filters.

rectangular towers are then used. The wastewater is distributed over the filter media by a series of channels with V-weirs at .3 to .5 m intervals. The water overflows into splash plates located .5 to .7 m below the weirs and .1 to .3 m above the filter surface. Periodic cleaning is required to remove slimes and particles which adhere to the weir surfaces and splash plates.

Low rate filters – These are used where a fully nitrified effluent is required. The filter is doused with wastewater intermittently at less than 5 minute intervals and at a rate of 10,000 to 40,000 m^3/ha-day. In terms of degradable organic matter the dousing rate varies from .12 to .4 kg of BOD_5/m^3 of media-day.

Odor readily develops in tropical countries as well as filter flies which are a nuisance, due to high ambient temperature. The utilization of this process should be limited to areas where sufficient open spaces are available to minimize the nuisance problem.

High rate filters – High rate filters have a hydraulic dousing rate approximately 10 times and a BOD_5 loading 3 to 5 times compared to the low rate filter. This filter is used as a roughing filter for waste discharge into sewers.

The effluent is recirculated at a rate varying from 1 to 4 times the influent flow. Recirculation may be done after the secondary clarifier or directly after the filter itself. The latter method returns viable organisms into the filter.

If the influent BOD_5 is very high, the filter may be operated in series. Fig. 7-2 shows the standard flow configurations used in high rate trickling filters. Due to high hydraulic loading, filter flies are continuously washed out before they reach the adult stage. However, since uniform wastewater distribution is seldom attained, few moist areas provide fertile breeding grounds for filter flies in the tropics.

Rotating filters – Large spaces are required for trickling filters as large void volume is necessary for air circulation and uniform distribution of the waste over the filter media is difficult to attain. Rotating filters have the same principles as trickling filters except that the media is immersed alternately in the wastewater and exposed to the air. In the trickling filter, wastewater and air circulate while the media is immobile.

In small treatment plants, an Imhoff tank is usually constructed below or beside the aerobic tank to hold and digest the sloughed-off slimes. This minimizes the need for continual sludge removal and the volume of sludge for disposal is reduced. In tropical conditions where the ambient temperature is within the mesophilic range of anaerobic digestion, methane could be recovered as shown by Nair (1971) and Ouano (1974). At this stage, methane recovery in this process is of academic interest only.

The most common type of contact media used is made of polyvinyl or polyethylene plastic discs attached to a shaft at 1 to 2 cm apart as used by Antonie (1972), Welch (1968), Popel (1962) and Marki (1964). In large units, supports are required to prevent warping and clogging of the area between the media. Hollow polyethylene plastic balls have been used by Euromatic Plastic Co. (1972), mesh wire sheets by Bruce (1973), and bamboo rods by Ouano (1974).

In terms of efficiency and utilization, the rotating filters are similar to the trickling filters. It has been used as polishing filter after septic tanks by Pretorius (1971) and as a roughing filter in feedlot wastes by Chittenden and Wells (1971).

A number of units are operating in Europe especially for small communities, although plants up to 10,000 m^3/day (2.5 mg) have been constructed in the United States as reported by Antonie (1974). Table 7-1 shows the power consumption of the different processes in relation to rotating filters.

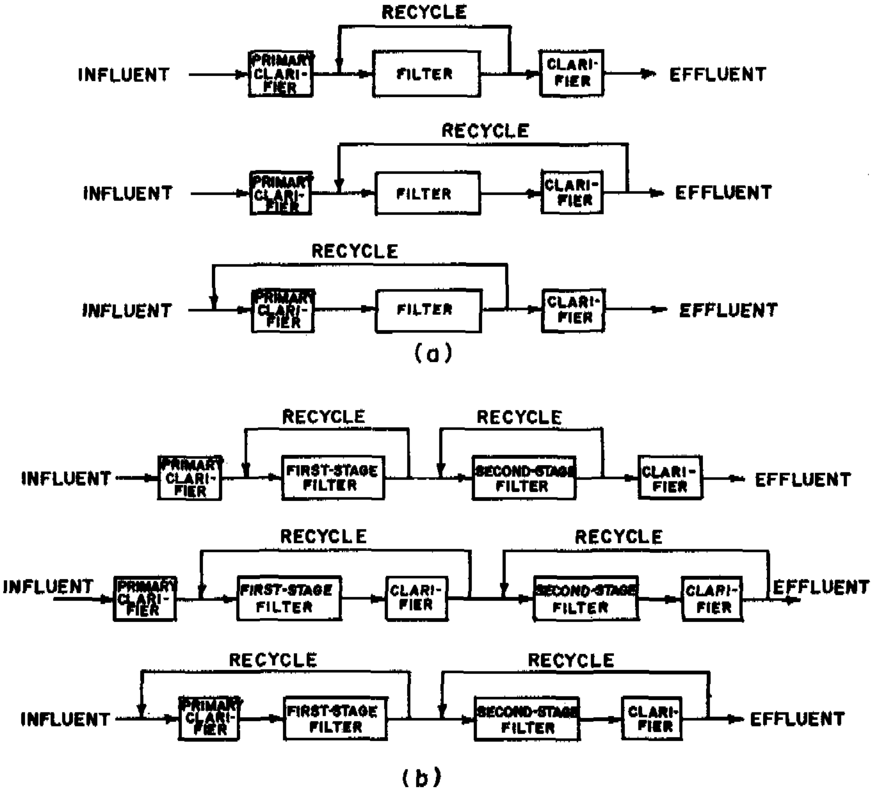


FIG. 7-2 **High rate trickling filter flowsheets with various recirculation patterns. a) Single stage filters, b) Two-stage filters.**

TABLE 7 – 1

Range of Estimated Power Requirements for Different Waste Treatment Processes

Process	Energy Required kw-hr/kg of influent BOD
Extended Aeration	2 to 4
Conventional Activated Sludge	1 to 3
Aerated Lagoons	0.5 to 1
High Rate Trickling Filter	.2 to .3
Rotating Filters	.05 to .1
Low Rate Trickling Filter	nil to .1
Lagoons	nil to 0.05

Anaerobic filters – Anaerobic microorganisms have slower growth rate than aerobic microorganisms. Hence, the high rate anaerobic digester is more difficult to operate due to the instability of the microbial population in relation to the food availability. The problem is further complicated due to low tolerance level of the methane forming organism to changes in pH, temperature and food supply. Anaerobic filters or fixed film reactors have recently been developed to maintain large active anaerobic population attached to suitable contact media in the biochemical reactor. The anaerobic bacteria in the slime is available almost all the time waiting for the food or BOD to be introduced to the biochemical reactor. The filter consists of a cylindrical column filled with rocks from 3 to 6 cm in diameter as shown in Fig. 7-3. The low strength waste flows upwards to the column and comes into contact with the anaerobic microorganism attached to the contact media. For tropical conditions, Centeno (1974) reported a removal efficiency up to 50% when loaded at the rate of .65 kg/m^3 – day, and up to 60% removal efficiency at .212 kg/m^3 – day.

Process kinetics

The bacterial population is controlled by protozoan predation, washout, precipitation and death. When the microbes die, lysis takes place recycling the nutrient and protoplasmic material into the waste stream which is helpful in treating nutrient deficient wastes. Death may be also caused by changes in environmental conditions, depletion of substrate or oxygen.

When the environmental conditions become unfavorable to a particular specie of microorganisms, population succession takes place. The original species will die off or become inhibited while new species will gain dominance and continue the degradation reaction. Since biological degradation requires an initial concentration of microorganism to proceed, the more diversified the

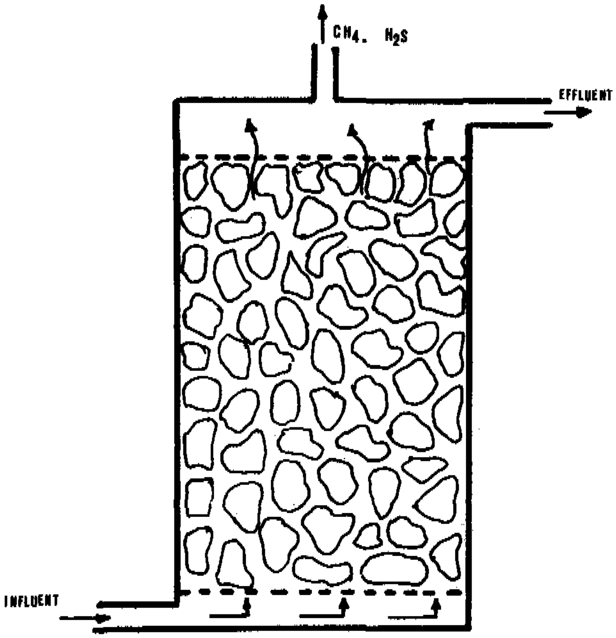


FIG. 7-3 Schematic diagram of an anaerobic filter.

species in the process, the more stable it becomes to changes in operating conditions. The trickling filter and bio-disc flora and fauna have been shown by Hawkes (1963) and Nair (1971) to be highly diversified compared to dispersed growth reactors such as activated sludge or Parseveer ditch. This explains the ability of the process to absorb shock loads.

The prey-predator relationship and higher form of organisms such as nematodes add to the stability of the rotating filters. At each stage of the food chain, further stabilization takes place as the food is converted to inert gases and energy. The end product of the conversion has a higher mineral and lower bio-degradable content than in the dispersed growth reactor. This leads to lower quantities of solids and better settling properties than encountered in dispersed growth reactors.

The Slime Layer – During the start up of a new filter, a very thin film composed of aerobic bacteria is formed. As the filter matures, the slime layer becomes thicker and oxygen in the innermost layer is limited because of diffusion. The aerobic bacteria at the inner layer die off and undergo lysis. Facultative and anaerobic bacteria then predominate at the inner layer. Under steady state, the aerobic layer thickness remains constant being limited by the oxygen diffusion, while the anaerobic layer continues to grow thicker until sloughed.

Sanders (1966) proposed a recirculation model of the nutrients and lysis products into the aerobic layer from the anaerobic layer as shown in Fig. 7-4. This signifies that as the anaerobic layer becomes thicker, recirculation of nut-

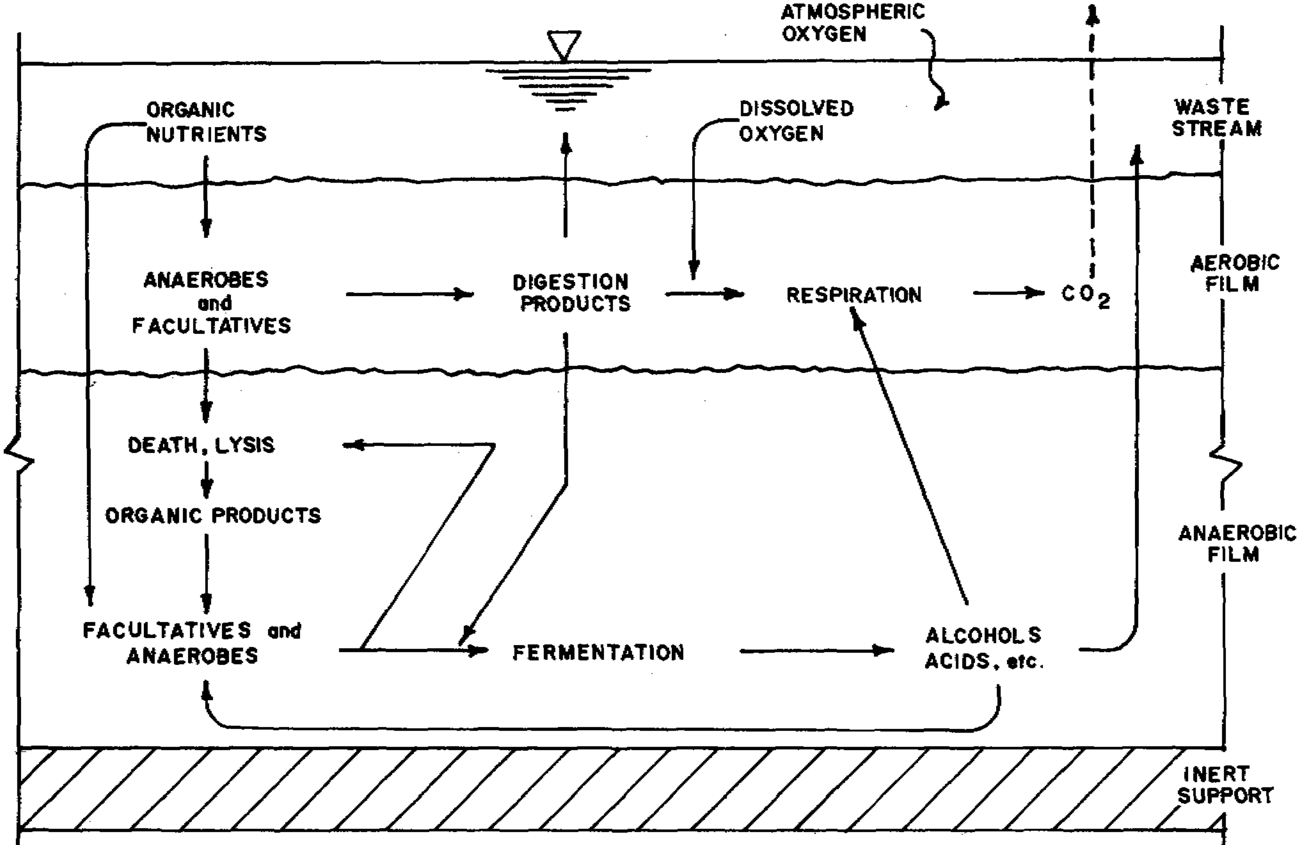


FIG. 7-4 Currently accepted concepts of the structure and metabolism of a mature biological film community.

rients occurs thereby decreasing the concentration gradient between the aerobic layer and the waste stream. A decline in efficiency is predicted.

Kornegay and Andrews (1968) found that the efficiency of the slime layer to remove nutrients remained constant after the nutrient uptake rate had reached its maximum. This led them to postulate that the anaerobic layer exists in a closed system in which the nutrients and substrates are consumed internally. The biodegradation in the anaerobic layer results in a more complete degradation of the waste resulting in sludge of lower quantities and higher specific gravity. This has been verified by Hoehn and Ray (1973).

Although the effective slime thickness has been qualitatively determined by Hawkes (1963) and McKinney (1962) as a very small portion of the total slime layer, a common misconception has prevailed in considering the whole slime layer as viable. Pretorius (1971) and Ellis and Banaga (1976) have determined the effective biomass in biofilter to be equivalent to 10,000 – 30,000 mg/l of MLSS in activated sludge process, which is an over-estimation considering the performance of the biofilters and dispersed reactors when treating the same wastes.

Since the activity of the biofilter is highly dependent on the surface available for bacterial growth, commercial development in the last 20 years has been directed at improving the specific surface area of the media and the wastewater circulation. The use of light plastic materials has made it possible to construct tall biofilters where land is limited. Plastic sheets are often corrugated to provide longer contact time and increase specific surface area. Examples of this are the commercial packings, Surfac, Dowpac, and Flocor.

At present, synthetic contact media could be classified as random packings or moulded sheets which are easier to install. Although they have a higher specific surface area when covered with slimes, some of the slime surfaces come into contact with each other, thereby decreasing the effective area. Cook (1971) concluded that the effect of increased surface area beyond 82 m^2/m^3 on the removal efficiency is minimal.

Oxygen Transfer and Utilization

Under normal operating conditions the biofilter effluent contains a dissolved oxygen concentration from 2.0 to 4.0 mg/1. Since it has been proven by Eckenfelder (1966) that oxygen is non-rate limiting in aerobic digestion above 0.5 mg/1, previous researches often neglected oxygen transfer and utilization.

Sanders et al. (1971) were able to establish the distribution of oxygen in the slime layer using microprobes. Mehta et al. (1972) proved that although the dissolved oxygen in the effluent is greater than .5 mg/l, the unequal distribution in the slime layer makes oxygen rate limiting. They were able to correlate the BOD_5 removal efficiency to the oxygen mass transfer rates. Recent studies by Williamson and McCarty (1976) show oxygen is rate limiting at COD levels of 88 mg/1 or more.

To improve oxygen utilization, the two main aspects to consider are *distribution of oxygen across the slime layer* and the *depth of oxygen penetration*. Redistribution may be brought about by using random packings or increased turbulence due to higher hydraulic rates. Oxygen penetration can be done by enriching the air with oxygen or by using dilute waste. The two aspects are satisfied by increased recirculation in trickling filters.

Trickling Filter – The oxygen transfers in the slime layer could be expressed as:

$$\frac{\partial O}{\partial t} = D_L \alpha \frac{\partial^2 O}{\partial X^2} - O_r \qquad (7-1)$$

where O is the dissolved oxygen concentration, mg/1; t is time, sec; D_L is diffusivity of oxygen in water, cm^2/sec; α is the oxygen transfer efficiency of the waste; X is the directional axis perpendicular to the slime surface, and O_r is the oxygen uptake rate in mg/1-sec. At steady state, equation 7-1 reduces to:

$$D_L \alpha \frac{d^2 O}{dX^2} = O_r \qquad (7-2)$$

Subject to the following boundary conditions:

$$\begin{array}{lll} X = 0.0 & O = O_s & \\ X = \delta & O = 0.0 & \\ X = \delta & dO/dx = 0.0 & (7-3) \end{array}$$

where δ is the depth of the aerobic layer, O_s is the oxygen concentration at saturation in mg/1 and the other terms are as defined before. Solving Eq. 7-2 yields:

$$O = \frac{O_r}{2D_L \alpha} X^2 - \frac{O_r}{D_L \alpha} X + O_s \qquad (7-4)$$

and

$$\delta = \sqrt{\frac{2D_L \alpha O_s}{O_r}} \qquad (7-5)$$

The mass transfer flux could be expressed as:

$$N_t = -\left(\frac{\partial C}{\partial x}\right)_{\delta=0} D_L \alpha \qquad (7-6)$$

where N_t is the oxygen mass transfer flux in mg/cm^2 –sec.

Then,

$$N_t = \sqrt{2 D_L \alpha O_s O_r} \qquad (7-7)$$

If the filter has a total surface area, A, for oxygen transfer, then the weight of oxygen transferred is:

$$\text{Wt of oxygen} = N_t A = A \sqrt{2 D_L \alpha O_s O_r} \qquad (7-8)$$

The weight of substrate removed is $O_r V$ in which V is the volume of wastewater. If oxygen is rate limiting, the quantity of substrate removed is equal to the mass of oxygen transferred, or:

$$\left(\frac{A}{V}\right) \sqrt{2 D_L O_s O_r \alpha} = (1\text{-}Y)\, O_r \qquad (7-9)$$

Since oxygen enrichment increases O_s, the depth of the aerobic layer increases at constant oxygen uptake rate, verifying the results by Torpey et al. (1972). Eckenfelder (1966) reported the same correlation for the depth of the slime layer in trickling filters. Fig. 7-5 shows the oxygen profile as determined by Sanders et al. (1971).

Rotating Filters – The oxygen transfer in rotating filters is more complicated than in trickling filters due to shearing of the liquid film as it dips into the liquid bulk. At high speed of rotation, a steady state may not occur. The oxygen mass balance in the rotating filter is as shown in Fig. 7-6.

At low speeds of rotation, Ouano and Pescod (1976) made the following assumptions on oxygen transfer and utilization:

1. The time for oxygen transfer and utilization to reach steady state is small compared to the total exposure time.
2. The shearing forces between the rotating media and the liquid bulk is sufficient to redistribute the dissolved oxygen across the aerobic layer.
3. The liquid bulk exists as a completely mixed stirred reactor. The liquid bulk acts as a reservoir for excess oxygen from the slime layer and as a source of oxygen just before atmospheric immersion. The oxygen transfer into the liquid bulk with the media acting as a stirrer could be expressed as:

$$\frac{dO}{dt} = \alpha K_L \frac{A_t}{V} (O_s - O) - O'_r \qquad (7-10)$$

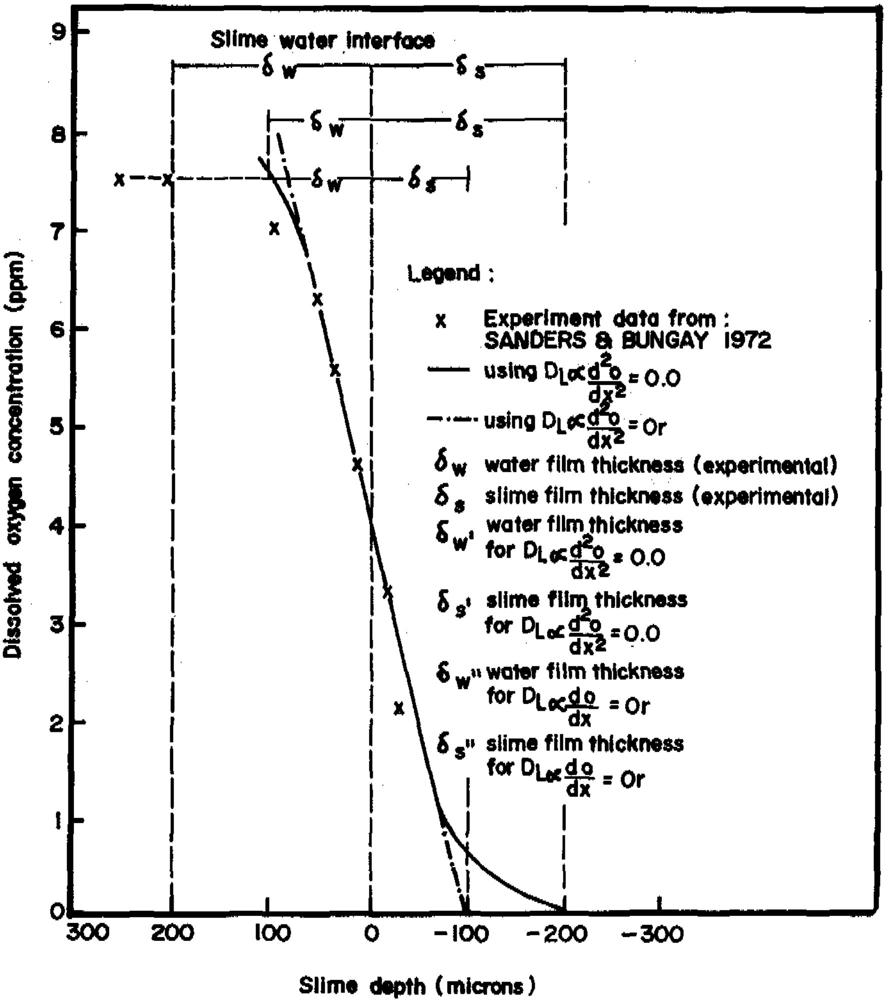
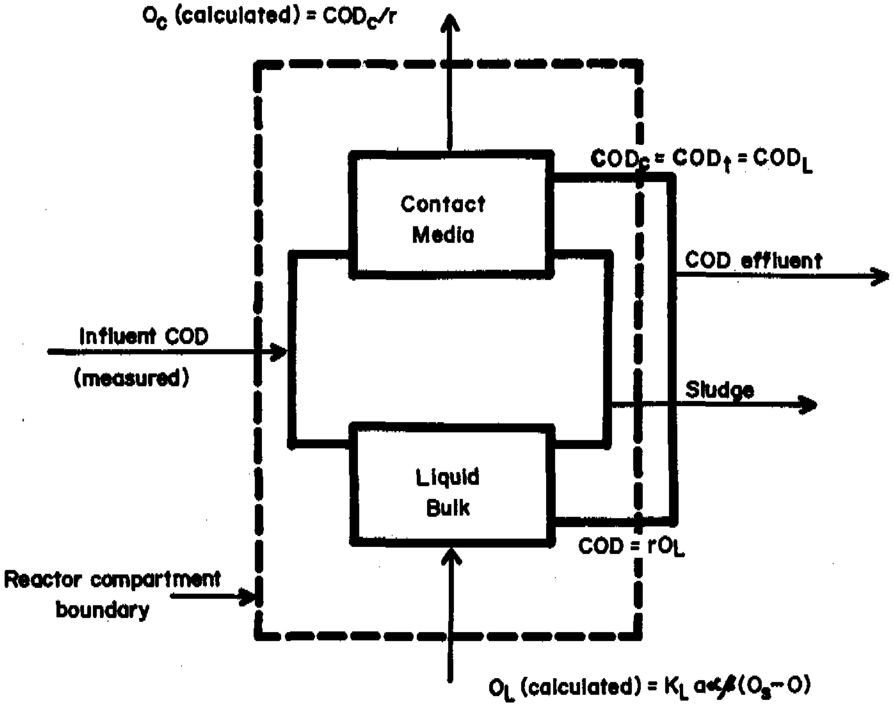


FIG. 7-5 Comparison of the different assumptions of oxygen utilization in the slime.



Legend :

r = O /COD ratio (respirometry)

COD = chemical oxygen demand

o = oxygen concentration

c = contact media

L = liquid bulk

t = total

FIG. 7-6 Material balance in slime layer and liquid bulk in rotating filters.

in which O'_r is the oxygen uptake during the immersion phase and A_t is the surface area of the tank, and α is the oxygen transfer efficiency in the liquid bulk. Combining Eq. 7-8 and Eq. 7-10, the change in dissolved oxygen concentration in each stage could be expressed as:

$$V \frac{dO^m}{dt} = K_L A_t (O_s - O^m) - (1-Y)O'_r V + f A\sqrt{2D_L \alpha O_s O_r} + q O^{m-1} \qquad (7-11)$$

where f is the fraction of surface area exposed; O^{m-1} is the dissolved oxygen concentration at the previous stage; O^m is the dissolved oxygen concentration at the stage under consideration and the other terms are as defined before.

When the oxygen is rate limiting in the slime layer, then Eq. 7-9 is used for O_r otherwise the substrate is taken as rate limiting. For scale up, K_L could be expressed by the following relationship:

$$\frac{K_L V/A_t}{D_L} = a \left(\frac{D^2 \omega \rho}{\mu} \right)^b \qquad (7-12)$$

in which D is the diameter of the media, m; ω is the speed of revolutions, rad/sec; ρ is the density, kg/m^3; and μ is the viscosity in kg/m^2-sec. Fig.1-10 shows the effect of scale up using various types of contact media.

From Eq. 7-11, the controversies on the oxygen utilization noted by Bruce et al. (1973) and Toypey et al. (1972) could be resolved.

Enriching the atmosphere with oxygen increases the depth of penetration of dissolved oxygen in the slime layer during the exposure stage. Except when the dissolved oxygen in the liquid bulk reaches zero, the good distribution that occurs across the slime layer during the immersion phase makes oxygen non-rate limiting. Hence, bubbling air into the liquid bulk will have minimal effect on the substrate degradation although the dissolved oxygen concentration will increase.

When using small units at very slow speeds of revolution, the shearing force could be insufficient to redistribute the dissolved oxygen across the slime layer. In which case, the oxygen transfer mechanism defined by Eq. 7-2, takes place in the liquid bulk with the dissolved oxygen concentration of the liquid bulk used in lieu of O_s. Since the dissolved oxygen concentration in the liquid bulk is less than or equal to O_s, then O_r' is less than or equal to O_r. In this case, exposure of a larger surface area will yield higher substrate removal as noted by Ellis and Banaga (1976).

Substrate and Nutrient Utilization

Although biofiltration techniques have been used in wastewater treatment for almost a hundred years, the mechanics of substrate utilization and degradation is just starting to be understood.

In the past empirical methods based on complied data and experimental results were used to develop a number of design criteria. Due to the lack of proper instruments and equipment to explore the slime layer in detail, experimentalists have often resorted to a "black box" concept of the trickling filter in which various combinations of outputs and inputs were measured and collated on the assumption that the same mechanisms holds true in all these ranges.

Current studies by Williamson and McCarty (1976) have shown the types of rate limiting species due to varying diffusivities and utilization rates. Most of the past experimental results were carried out with different rate limiting conditions and should be interpreted for a specific range of operating conditions rather than in a generalized context. Ordon (1968) reviewed the three most common design equations used for trickling filters and found the results varied sometimes by a factor of 13 or *1,300% error.* Baker and Graves (1968) found those equations to give dissimilar trends which makes extrapolation a delicate exercise into uncertainty. For large treatment works, pilot plant testing must be done before a reasonable design can be carried out. For small industrial waste treatment plants, data from literature treating similar waste coupled with a reasonable factor of safety and past experience is usually sufficient. At present, there are almost a dozen design equations for trickling filters, half a dozen for rotating filters and two or three for anaerobic filters.

Design Parameters

The rate of BOD degradation could be expressed as:

$$\frac{dC}{dt} = \frac{-KCX_a}{k_m + C} \tag{7-13}$$

where the terms have been previously defined. The weight of the microbial mass in the fixed film reactor could be expressed as:

$$X = A_v \rho_s \delta \tag{7-14}$$

where A_v is the specific surface area of the contact media, m^2/m^3; V is the volume of the reactor, m^3; δ is the thickness of the slime layer, m; ρ_s is the density of the microbial mass, kg/m^3. Rearranging Eq. 7–14 yields:

$$X_a = \frac{X}{V} = A_v \rho_s \delta / V \tag{7-15}$$

Substituting Eq. 7–15 in Eq. 7–13 yields:

$$\frac{dC}{dt} = \frac{-kC A_v \rho_s}{k_m + C} = -k \rho_s \frac{A_v C}{k_m + C} \tag{7-16}$$

When $k_m >> C$, then Eq. 7–16 simplifies to:

$$\frac{dC}{dt} = -k\rho_s A_v C = K'A_v C \qquad (7-17)$$

where

$$K' = \frac{k\rho_s}{k_m}$$

When $k_m << C$, Eq. 7–16, simplifies to:

$$\frac{dC}{dt} = -k'' A_v \qquad (7-18)$$

where

$$k'' = k\rho_s \qquad (7-19)$$

In some instances, the inhibitory term, $k_m + C$ takes the form $k_m + C_o$ where C_o is the initial BOD concentration. Under this condition, C_o is normally much higher than k_m. Hence, Eq. 7–13 simplifies to:

$$\frac{dC}{dt} = \frac{-K''A_v C}{C_o} \qquad (7-20)$$

Integrating Eq. 7–20, yields;

$$\int_{C_o}^{C} dC = \int_{0}^{\Theta} \frac{-K''A_v}{C_o}\, dt$$

$$C = C_o \exp\left(\frac{-K'''A_v\Theta}{C_o}\right) \qquad (7-21)$$

Experimental studies by Howland in 1957 shows the detention time Θ, of the liquid in the trickling filter could be expressed as:

$$\Theta = h^{b}\left(\frac{A}{q}\right)^{a} K_1 \qquad (7-22)$$

where K_1 is a constant, h is the filter depth in m; A is the cross sectional area of the filter, m^2; q is the influent flow rate, m^3/day, a and b are constants whose values are close to one. Substituting Eq. 7–22 into Eq. 7–21 results to:

$$C = C_o \exp\left(\frac{-k'' k_1 A_v}{C_o}\left(\frac{A}{q}\right)^{a} h^{b}\right) \qquad (7-23)$$

The terms inside the exponential terms of Eq. 7-23 could be rearranged as:

$$h\left(\frac{K'' k_1}{C_o}\right)\left(\frac{A}{q}\right)(A_s) = K'' k_1 \; A_v\left(\frac{A}{C_o q}\right) h = K'' k_1\left(\frac{A_V}{C_o}\right)\left(\frac{Ah}{q}\right)$$

$$= \left(\frac{Ah}{C_o q}\right) k'' k_1 A_v \qquad (7-24)$$

The different terms in brackets shown in Eq. 7-24 are the design parameters of the trickling filter. The terms q/A is known as hydraulic loading rate, m^3/m^2–day; q C_o/A is the areal loading rate, kg of BOD/m^2–day; q/Ah is the volumetric loading rate, m^3 of wastewater/m^3 of filter-day; and C_oq/Ah is the organic loading rate, kg of BOD/m^3 –filter–day.

Velz Equation – Velz (1949) studied the effect of filter depth on the BOD_5 removal efficiency of trickling filters with rock media treating domestic sewage. Since the initial BOD concentration of domestic sewage does not vary drastically, and for rock the specific surface is approximately 47 m^2/m^3, then for constant hydraulic loading rate, Eq. 7–23 simplifies into:

$$C = C_o \exp(-Kh) \qquad (7-25)$$

At 20°C, K is approximately 0.45/m. Not all the soluble BOD could be stabilized by the trickling filter microorganism. For purposes of calculation, C_o is taken at less than 0.9 of the iniitial BOD.

Example 7–1. Calculate the filter depth required to treat domestic sewage with an initial BOD concentration of 200 mg/l to meet an effluent BOD of 40 mg/l at 20°C.

Solution: The non-degradable BOD is 0.1 x 200 or 20 mg/1. Hence, the soluble BOD in the effluent is (40-20) or 20 mg/1. Substituting into Eq. 7–26 the values of C, C_o and k, yields,

$$20 = 0.9\,(200)\ \exp\ (-0.345\,h)$$

or

$$h = \frac{-1}{0.345}\ \ln\left(\frac{10}{90}\right) \qquad (7-26)$$

$$= 6.1\,m \qquad \text{use two towers in series of 3 m depth}$$

Velz' equation is a very simplified model of the BOD removal in trickling filters. By varying the cross sectional area and the wastewater flow rate, q, Bruce (1970) found out that the hydraulic loading rate, q/AH is a more appropriate parameter. This conclusion is evident from Eq. 7–23 which could be rearranged as:

$$C_o\ \exp\left(-K\frac{Ah}{q}\right) = C_o\ \ \exp\,(-K/H) \qquad (7-27)$$

where K is equal to ($K''k\ A_v/C_o$) and H is the volumetric loading rate m^3/m^3–day. Eq. 7–27 is valid if C_o and the specific surface are constant or the inhibitory term is not equal to $K_m + C_o$ However, it is erroneous to conclude as Tuan and Thanh (1978) has made that the BOD removal efficiency is dependent on the hydraulic loading rate and independent of h, since, the hydraulic loading rate is a function of h as given in Eq. 7–24.

Bruce Equation – With the development of plastic materials for the filter contact media, the specific surface becomes an important design consideration. Specific surface areas as high as 300 m^2/m^3 have been developed although current commercial practice limits the specific surface area to 190 m^2/m^3. Poor ventilation, clogging and odor are prevalent in contact media with very high specific surfaces. Hence, Eq.7–25 could be simplified as:

$$C = C_o\ \ \exp\,(-K\,A_v/H) \qquad (7-28)$$

if the inhibitory term is independent of C_o as suggested by Bruce and Boon (1971).

In most instances, the constants a and b of Eq. 7–22, may differ from one especially when plastic contact media are used. C.I.R.I.A. (1973) suggested the following modification to Eq. 7-28:

$$C = C_o\ \exp\ (-K\,A_v/H^{.76}\ ) \qquad (7-29)$$

Eq. 7–28 and 7–29 are popular in British practice. The value of K for temperature above 15°C could be calculated as:

$$K_T = (1.08)^{T-15} \quad (.0203) \qquad (7-30)$$

For domestic sewage at 15°C, K is approximately .0203.

Theoretically, the trickling filter volume required using plastic contact media, V_p, could be estimated from the trickling filter volume required using crushed rocks filter media V_r as:

$$\frac{V_p}{V_r} = \frac{40}{A_v} \qquad (7-31)$$

In practice the higher the specific area,the higher the dead zone, the unwetted area and the ineffective portions are. Eq. 7-31 is modified as:

$$\frac{V_p}{V_r} = \frac{40}{12 + .7A_v} \qquad (7-32)$$

In trickling filters using crushed rocks for contact media, the minimum hydraulic loading rate is 4.8 m^3/m^2 day. To maintain sufficient wetting of the contact media, the minimum hydraulic loading rate must be adjusted in proportion to the specific surface area. Hence, the minimum hydraulic loading rate for trickling filter using plastic contact media is adjusted as:

$$(q/A) = 4.8\ A_v/40 = 0.12\ A_v \qquad (7-33)$$

Example 7–2. a) Determine the trickling filter volume required to treat 10,000 m^3/day of domestic sewage with an initial BOD of 200 mg/l to satisfy an effluent standard of 40 mg/l using crushed rock as filter media. b) Calculate the volume required if plastic media with a specific surface area of 150 m^2/m^3 are used instead of crushed rock. c) Compare the volume required for the trickling filter media if Velz' equation is used. Temperature is 20°C.

Solution: a) From Eq. 7–30;

$$K = (1.08)^{20-15} \quad (0.0203)$$
$$= 0.03$$

For crushed rocks, $A_r = 40m^2/m^3$, hence, the volumetric loading rate is calculated from Eq. 7–28.

$$40 = 200 \exp(-.03(40)/H^{.76})$$

$$H = 0.75$$

The volume of the filter is:

$$\frac{V}{q} = \frac{Ah}{q} = \frac{1}{H}$$

$$\therefore V = 10{,}000/.75 = 13{,}333 \text{ m}^3$$

The minimum dousing rate or hydraulic rate is 4.8 m^3/m^2 – day. Hence, the maximum cross sectional area of the filter is:

$$q/_A = 4.8 \quad \text{or} \quad A = 10{,}000/4.8 = 2083 \text{ m}^2 \text{ say } 2000$$

The depth of the filter is 13,333/2000 = 6.66 m say 6.7 m

b) From Eq. 7–32

$$V_p = 13{,}333 \times 40/(12 + .7\ (150))$$

$$= 4{,}558$$

$$q/A = 4.8 \times 150/40 = 18 \text{ m}^3/\text{m}^2 - \text{day}$$

$$\therefore A = 10{,}000/18 = 555 \text{ m}^2$$

$$h = 4{,}558/555 = 8.2 \text{ m}$$

c) From Example 7–1, h = 6.4 m. Since, q/A is 4.8 m^3/m^2 –day, then

$$A = 10{,}000/4.8 = 2083 \text{ m}^2$$

$$V = Ah = 2083 \times 6.4 = 13{,}333 \text{ m}^3$$

Eckenfelder Equation – The depth of the filter is usually very small compared to the diameter specially if crushed rocks are used as contact media. The mixing pattern in the trickling filter could approximate the completely mixed system. A mass balance of the incoming, outgoing, accumulated and degraded organic wastes yields:

$$q\,C_o = q\,C - \frac{k' A_v C V'}{C_o} + V' \frac{dC}{dt} \qquad (7-34)$$

where V' is the effective reactor volume, m^3

At steady state, dC/dt is equal to zero, hence, Eq. 7–34 simplifies to:

$$C = \frac{C_o}{1 + \left(\frac{k A_v}{C_o}\right)\left(\frac{V'}{q}\right)} \qquad (7\text{–}35)$$

The ratio V'/q is the reaction time, which is almost equal to the wastewater detention time in trickling filters. Substituting Eq. 7–22 for V'/q into Eq. 7–35 yields:

$$C = \frac{C_o}{1 + \left(\frac{kA_v k_1 h^b}{C_o}\right)\left(\frac{A}{q}\right)^a} \qquad (7-36)$$

When the range of initial BOD is very narrow then Eq. 7-36 takes the form

$$C = \frac{C_o}{1 + K h^b \left(\frac{A}{q}\right)^a} \qquad (7-37)$$

where K is equal to $k\, k_1\, A_v$.

Eckenfelder recommended a K value of 52.2. The constants a and b are equal to 0.5 and 0.67 respectively. Hence, Eq. 7–37 takes the form

$$C = \frac{C_o}{1 + 5.22\, h^{0.67} \left(\frac{A}{q}\right)^{0.5}} \qquad (7\text{–}38)$$

where h is in m, q is in m^3/day, and A is in m^2. The original value of K was 2.5 where A, q and h are in English units.

> Example 7–3. Calculate the trickling filter volume required to treat the domestic sewage whose characteristics are given in Example 7–2 using Eckenfelder equation to meet an effluent quality of 40 mg/1.
>
> Solution: Substituting the values of C_o, C, A and q into Eq. 7–38, yields:

$$40 = \frac{200}{1 + 5.22\left[\dfrac{h^{0.67}\,(A)^{0.5}}{(10{,}000)^{0.5}}\right]}$$

or

$$h^{0.67}\;A^{0.5} = 7.46$$

For crushed rocks, the minimum q/A is 4.8 m^3/m^2–day to provide sufficient wetting of the filter surfaces.
Hence,

$$A = q/4.8 = (10{,}000\ m^3/day)\ /\ (4.8\ m^3/m^2\ –day)$$

$$A = 2083\ m^2 \text{ say } 2000\ m^2$$

Therefore,

$$h = (7.46\ /\ 2000)^{1.33}$$

$$= 2.1\ m$$

The trickling filter volume is then equal to

$$Ah = 2.1 \times 2000 = 4{,}200\ m^3$$

It should be noted from Eq. 7–38, that the BOD removal efficiency is inversely proportional to the 0.67 power of the trickling filter depth and 0.5 power of the trickling filter cross sectional area. Hence, it would be advisable to use the maximum practical trickling filter height.

For example, if the maximum filter height is 6m, then

$$A = 76.6/\ (6)^{.67}\ m^2$$

$$= 531\ m^2$$

V = 531 x 6 = 3186 which is lower than the 4,200 m^3 calculated above.

NRC Equation – The National Research Council conducted a study on the removal efficiency of trickling filters for domestic sewage from military camps. By means of regression analysis, the following recommended formula was obtained:

$$\frac{C_o - C}{C_o} = \frac{1}{1 + 7.1\left(\dfrac{qC_o}{Ah}\right)^{.5}} \qquad (7-39)$$

The term C_oq/Ah is in kg of BOD/m^3–day.

Example 7–4. Calculate the filter volume required to treat he domestic sewage whose characteristics are given in Example 7-2 using the NRC equation to meet an effluent standard of 40 mg/1.

Solution:

The weight of BOD applied is equal to:

$$C_o q = 10{,}000 \frac{m^3}{day} \times 200 \frac{mg}{1} \times 10^{-3} \frac{kg\text{–}1}{mg\text{-}m^3}$$

$$= 2000 \text{ kg/day}$$

Substituting the values of C_o,C and C_oq into Eq. 7–33 yields:

$$\frac{200 - 40}{200} = \frac{1}{1 + .45(2000/V)}$$

$$V = 6{,}480 \text{ m}^3$$

The volume is only 50% of the volume calculated from Example 7–2 using the Velz and Bruce equation.

Eq. 7–39 could be derived also from Eq. 7–36 if a and b are equal to 0.5. With those values, Eq 7–36 could be rearranged as:

$$C = \frac{C_o}{1 + K\left(\frac{hA}{C_o q}\right)^{.5}\left(\frac{1}{C_o}\right)^{.5}} \tag{7 – 40}$$

When the range of initial BOD values tested are very narrow as could happen with domestic sewage in military camps the square root of the initial BOD value will not vary appreciably. Hence the square root of C_o in Eq. 7–40 could be treated as a constant and lumped with K constant to give the NRC equation.

Recirculation

For strong wastewater, the trickling filter has a tendency to turn anaerobic. The incoming wastewater is diluted by recycling the effluent although in some instances the settled solids are also recycled. Recirculation improves the wastewater distribution in the filter thereby minimizing the dead spaces of the contact media.

The influent BOD is diluted by the recycled flow. Hence, the effective influent BOD, C_o' after mixing the raw wastewater and the recyled flow is equal to:

$$C_o' = \frac{q\,C_o + q_r\,C}{q + q_r} = \frac{C_o + R\,C}{1 + R} \qquad (7-41)$$

where R is the recirculation ratio and the other terms are as defined before. Velz equation (Eq. 7–25) is then modified as:

$$C = \frac{C_o + R\,C}{1 + R} \exp(-k\,h) \qquad (7-42)$$

Rearranging Eq. 7 – 42, yields:

$$C = \frac{C_o \exp(-k\,h)}{1 + R - R\exp(-kh)} \qquad (7-43)$$

Similarly, the NRC equation is modified as:

$$\frac{C_o - C}{C_o} = \frac{1}{1 + 7.1\left(\frac{q\,C_o}{AhF}\right)^{.5}} \qquad (7\text{–}44)$$

where

$$F = \frac{1 + R}{(1 + .1R)^2}$$

The major deficiency of the current design equation for trickling filter with recirculation is the lack of proper analysis in adapting the design equation obtained from trickling filter without recirculation.

Mass balance of the different BOD stream is neglected resulting in erroneous conclusion. If a mass balance around the trickling filter is made, this results to:

$$q\,C_o + q_r C = (q + q_r)\,C + K\,A_v C\,V' + V'\frac{dC}{dt} \qquad (7\text{-}45)$$

where V' is the effective trickling filter volume and the other terms have been defined before. At steady state, $V' dC/dt$ is equal to zero.Solving for C yields:

$$C = \frac{C_o}{1 + K A_v \left(\frac{V'}{q}\right)} \qquad (7-46)$$

Hence, if V'/q is the effective hydraulic detention time defined by Howland in Eq. 7–22, then Eq. 7–46, is modified:

$$C = \frac{C_o}{1 + K A_v \left(\frac{A}{q}\right)^a \left(h\right)^b} \qquad (7-47)$$

where K is equal to $K_1 A_v K'$.

If a is equal to 0.5 and b is equal to 0.67, then Eq. 7–47 reduces to Eq. 7–39. When the inhibitory term is defined by $k_m + C_o$, where $C_o >> k_m$, then Eq. 7–47 reduced to:

$$C = \frac{C_o}{1 + k_1 \left(\frac{Ah}{C_o q}\right)^{.5} \frac{1}{C_o^{0.5}}} \qquad (7-48)$$

If C_o is taken as constant, then Eq. 7–48 reduces to the NRC equation without recirculation. The analysis could be extended to the plugflow equations of the trickling filter. Recirculation of the effluent merely equalizes the variations of the influent quality and quantity thereby improving the utilization of the trickling filter volume.

The effect of recirculation on the trickling filter efficiency is minor as shown in Eqs. 7-46 and 7–48. In fact the dilution of the influent BOD reduces the rate of organic degradation if it conforms to a first order biochemical degradation. Hence, the reduction of the trickling filter volume due to the recirculation will not be significant compared to those predicted by Eq. 7 –43 or 7 –44.

The major shortcoming in the derivation of Eq. 7–43 is the change in the effective influent BOD concentration without considering the effect of the total flow rate $(q + q_r)$ on the detention time. Hence, q in Eq. 7–44 should be replaced by $q + q_r$ for the equation to be correct.

Ordon (1968) conducted a sensitivity analysis on the effect of effluent recirculation on the required trickling filter volume, in cu.m., to treat 3781

m^3/day of domestic sewage at 20°C with an enfluent BOD of 150 mg/l and effluent of 20 mg/l. The trickling filter volumes required at various recirculation ratios using the different design equation are tabulated below.

Recirculation Ratio	NRC	Design Equation Eckenfelder	Galer & Gotaas
0	1133	1644	8503
1	708	425	652
2	567	198	207
3	510	113	122
4	418	71	102
5	453	51	85
6	–	40	–

TABLE 7–1. Ordon's Calculation on the Trickling Filter Volume Reduction due to Recirculation

The trickling filter volume is reduced by almost 97.5% when the recirculation ratio is equal to six in comparison to the trickling filter volume operated without any recirculation. If the trickling filter volume is fixed at 1644 m^3, the same trickling filter is expected to give the same BOD removal efficiency when the wastewater flow rate is increased from 3787 m^3/day to 137,490 m^3/day with a recirculation ratio of six. Such performance can not be expected from any trickling filter.

Recirculation should be limited to attain maximum wetting of the contact media surface. For domestic sewage the recirculation ratio is limited within a range from one to two. For strong industrial wastewater higher recirculation rates may be utilized. The optimum recirculation ratio should depend on the geometric configuration of the contact media. Reynolds and Chipperfield (1970) found the trickling filter efficiency to decrease when the contact surface are substantially wetted. Those findings contradict those predicted by the different trickling filter equations. Recirculation of the trickling filter effluent is expected to improve the BOD removal efficiency slightly and not as predicted by Eq. 7–42, and 7–44 and other trickling filter equations.

Rotating Filters

The use of rotating filters for wastewater in small communities has been effective in Europe and U.S.A. due to their ability to function with minimal maintenance, low power cost and high stability. One of the major shortcomings of the process is its high initial cost which, however, may be minimized by using other types of contact media available locally or which could be shipped more conveniently.

Design Parameters – The design of RBF units could be approached using empirical design criteria used in the design of trickling filter and activated sludge process or with the use of substrate degradation kinetics and mass transfer principles. The design procedure from the former method is simpler but it requires intensive pilot plant evaluation technique to determine the optimal combination of operating variables.

When designing wastewater treatment facilities empirical approach is advantageous due to its simplicity. However, in instances where industrial or agricultural wastes are treated simultaneously with domestic waste, the application of substrate degradation kinetics and mass transfer principles is preferable as it requires less laboratory work, and better control of trade offs among the different design parameters.

Empirical Approach – The major design parameters in RBF design are areal loading, volumetric loading, hydraulic loading, retention time and peripheral velocity. Areal loading is the quantity of BOD_5 or COD which could be removed per unit disc area per unit time. Since substrate degradation rate is a first order reaction the first stage of the RBF unit has a higher areal loading rate compared to the units at the effluent end. Some manufacturers rate their RBF using an average value of the areal loading for the whole unit while others use the first stage areal loading rates only. It is therefore advisable to verify the basis on which the areal loading rates have been established or determined. Table 7–2 shows the range of values reported in literature for the different areal loading rates and other design paramaters. In most instances the authors quoted in Table 7–2 did not elucidate on their procedure of caculating the areal loading rates.

Volumetric loading is the quantity of matter degraded per unit volume of reactor per unit time. This parameter is borrowed from the design of activated sludge process. The parameter is significant only if a large quantity of dispersed growth organism exists. Some authors neglect this parameter in reporting their studies as shown in Table 7–2.

The retention time varies from 45 mins in RBF units with sludge recirculation to four hours in RBF unit with simultaneous sludge removal as reported by Joost (1969) and Bruce (1973) respectively. Table 7–2 shows the detention time reported by other authors in wastewater treatment literatures.

The speed of revolution of RBF units has been limited from 0.5 to 15 rpm (Pretorius 1975) although speeds up to 45 rpm have been tried by Welch (1969). For scale up, peripheral velocity has been used although no correlation has been established between the process performance and efficiency as reported by Chesner and Molof (1976). Peripheral velocity has been reported to vary from 0.1 to 2 ft/sec. The major consideration in the choice of peripheral velocity has been the premature sloughing of the slime layer due to high shearing forces created between the wastewater bulk and the immersed contact media.

Hydraulic loading which is defined as the quantity of waste volume

TABLE 7–3 – Summary of Empirical Parameters Used in RBF Design

References	Areal Loading	Volumetric Loading	Retention Time	Speed of Revolution	Type of Wastes	Efficiency	No. of Stages	Hydraulic Loading
	Kg of BOD/ m^2	Kg of BOD/ m^3	hr	rpm		% BOD_5 Removed		m^3/m^2 – day
Bruce et al (1973)	.01	–	6.1	1	sewage	90-94%	5	–
Joost (1969)	–	–	.75	–	sewage	90%	4	–
Pertorius (1971)	.022*	–	11–16	15–15	sewage anaerobically pretreated	nitrification	9	–
Pescod (1972)	.023*	12*	6	5–10	bottling plant wastes	80*	6	–
"	.024*	5.2*	2–12	10	sewage	60-80*	6	–
Burm et al (1972)	.01–.1*	–	4.1–8.7	4	cannery wastes	80-90	2	–
Torpey at al (1972)	–	–	1–1.2	10	sewage	93	10	12
Birks et al (1971)	–	–	7.5–8.0	62 fpm peripheral	cheese waste	95	4	–
Antonie et al (1971)	–	–	–	.75–2	sewage	95%	4	.11

* COD instead of BOD

doused per unit area of contact media per unit time (m^3/m^2 –day) have been suggested as a better design parameter than areal loading (Pescod, 1972).

Empirical Design Equation – Various design equations have been proposed to correlate the RBF waste regradation efficiency with the various design parameters mentioned above. The regression equation will allow trade offs between different levels and variables affecting the process efficiency as well as minimize the cost and data required from pilot plant studies. However, wastewater degradation could take place through several metabolic pathways as described limiting the range of applicability of the regression equations. It would not be surprising if the values predicted by the different models vary by a factor of 1300% as in the case of trickling filter design equations. Since the study on rotating filter has just started, there are very few regression equations available.

Joosts (1969) proposed correlation of the BOD_5 reduction per stage with respect to the different design parameters as:

$$\frac{\text{Percent BOD}_5\text{ Reduction}}{\text{Stage}} = k\,C^a\,R^b\,T^c\,\Theta^d \qquad (7-49)$$

where k is the treatability constant of the waste material, C is the concentration of waste material in stage, R is the physical configuration constant depending on the disc diameter, spacing, thickness, and submergence, T is the temperature of the waste material and Θ is the reactor residence time. However, Joost merely proposed the above equation without any experimental verification or analytical studies to establish the values of the different constants.

Hsieh (1972) suggested the following equation to correlate the percentage COD reduction:

$$P = 100\,(1 - e^{-bN\Theta}) \qquad (7-50)$$

and

$$N = \frac{V_t V_L}{A_L n} \qquad (7-51)$$

where P is the percentage COD reduction, b is a constant, N is the disc number, Θ is the detention time, hr., V_t tank volume per stage, m^3; V_L is the volumetric loading, Kg of COD/m^3 day; A_L is the areal loading in Kg of COD/m^2 –day and n is the surface per disc, m^2/disc.

Weng and Molof (1974) suggested the following equation to correlate the RBF nitrification efficiency:

$$F = \frac{0.0545_{\omega}^{0.644}\ Q^{.414}}{C^{0.530}\ A_{w}^{1.276}} \qquad (7-52)$$

where F is the intrification efficiency, A_w is the surface area immersed in water m^2; Q is the wastewater flow rate, m^3/hr; w is the speed of revolution, rpm and C is the substrate concentration in mg/1.

Except for the above mentioned equations very little work has been done on RBF to correlate the systems performance and the different design parameters.Extensive pilot plant equations have to be carried before any design could be done with confidence. Fig. 7-7 shows a typical curve for sewage design.

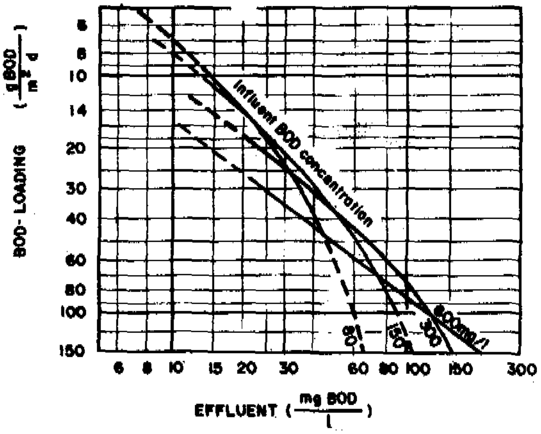


FIG. 7-7 Efficiency of rotating discs after Rincke (1976)

Example 7–5. Design a treatment plant for a small community with a sewage flow rate of 400 m^3/day with a BOD of 300 mg/1. The required stream standard is 30 mg/1 of BOD.

Solution: From Fig. 7–7, the required areal loading is 25 g of BOD/m^2–d. Since the total BOD discharge is (300 x 10^{-3} x 400 x 10^3) = 120,000 gm/day the required contact area is of the filter is:

Area = 120,000/25 = 4800.0 m^2

Rincke (1976) found that the filter area required using Fig. 7–7 is over optimistic and hence should be increased by 33%. Therefore, the required area = 4,800 x 1.33 = 6400 m^2

If a settling tank is required underneath then A/V = 150 m^2/m^3. Hence the tank volume is:

$$V = \frac{6{,}400}{150} = 43 \text{ m}^3$$

If the pheripheral velocity is fixed at 18 m/min, assuming a 3 m diameter as structurally sound for the disc, then the speed of revolution

$$\text{RRM} = \frac{18.0}{(1.5)\ (3.28)(2)\ (\pi)} = 1.94 \text{ rpm say 2 rpm}$$

The procedure does not give any indication on the effect of the number of stages nor on the D.O. level in the tank.

Kinetic Approach – Instead of using intuition the mechanism responsible for substrate degradation is studied in detail, the resulting equation will yield a more rational representation of the problem. The two major processes to be determined are the rate of substrate degradation and the rate of oxygen mass transfer.

Substrate Degradation Rate – The rate of substrate degradation could be expressed by Monod's equation as:

$$\frac{dC}{dt} = -\frac{1}{Y}\ \frac{\mu_{max}\ C}{K_m + C}\ X \qquad (7-53)$$

where K_m is the Monod constant, mg/1; X is the microbial mass concentration, mg/1; μ_{max} is the maximum specific growth, rate, hr^{-1}; Y is the yield coefficient, mg of microbial mass produced/mg of substrate degraded; t is the time hr.

In the rotating filter the microbial mass could be expressed as the sum of the fixed film biomass and the dispersed biomass or

$$VX = A\rho_m\delta \quad + X_dV \qquad (7-54)$$

where V is the volume of wastewater, m^3; A is the filter surface area, m^2; ρ_m is the density of biomass, mg/m^3; δ is the thickness of the aerobic layer, m, X_d is the dispersed biomass concentration, mg/1; and the other terms are as defined before.

Substituting Eq. 7-54 into Eq. 7–53 yields:

$$\frac{dC}{dt} = \frac{\mu_{max}\quad C}{Y\,(K_m\ + C)} \left\{ \frac{A\rho_m\delta}{V} \quad + X_d \right\} \qquad (7-55)$$

When $K_m \gg C$ and $X_d \rightarrow 0.0$, then Eq. 7–55 simplifies to:

$$\frac{dC}{dt} = - K_1 C \left(\frac{A}{V}\right) \qquad (7-56)$$

where $k_1 \approx \frac{\rho' m \mu_{max}}{Y K_m}$

Hence, a mass balance around the first stage of the bio disc yields:

$$\frac{VdC_1}{dt} = qC_o - qC_1 - K_1 C_1 \left(\frac{A}{V}\right) \qquad (7-57)$$

At steady state when $dC_1/dt = 0.0$, the effluent concentration of the first stage could be expressed as:

$$C_1 = \frac{C_o}{\left(1 + K_1 \frac{A}{Q}\right)} \qquad (7-58)$$

By making similar mass balance the effluent concentration for the nth stage could be expressed as:

$$C_n = \frac{C_o}{(1 + k\ A/Q)^n} \qquad (7-59)$$

where n is the number of stages and the other terms are as defined before.

It is worthwhile to note that using Eq. 7–59, the substrate degradation efficiency could be expressed as a function of the hydraulic loading rate, A/Q as determined empirically by Popel (1964). Hence, the kinetic equation gives a more general concept of the waste stabilization mechanism in the rotation biological filter which is approximated by the empirical equations at different boundary conditions.

For a particular type of contact media there exists a maximum limit of surface area that could be accommodated per unit volume of wastewater. This parameter is called areal density (A/V) which remains constant during scale up. The areal density for disc varies from 160-200 m^2/m^3–day, for rods from 180-250 m^2/m^3– day, for hollow 1" spheres 120-150 m^2/m^3–day. The upper values are used for units with separate settling tanks. Hence Eq. 7–59 could be re-arranged as:

$$C_n = \frac{C_o}{(1 + K\frac{A}{V}\Theta)^n} \qquad (7-60)$$

Example 7–6. The Pepsi-Cola Bottling in Bangkok produces a wastewater with the following characteristics:

Flow rate	1,000 m^3/day or 41.5 m/hr
COD (C_o)	700-1,000 mg/l
pH	6.0-8.0
Total N	less than 5 mg/l
DO	5–6 mg/l
Suspended solids less than	10 mg/l

From the study, the following kinetic constants for this type of plant waste are obtained:

Substrate degradation	k_o	0.2000
O_2 COD	k_s	.56
Oxygen transfer efficiency of the liquid bulk		1.2

From the model studies, the areal densities of the different contact media used are as follows:

Spheres	1.98 cm^2/cm^3 at 20 mm diameter
Discs	1.78 cm^2/cm^3 at a thickness of 3 mm
Rods	1.58 cm^2/cm^3

From the physical characteristics of water at 20°C the following properties apply:

Viscosity	.862 x 10^{-2} poise
Density	.99 gm/cm^3 say 1.0 gm/cm^3
Saturation concentration of oxygen	7.9 mg/l
Diffusivity of oxygen	.1 cm^2 /hr

Assuming that the required plant effluent standards are as follows:

COD = C	less than 100 mg/l
DO	greater than 4 mg/l
pH	5-8
Total N	less than 20 mg/l

Solution: *Design of the Aerobic Zone*

Step 1 The removal efficiency required to meet the effluent standard is:

$$P = 100 \frac{1000-100}{1000} = 90\%$$

It is apparent that the total volume of the treatment plant required to achieve a removal efficiency of 90% drops considerably as the number of stages used is increased from 1 to 4, if the detention time is fixed. After the fourth stage, the incremental decrease in the volume of the treatment plant is very small. Therefore the number of stages is taken as 4.

Step 2 In Chapter III, it was found that, for optimal conditions, the product of the areal density and the detention time per stage should be equal, hence:

$$\frac{A}{V}\,\Theta = \frac{1}{k_o}\left[\left(\frac{C}{C_o}\right)^{1/4} - 1\right] = \frac{1}{.2}\left[(10)^{1/4} - 1\right] = 3.9$$

Step 3 If the contact medium is disc, the areal density may be approximated as 1.78 cm^2/cm^3. Therefore the detention time per stage is:

$$\Theta = 3.9/1.78 = 2.19 \text{ hr}$$

For structural reason, the maximum disc diameter is taken at 2 m. Allowing for .05 m clearance between the tank's edge and the disc, the width of the tank W is often equal to 2.1 cm. If the slope of the hopper is taken at 30°, the volume of the aerobic zone per stage is equal to:

$$V = \frac{W^2 L}{4} + 2L \tan 30^\circ = \frac{(2.1)^2}{4} L\left(2.576\right) = 2.84 \text{ L}$$

Step 4 The volume per stage should be equal to the product of the detention time per stage and the flow rate. To avoid constructing very long tanks, and to provide for flexibility of operation in case of a breakdown of the equipment, the wastewater flow is divided into 4 units. Hence, the flow rate into one unit is one fourth of the total wastewater flow. The volume per stage is then:

$$V = \frac{Q\Theta}{4} = \frac{41.5}{4} \times 2.19 = 22.72 \text{ m}^3$$

Adding a 15% allowance for the volume of the disc, then the total volume per stage of the aerobic zone should be:

$$V = 22.72 \times 1.15 = 26.1 \text{ m}^3$$

Step 5 The length of the aerobic chamber can be determined by equating the volume obtained in Steps 3 and 4, which results in:

$$L = 26.1/2.84 = 9.2 \text{ m}$$

To minimize sagging of the shaft, a support with a thickness of 20 cm is provided along the shaft. Therefore, the total length of the reactor occupied by the discs will be equal to 9.0 m. If PVC discs are used, the thickness will be approximately 3 mm. Allowing a distance of 1.1 cm between the disc surfaces to provide sufficient ventilation for aeration, then the total spacing between the discs is 1.4 cm. The total number of discs that could be placed throughout each stage is:

$$N = 900/1.4 = 642$$

Step 6 The total surface area of the discs is equal to:

$$A = 2 \times 642 \times \frac{\pi}{4} (2)^2 = 4032 \text{ m}^2$$

The areal density is then equal to:

$$4032 \text{ m}^2/22.72 \text{ m}^3 = 1.77 \text{ m}^2/\text{m}^3 = 1.77 \text{ cm}^2/\text{cm}^3$$

Since the assumed areal density of 1.78 is only 0.56% off the calculated value, then there is no need for further refinement. Otherwise, Step 3 should be repeated with the areal density calculated in Step 6.

Step 7 The total volume of the discs is:

$$V_d = \frac{\pi}{4} D^2 = 642 \times \frac{\pi}{4}(2)^2 = 6.04 \text{ m}^3$$

Assuming that the discs are 50% submerged, then the volume of the aerobic zone occupied by the disc is 3.02 m.^{3} The required volume of the aerobic tank is the sum of the volumes occupied by the discs and the wastewater, which is 3.02 m^3 + 22.72 m^3 or 25.75 m^3. Comparing this with the volume of the aerobic zone calculated in Step 4, it is apparent that the difference is only 1.34%. Hence, there is no need for recalculation of the allowance of 15% assumed in Step 4. Therefore, the volume of the aerobic zone may be taken as 26.1 m^3.

The volume of the wastewater is greater than that calculated in Step 4, being the difference between the aerobic zone volume and the volume of the discs in the aerobic zone, or 26.1-3.02 m. The actual detention time is then equal to:

$$\Theta = \frac{23.08 \text{ x } 4}{41.5} = 2.22 \text{ hr}$$

As a check, the product of the detention time and the areal density is equal to 3.93 which is very close to the required detention time calculated in Step 2.

Step 8 In the first stage, the concentration of COD is:

$$C_1 = \frac{C_o}{(1 + k\ A/V\ \Theta)} = \frac{1000}{(1.786)} = 560 \text{ mg/1}$$

$$O_r = \frac{K_{s_1}\ (C_o - C_1)}{\Theta} = \frac{(1000 - 560)\ (.50)}{2.22} = 111 \text{ mg/1-hr}$$

Step 9 The surface area of the tank is then equal to:

$$A_t = WL = 2.1 \text{ x } 9.2 = 19.32 \text{ m}^2$$

The ratio A_t/V is equal to:

$$19.32 \text{ m}^2/22.72 \text{ m}^3 = .85 \text{ m}^2/\text{m}^3 = 85 \text{ x } 10^{-2} \text{ cm}^2/\text{cm}^3$$

It was noticed in the experiments that the first stage of the aerobic zone supported filamentous organisms which were displaced by more compact slime formations as the stage number increased. Tomlinson and Snaddon (1965) found the oxygen transfer efficiency of the filamentous slimes to be 9 times that of tap water. As estimated, the oxygen mass transfer efficiency of the slime layer in the first stage is taken as 9, in the second stage as 4, in the third stage as 1.0 and in the fourth stage as 0.89.

Step 10 Substituting all the values required in Eq. 7-11 as determined in the previous step yields the following equation if the dissolved oxygen is taken at 0.5 mg/1 at this particular stage.

$$.5 = \frac{2.22}{(1 + 0.0085 \text{ x } 1.2\ K_L)} \left\{ \frac{5.00}{2.22} + 1.77\ \ 2 \text{ x } 7.9 \text{ x } 111 + (1.2)\ (7.9)\ \ (.0085)\ K_L - 111 \right\}$$

After solving for K_L:

$$K_L = 481$$

Step 11 The dimensionless term:

$$\frac{K_L \quad V/A_t}{D_L} = \frac{481}{(.1)(.0085)} = 5.65 \times 10^5$$

From Fig. 1–3, the modified Reynold's number is equal to:

$$\frac{D^2 \rho \omega}{\mu} = 2.7 \times 10^6$$

Solving for the angular velocity:

$$\omega = .58 \text{ rad/sec}$$

in rpm:

$$\frac{158 \times 60}{2\pi} = 5.5 \text{ rpm}$$

say 6 rpm.

Step 12 The concentration of the substrate in the second, third and fourth stages may now be determined. The dissolved oxygen level is set at 1.0 mg/1 in the second stage, 2.0 mg/1 in the third stage and 4.0 mg/1 in the fourth stage.

The different speeds of revolution of the contact media are calculated as shown in Steps 8 to 11. The results are as shown in Table 7–4 . Summary of the dimensions of the aerobic zone is tabulated in Table 7–5 .

Anaerobic Zone Design

In the study, the following aerobic sludge properties were determined:

Volatile solids in dry sludge – 85% of the total solids
Non-volatile solids in dry sludge – 15% of the total solids
Total solids in fresh sludge – 29%
Total solids in the settled sludge – 5%

Step 13 The rate of sludge formation can be estimated by subtracting from the COD the portion which is oxidized. This gives for the first stage, a sludge production rate of:

$$q_s = (1 - k_s) \text{ COD } Q = \frac{.44 \times .430 \times 1000}{4} = 50 \text{ kg/day}$$

In terms of the volume of the sludge, assuming a sludge density of 1 gm/cm^3, then:

$$q_s = \frac{50}{.02} \times 10^{-3} = 25 \text{ m}^3/\text{day}$$

Step 14 Taking a detention time of 60 days in the anaerobic digester, which is located underneath the aerobic zone, the depth of the tank can be calculated.

$$d = \left\{ 1.05 + 1.05 \tan 30^{o} + 1.0 + \frac{3\ (2.5)}{2.1 \times 9.2} + \frac{60\,(2.5)\,(.02)\,(.85)}{(.05)(2.1)\,(9.2)} \right\} = 5.7\text{m}$$

TABLE 7–4

Summary of COD Concentration DO Level and Speed of Revolution of the Contact Media in each Stage

Stage No.	COD mg/1	COD mg/1	Oxygen Uptake Rate	D. O. mg/1	Speed of Rev. of Media
Influent	1000		mg/1-hr	5	(rpm)
1	560	440	111	.5	6
2	314	246	63	1	4
3	176	138	35	2	3
4	98	78	19	4	1

TABLE 7 – 5

Summary of Aerobic Section Design

Number of units	–	4
Capacity of each unit	–	10.4 m^3/hr
Total number of stages per unit	–	4
Contact media	–	PVC discs
Diameter	–	2 m
Thickness	–	3 mm
Spacing between disc surfaces	–	11 mm
Total number of discs per stage	–	642
Areal density	–	1.77 cm^2/cm^3
Speed of revolution:		
1st stage	–	6 rpm
2nd stage	–	4 rpm
3rd stage	–	3 rpm
4th stage	–	2 rpm
Detention time per stage	–	2.22 hrs
Aerobic Tank:		
Length	–	9.2 m
Width	–	2.1 m
Depth of side	–	1.05 m
Total depth of aerobic chamber	–	1.6 m
Opening at hopper bottom	–	20 cm
Slope of hopper	–	30°
Spacing between aerobic tank side and discs side	–	5 cm

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APPENDIX

TABLE A – 1 * Areas under the Normal Curve from K_α to ∞

$$P\{\text{normal} \geq K_\alpha\} = \int_{K_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\, dx = \alpha$$

$K\alpha$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.00990	.00964	.00939	.00914	.00889	.00866	.00842
2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139

$K\alpha$	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
3	.00135	$.0^{3}968$	$.0^{3}687$	$.0^{3}483$	$.0^{3}337$	$.0^{3}233$	$.0^{3}159$	$.0^{3}108$	$.0^{4}723$	$.0^{4}481$
4	$.0^{4}317$	$.0^{4}207$	$.0^{4}133$	$.0^{5}854$	$.0^{5}541$	$.0^{5}340$	$.0^{5}211$	$.0^{5}130$	$.0^{6}793$	$.0^{6}479$
5	$.0^{6}287$	$.0^{6}170$	$.0^{7}996$	$.0^{7}579$	$.0^{7}333$	$.0^{7}190$	$.0^{7}107$	$.0^{8}599$	$.0^{8}332$	$.0^{8}182$
6	$.0^{9}987$	$.0^{9}530$	$.0^{9}282$	$.0^{9}149$	$.0^{10}777$	$.0^{10}402$	$.0^{10}206$	$.0^{10}104$	$.0^{11}523$	$.0^{11}260$

TABLE A – 2 (Continued)

P {chi square with ν degrees of freedom $\geq$ tabled value} $= \alpha$

ν \ α	0.25	0.10	0.05	0.025	0.01	0.005
1	1.323	2.706	3.841	5.024	6.635	7.879
2	2.773	4.605	5.991	7.378	9.210	10.597
3	4.108	6.251	7.815	9.348	11.345	12.838
4	5.385	7.779	9.488	11.143	13.277	14.860
5	6.626	9.236	11.071	12.833	15.086	16.750
6	7.841	10.645	12.592	14.449	16.812	18.548
7	9.037	12.017	14.067	16.013	18.475	20.278
8	10.219	13.362	15.507	17.535	20.090	21.955
9	11.389	14.684	16.919	19.023	21.666	23.589
10	12.549	15.987	18.307	20.483	23.209	25.188
11	13.701	17.275	19.675	21.920	24.725	26.757
12	14.845	18.549	21.026	23.337	26.217	28.299
13	15.984	19.812	22.362	24.736	27.688	29.819
14	17.117	21.064	23.685	26.119	29.141	31.319
15	18.245	22.307	24.996	27.488	30.578	32.801
16	19.369	23.542	26.296	28.845	32.000	34.267
17	20.489	24.769	27.587	30.191	33.409	35.718
18	21.605	25.989	28.869	31.526	34.805	37.156
19	22.718	27.204	30.144	32.852	36.191	38.582
20	23.828	28.412	31.410	34.170	37.566	39.997
21	24.935	29.615	32.671	35.479	38.932	41.401
22	26.039	30.813	33.924	36.781	40.289	42.796
23	27.141	32.007	35.172	38.076	41.638	44.181
24	28.241	33.196	36.415	39.364	42.980	45.559
25	29.339	34.382	37.652	40.646	44.314	46.928
26	30.435	35.563	38.885	41.923	45.642	48.290
27	31.528	36.741	40.113	43.194	46.963	49.645
28	32.620	37.916	41.337	44.461	48.278	50.993
29	33.711	39.087	42.557	45.722	49.588	52.336
30	34.800	40.256	43.773	46.979	50.892	53.672
31	35.887	41.422	44.985	48.232	52.191	55.003
32	36.973	42.585	46.194	49.480	53.486	56.328
33	38.058	43.745	47.400	50.725	54.776	57.648
34	39.141	44.903	48.602	51.966	56.061	58.964
35	40.223	46.059	49.802	53.203	57.342	60.275
36	41.304	47.212	50.998	54.437	58.619	61.581
37	42.383	48.363	52.192	55.668	59.892	62.883
38	43.462	49.513	53.384	56.896	61.162	64.181
39	44.539	50.660	54.572	58.120	62.428	65.476
40	45.616	51.805	55.758	59.342	63.691	66.766
41	46.692	52.949	56.942	60.561	64.950	68.053
42	47.766	54.090	58.124	61.777	66.206	69.336
43	48.840	55.230	59.304	62.990	67.459	70.616
44	49.913	56.369	60.481	64.201	68.710	71.893
45	50.985	57.505	61.656	65.410	69.957	73.166

TABLE A-3 (Continued)

$P\{\text{Student's } t \text{ with } \nu \text{ degrees of freedom} \geq \text{tabled value}\} = \alpha$

α ν	0.25	0.10	0.05	0.025	0.01	0.005
48	0.6796	1.2994	1.6772	2.0106	2.4066	2.6822
49	0.6795	1.2991	1.6766	2.0096	2.4049	2.6800
50	0.6794	1.2987	1.6759	2.0086	2.4033	2.6778
51	0.6793	1.2984	1.6753	2.0076	2.4017	2.6757
52	0.6792	1.2980	1.6747	2.0066	2.4002	2.6737
53	0.6791	1.2977	1.6741	2.0057	2.3988	2.6718
54	0.6791	1.2974	1.6736	2.0049	2.3974	2.6700
55	0.6790	1.2971	1.6730	2.0040	2.3961	2.6682
56	0.6789	1.2969	1.6725	2.0032	2.3948	2.6665
57	0.6788	1.2966	1.6720	2.0025	2.3936	2.6649
58	0.6787	1.2963	1.6716	2.0017	2.3924	2.6633
59	0.6787	1.2961	1.6711	2.0010	2.3912	2.6618
60	0.6786	1.2958	1.6706	2.0003	2.3901	2.6603
61	0.6785	1.2956	1.6702	1.9996	2.3890	2.6589
62	0.6785	1.2954	1.6698	1.9990	2.3880	2.6575
63	0.6784	1.2951	1.6694	1.9983	2.3870	2.6561
64	0.6783	1.2949	1.6690	1.9977	2.3860	2.6549
65	0.6783	1.2947	1.6686	1.9971	2.3851	2.6536
66	0.6782	1.2945	1.6683	1.9966	2.3842	2.6524
67	0.6782	1.2943	1.6679	1.9960	2.3833	2.6512
68	0.6781	1.2941	1.6676	1.9955	2.3824	2.6501
69	0.6781	1.2939	1.6672	1.9949	2.3816	2.6490
70	0.6780	1.2938	1.6669	1.9944	2.3808	2.6479
71	0.6780	1.2936	1.6666	1.9939	2.3800	2.6469
72	0.6779	1.2934	1.6663	1.9935	2.3793	2.6459
73	0.6779	1.2933	1.6660	1.9930	2.3785	2.6449
74	0.6778	1.2931	1.6657	1.9925	2.3778	2.6439
75	0.6778	1.2929	1.6654	1.9921	2.3771	2.6430
76	0.6777	1.2928	1.6652	1.9917	2.3764	2.6421
77	0.6777	1.2926	1.6649	1.9913	2.3758	2.6412
78	0.6776	1.2925	1.6646	1.9908	2.3751	2.6403
79	0.6776	1.2924	1.6644	1.9905	2.3745	2.6395
80	0.6776	1.2922	1.6641	1.9901	2.3739	2.6387
81	0.6775	1.2921	1.6639	1.9897	2.3733	2.6379
82	0.6775	1.2920	1.6636	1.9893	2.3727	2.6371
83	0.6775	1.2918	1.6634	1.9890	2.3721	2.6364
84	0.6774	1.2917	1.6632	1.9886	2.3716	2.6356
85	0.6774	1.2916	1.6630	1.9883	2.3710	2.6349
86	0.6774	1.2915	1.6628	1.9879	2.3705	2.6342
87	0.6773	1.2914	1.6626	1.9876	2.3700	2.6335
88	0.6773	1.2912	1.6624	1.9873	2.3695	2.6329
89	0.6773	1.2911	1.6622	1.9870	2.3690	2.6322
90	0.6772	1.2910	1.6620	1.9867	2.3685	2.6316

TABLE A-4 Table of Random Digits*

09656	96657	64842	49222	49506	10145	48455	23505	90430	04180
24712	55799	60857	73479	33581	17360	30406	05842	72044	90764
07202	96341	23699	76171	79126	04512	15426	15980	88898	06358
84575	46820	54083	43918	46989	05379	70682	43081	66171	38942
38144	87037	46626	70529	27918	34191	98668	33482	43998	75733
48048	56349	01986	29814	69800	91609	65374	22928	09704	59343
41936	58566	31276	19952	01352	18834	99596	09302	20087	19063
73391	94006	03822	81845	76158	41352	40596	14325	27020	17546
57580	08954	73554	28698	29022	11568	35668	59906	39557	27217
92646	41113	91411	56215	69302	86419	61224	41936	56939	27816
07118	12707	35622	81485	73354	49800	60805	05648	28898	60933
57842	57831	24130	75408	83784	64307	91620	40810	06539	70387
65078	44981	81009	33697	98324	46928	34198	96032	98426	77488
04294	96120	67629	55265	26248	40602	25566	12520	89785	93932
48381	06807	43775	09708	73199	53406	02910	83292	59249	18597
00459	62045	19249	67095	22752	24636	16965	91836	00582	46721
38824	81681	33323	64086	55970	04849	24819	20749	51711	86173
91465	22232	02907	01050	07121	53536	71070	26916	47620	01619
50874	00807	77751	73952	03073	69063	16894	85570	81746	07568
26644	75871	15618	50310	72610	66205	82640	86205	73453	90232

TABLE A-5

Typical Radius of Influence for Surface Aerators for Complete Mixing

HP	Depth, ft.		Width or Length, ft.	
	min.	max.*	min.	max.*
5	5	10	20	40
7.5	5	12	20	40
10	5	14	20	40
15	6	16	25	55
20	6	16	30	60
25	8	18	35	65
30	8	18	35	70
40	8	18	35	75
50	10	20	35	80
60	10	20	40	80
75	12	22	40	90
100	12	24	55	110

*For complete mix conditions

Subject Index