THEORETICAL STUDY OF HIGH-RATE SEDIMENTATION

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The term high-rate sedimentation used here refers to the use of shallow gravitational settlers with detention periods of not more than 15 min to achieve comparable or better settling efficiencies normally obtained in conventional rectangular settling tanks having detention periods of usually more than 2 hr. The settlers can be rectangular, circular, or any other convenient shape. They can be either horizontal or inclined. One obvious advantage is the significant saving in capital cost of construction, land, and piping. The short detention periods required make the high-rate settlers extremely attractive in wastewater treatment. For instance, a short detention in final settling in the activated sludge process could eliminate the possibility of developing anaerobic conditions which may be detrimental to the aerobic microorganisms.

The main feature of a high-rate settler is its shallow "depth" which is usually not more than a few inches. The idea appears to be suggested originally by Hazen in 1904 (1) and was explored by Camp in 1946 (2). Hansen and Culp successfully demonstrated that, by using circular tubes of 0.5 to 4 in. in diameter (1.3 to 10.2 cm) and up to 8 ft in length (2.44 m), turbidity removal as high as 96 percent could be obtained (3). A number of installations using high-rate settlers actually have been constructed by industries and municipalities (4).

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The development is based theoretically on the concept advanced by Camp (2). It is significant to note that the model Camp used to develop the concept was an ideal open rectangular tank with a uniform flow across the tank cross section. In his model, all the suspended particles would follow a straight path. In the small conduits used as high-rate settlers, laminar flow may develop and the velocity distribution can be quite different from uniform. As a result, the particle paths are not straight lines. With Camp's model, the overflow rate of a settling tank expressed in rate of flow per unit tank area represents the critical fall velocity of the suspended particles. Theoretically, suspended particles with fall velocities greater than or equal to this critical value will be completely removed in the tank. Here again, difficulties arise. First, no information is available whether the parameter overflow rate still retains the same physical significance for settlers other than those rectangular in shape. Second, nothing is known as to how to calculate the overflow rate for settlers such as inclined circular tubes. These facts indicate that Camp's model is in need of extensive generalization if it is to be applied to high-rate sedimentation. The purposes of the present study are:

1. To conduct basic theoretical research on the characteristics of high-rate settlers and the governing physical properties of high-rate settling systems.
2. To provide information as general guidance for practical design and further laboratory investigations.

General Equation

It is assumed that the flow is laminar and one-dimensional in a high-rate settler and that the suspended particles are discrete particles which do not aggregate. Consider a single suspended particle moving in a laminar flow. The equation of motion is as follows:

$$m \frac{d\mathbf{v}_p}{dt} = (\rho_p - \rho)V\mathbf{g} - \mathbf{F}_r \quad \ldots\ldots 1$$

where m is the mass of the particle, v_p is the velocity of the particle, t is the time, ρ_p and ρ are the densities of the particle and fluid, respectively, g is the gravitational acceleration, V is the volume of the particle, and F_r is the fluid resisting force. Quantities shown in boldface letters are vectors. For laminar flow, Stokes' law can be used for the resisting force as follows (5):

$$F_r = 3\pi\mu d_p(\mathbf{v}_p - \mathbf{v}) \quad \ldots\ldots 2$$

where μ is the dynamic viscosity of the fluid, d_p is the hydraulic diameter of the particle and v is the local fluid velocity. Ignoring the inertial effect $\left(m\frac{d\mathbf{v}_p}{dt} = 0\right)$, substituting Equation 2 into Equation 1 and rearranging,

$$\mathbf{v}_p - \mathbf{v} = \frac{(\rho_p - \rho)V\mathbf{g}}{3\pi\mu d_p} = \mathbf{v}_s \quad \ldots 3$$

where $\mathbf{v}_s$ is the fall velocity of the particle and is in a vertically downward direction.

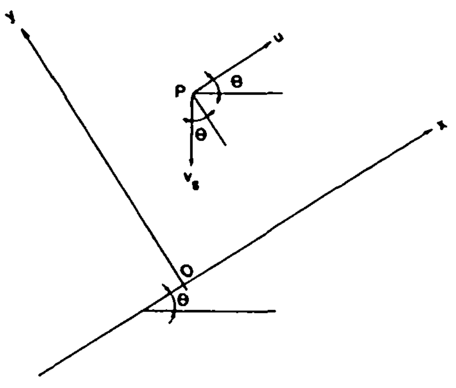

FIGURE 1.—Coordinate system.

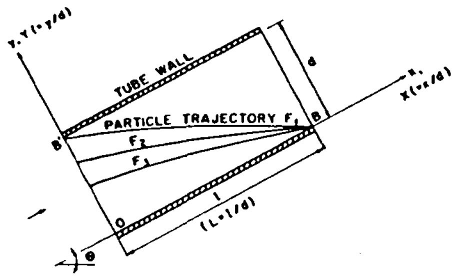

FIGURE 2.—High-rate settling system using a circular tube settler.

Figure 1 presents the coordinate system used for the present study. The x-axis is parallel to the direction of flow and the y-axis is normal to the direction of flow. θ is the angle between the x-axis and a horizontal line or the angle of inclination and u is the local fluid velocity in the x-direction. Equation 3 can be written into the two non-vectorial equations as follows:

$$v_{px} = u - v_s \sin\theta \quad \ldots\ldots 4$$

$$v_{py} = -v_s \cos\theta \quad \ldots\ldots 5$$

where v_{px} and v_{py} are the velocity components of v_p in the x and y directions, respectively. By definition,

$$v_{px} = \frac{dx}{dt} \quad v_{py} = \frac{dy}{dt} \quad \ldots\ldots 6$$

where x and y are the coordinates in the x and y directions, respectively. Combining Equations 4, 5, and 6,

$$\frac{dy}{dx} = \frac{-v_s \cos\theta}{u - v_s \sin\theta} \quad \ldots\ldots 7$$

Equation 7 is the differential equation of the particle trajectory resulting from the combined effects of fluid drag and gravitational settling.

Integrating Equation 7,

$$\int u\,dy - v_s y \sin\theta + v_s x \cos\theta = C_0 \ldots 8$$

where C_0 is the integration constant.

Dividing Equation 8 with v_0, the average flow velocity, and d, the depth of flow measured normal to the direction of flow,

$$\int \frac{u}{v_0}\,dY - \frac{v_s}{v_0}\,Y \sin\theta + \frac{v_s}{v_0}\,X \cos\theta = C_1 \quad \ldots 9$$

where C_1 is the adjusted integration constant, $Y = y/d$ and $X = x/d$. Equation 9 is the general equation of the particle trajectory. C_1 and $\int \frac{u}{v_0}\,dY$ can be evaluated for a particular particle trajectory in a given high-rate settling system.

Circular Tube Settlers

Figure 2 presents a high-rate settling system, using circular tube settlers. The x-axis coincides with the bottom of the tube and d and l are the diameter and length of the settler, respectively. The sectional plane shown contains the axis of the tube.

For laminar flow in a circular tube (5),

$$\frac{u}{v_0} = S[Y - Y^2] \quad \ldots\ldots\ldots 10$$

Substituting Equation 10 into Equation 9 and carrying out the integration,

$$S\left[\frac{Y^2}{2} - \frac{Y^3}{3}\right] - \frac{v_s}{v_0}\,Y \sin\theta + \frac{v_s}{v_0}\,X \cos\theta = C_1 \quad \ldots 11$$

Equation 11 is the general equation for the trajectories of suspended particles in laminar flow through a circular tube. The constant C_1 can be evaluated if the coordinates of any point on a given trajectory are known. Consider a family of trajectories such as F_1, F_2, F_3, all of which pass through B, the bottom point at the exit end of the tube (See Figure 2). The coordinates for this point expressed in dimensionless form are

$$X = L, \quad Y = 0 \quad \ldots\ldots\ldots 12$$

where L = the relative length of the settler = l/d. C_1 is found to be

$$C_1 = \frac{v_s}{v_0}\,L \cos\theta \quad \ldots\ldots\ldots 13$$

Substituting Equation 13 into Equation 11 and rearranging,

$$S\left[\frac{Y^2}{2} - \frac{Y^3}{3}\right] - \frac{v_s}{v_0}\,Y \sin\theta + \frac{v_s}{v_0}\,(X - L) \cos\theta = 0 \quad \ldots 14$$

Equation 14 is the equation for the family. The actual path a particular suspended particle will take depends on the magnitude of v_s/v_0 for the particle.

Among this family of trajectories, there is a limiting trajectory which starts at B′, the top point of the tube at its entrance end, and represents the uppermost trajectory in the family. The physical significance of the limiting trajectory is that it defines the critical particle fall velocity, v_{sc}, for a given system. Any suspended particle with its fall velocity greater than or equal to this critical fall velocity would be completely removed in the settler of the given system. A useful relationship is obtained from Equation 14 by using the coordinates of B′ as follows:

$$X = 0, \quad Y = 1 \quad \ldots\ldots\ldots 15$$

The result is:

$$\frac{v_{sc}}{v_0}(\sin\theta + L\cos\theta) = \tfrac{4}{3} \quad \ldots\ldots 16$$

Equation 16 indicates that the performance of a high-rate settling system can be characterized by a parameter S with

$$S = \frac{v_s}{v_0}(\sin\theta + L\cos\theta) \quad \ldots\ldots 17$$

The critical S-value, S_c, for circular tube settlers is $\frac{4}{3}$. Any suspended particle in such a system with its S-value greater than or equal to $\frac{4}{3}$ would be completely removed, in theory at least, from the flow without the need of knowing the critical fall velocity of the system at all.

Other Types of Settlers

Following a similar procedure as for the case of circular tubes, the corresponding results obtained for other types of settlers are presented below. Ideal uniform flow also is considered for comparison purposes.

Flow between Parallel Plates (5)

$$\frac{u}{v_0} = 6(Y - Y^2) \qquad \text{. . . 18}$$

$$S_c = \frac{v_{sc}}{v_0}(\sin\theta + L\cos\theta) = 1 \qquad \text{. . 19}$$

Square Conduits (6)

$$\frac{u}{v_0} = \frac{-\frac{1}{8} - \sum_{m=1}^{\infty} \frac{2}{m^3\pi^3}(\cos m\pi - 1)\sin\frac{m\pi}{2}\left(\cosh m\pi Y - \frac{\cosh m\pi - 1}{\sinh m\pi}\sinh m\pi Y\right)}{-\frac{1}{12} + \sum_{m=1}^{\infty} \frac{2}{m^5\pi^5}(\cos m\pi - 1)^2\left[\sinh m\pi - \frac{(\cosh m\pi - 1)^2}{\sinh m\pi}\right]} \qquad \text{. . 20}$$

$$S_c = \frac{v_{sc}}{v_0}(\sin\theta + L\cos\theta) = \frac{11}{8} \qquad \text{. . . 21}$$

Shallow Open Tray (5)

$$\frac{u}{v_0} = \tfrac{3}{2}[2Y - Y^2] \qquad \text{. . . 22}$$

$$S_c = \frac{v_{sc}}{v_0}(\sin\theta + L\cos\theta) = 1 \qquad \text{. . 23}$$

Uniform Flow

$$\frac{u}{v_0} = 1 \qquad \text{. . . 24}$$

$$S_c = \frac{v_{sc}}{v_0}(\sin\theta + L\cos\theta) = 1 \qquad \text{. . 25}$$

General Discussion

It is interesting to note that S_c is identical for systems using parallel plates or shallow trays and with uniform velocity distributions. On the other hand, the values of S_c for circular tubes and square conduits are appreciably different from those for the above three cases. It is important to point out that identical S_c may not mean identical behaviors of the systems. This is indicated in Figure 3, showing the limiting trajectories for various types of settlers, assuming $\theta = 0$ for simplicity. The patterns of the limiting trajectory are rather different for shallow trays, parallel plates, and uniform flow distributions. On the other hand, the pattern for circular tubes is identical with that for parallel plates even though the values of S_c for the two types of settlers are not the same.

Overflow Rate

The design of settling tanks for water and wastewater treatment usually is based on the parameter overflow rate expressed as rate of flow per unit tank area. The concept was originated from the fact that, for an ideal uniform flow in a horizontal flow rectangular settling tank, the overflow rate represents the critical particle fall velocity. Theoretically, any suspended particle having a fall velocity greater than or equal to this critical value would be removed completely in the tank. This same concept is readily adaptable to high-rate settling systems, since the critical fall velocity can be estimated easily from S_c, the critical S-value. The following equation should be adequate for this purpose:

$$\text{Overflow rater } (= v_{sc}) = CK\frac{v_0}{L} \qquad \text{. . 26}$$

where

$$K = S_c \frac{L}{\sin\theta + L\cos\theta}$$

where C is a constant and its magnitude depends on the units used for the various terms in Equation 26. In British units with v_0 in fps and overflow rate in U. S. gpd/sq ft, $C = 6.54 \times 10^5$. In metric units with v_0 in cm/sec and overflow rate in cu m/day/sq m, $C = 8.64 \times 10^2$.

With Equation 26, a high-rate settling system can be designed just as any conventional settling tank by selecting an appropriate overflow rate to begin with. Equation 26 also provides the common basis for comparing the performance of different settling systems since, theoretically, systems with the same overflow rate should have comparable performance.

Influence of L on Settler Performance

The critical particle fall velocity for a given high-rate settling system can be expressed in the following form:

$$\frac{v_{sc}}{v_0} = \frac{S_c}{\sin\theta + L\cos\theta} \quad \ldots\ldots 27$$

Figure 4 is the plotting of Equation 27, assuming $\theta = 0$, for systems using circular tubes and parallel plates. For a fixed v_0, v_{sc} decreases rapidly with L, the relative length of the settler, when L is relatively small. This indicates that suspended particles with much smaller fall velocities are removed completely as L increases. The rate of decrease in v_{sc} drops appreciably after L reaches 20 and becomes rather insignificant with L greater than 40. Hence L should be kept below 40 and preferably around 20. Figure 5 presents the plotting of Equation 27 for parallel plates at $\theta = 20°$ and 40°. The general pattern is about the same as for $\theta = 0$.

Influence of θ on Settler Performance

By differentiating Equation 27 with respect to θ and setting the result to zero, the following relationship is obtained:

$$\theta = \tan^{-1}\frac{1}{L} \quad \ldots\ldots\ldots\ldots 28$$

The second derivative of Equation 27 with respect to θ is as follows:

$$\frac{d^2\frac{v_{sc}}{v_0}}{d\theta^2} = \frac{2S_c(\cos\theta - L\sin\theta)^2}{(\sin\theta + L\cos\theta)^3} + \frac{S_c}{\sin\theta + L\cos\theta} \quad \ldots 29$$

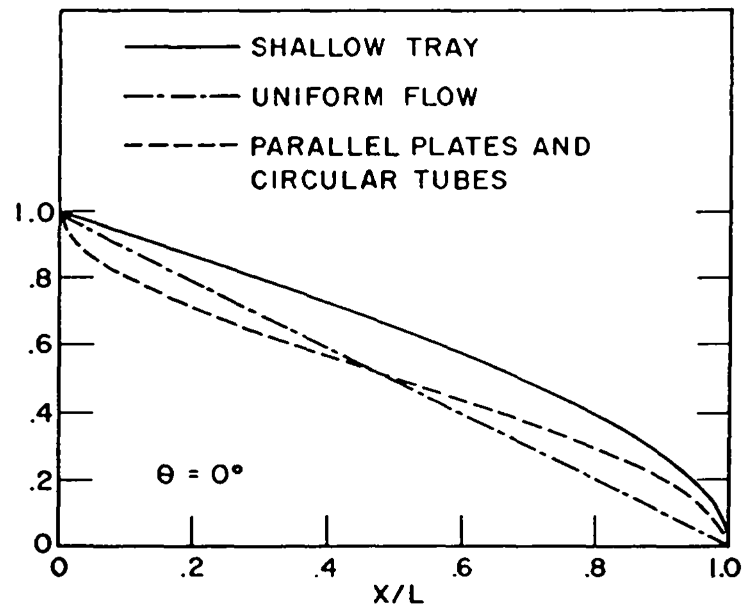

FIGURE 3.—Limiting trajectories in different types of settlers.

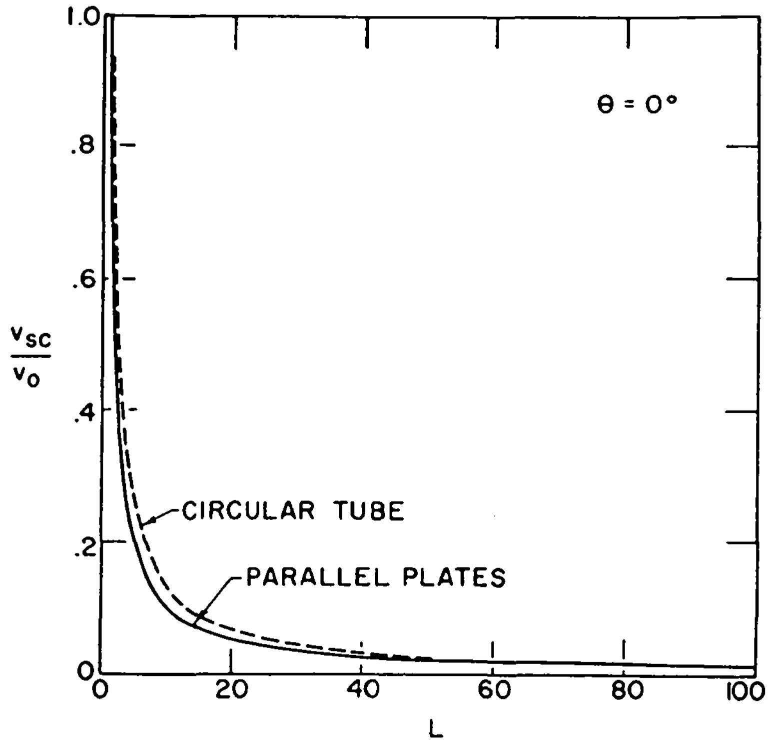

FIGURE 4.—Relative settler length vs. performance (θ = 0°).

Since the angle of inclination, θ, can be no more than 90°, the right side of Equation 29 is always positive. Hence, the relationship expressed in Equation 28 gives the minimum v_{sc}. For instance, for $L = 20$, $\theta = 2°54'$ would give the best theoretical performance if all other parameters are kept constant.

Figure 6 presents the variation of settler performance with θ at three different values of L. The ordinate is expressed as the ratio of v_{sc} at $\theta = \theta$ to v_{sc} at $\theta = 0°$. v_0 is assumed to be constant. It can be seen from Figure 6 that the performance of the settler deteriorates rapidly after θ reaches about 40°. This is indicated by the rapid increase in v_{sc}. Figure 6 also shows that there is little change in performance when L is increased from 30 to 60 and systems having larger L tend to be more sensitive to changes in θ.

Fractional Removal Efficiency

For suspended particles with S-values less than the critical S-value (S_c) of a given high-rate settling system, only a fraction is removed in the settler. This is referred to as fractional removal. Only systems with horizontal parallel plates and circular tubes are considered here.

Figure 7 shows a high-rate settling system using horizontal parallel plates. It is assumed that all the suspended particles have the same fall velocity and the corresponding S-value is less than S_c of the system. Consider the particle trajectory J which starts at E_0 at the entrance side and ends at E_2, the bottom point of the settler at the exit side. q_1 is the portion of the total flow Q entering the settler below E_0 and q_2 is the remaining portion entering above E_0 (See Figure 7). Suspended particles in q_1 will be removed completely in the settler since their trajectories must end

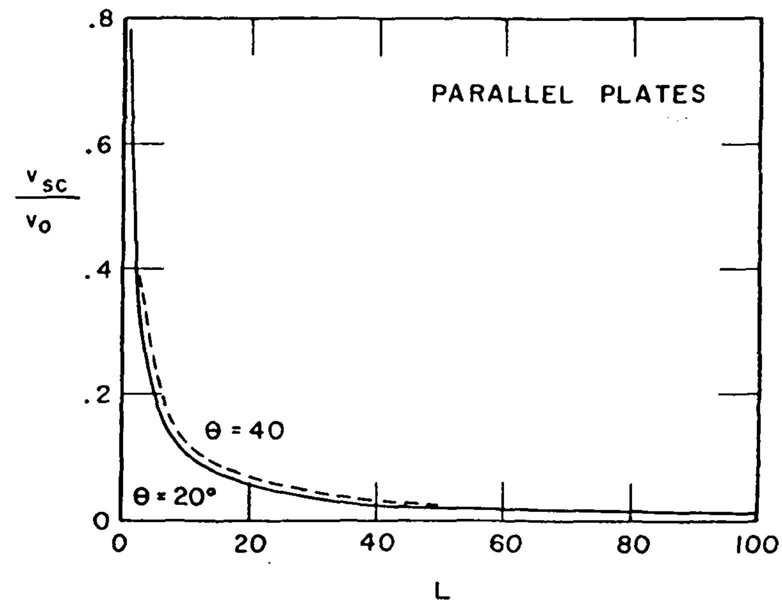

FIGURE 5.—Relative settler length vs. performance (θ = 20°, 40°).

up between E_1 and E_2. On the other hand, suspended particles in q_2 will remain in the flow. The fractional removal efficiency (Eff) is therefore,

$$\text{Eff} = \frac{\int_0^y u\,dy}{v_0 d} = \int_0^Y \frac{u}{v_0}\,dY \quad \ldots 30$$

Substituting Equation 18 into Equation 30 and integrating,

$$\text{Eff} = 3Y^2 - 2Y^3 \quad \ldots\ldots 31$$

Substituting the following boundary conditions for the trajectory J into Equation 9,

$$X = 0, \quad Y = Y \quad \ldots\ldots 32$$

$$X = L, \quad Y = 0 \quad \ldots\ldots 33$$

the following result is obtained:

$$3Y^2 - 2Y^3 = \frac{v_s}{v_0} L \quad \ldots\ldots 34$$

Hence, for horizontal parallel plates,

$$\text{Eff} = \frac{v_s}{v_0} L = S \quad \ldots\ldots 35$$

Equation 35 indicates that the fractional removal efficiency equals the S-value of the suspended particles.

Thomas found the theoretical fractional removal efficiency of a horizontal circular tube settler as follows (7):

$$\text{Eff} = 1 + \frac{2}{\pi}\left(2\alpha^3\beta - \alpha\beta - \sin^{-1}\beta\right) \quad ..36$$

where

$$\alpha = \left(\tfrac{3}{4}S\right)^{\frac{1}{3}}$$

$$\beta = \sqrt{1 - \alpha^2}$$

$$S = \frac{v_s}{v_0} L \quad (\text{Since } \theta = 0)$$

Equation 36 indicates that the fractional removal efficiency is a function of the S-value only. This shows the usefulness of the S-value for characterizing the theoretical performance of high-rate settling systems.

Figure 8 is the plotting of Equations 35 and 36. Notice that the fractional removal efficiency is less than unity for $S = 1$ in the case of circular tube settlers since $S_c = \frac{4}{3}$ for these systems. In addition, circular tube settlers tend to have better fractional efficiencies than parallel plates for lighter and smaller suspended particles.

Establishment of Laminar Flow

Laminar flow has been assumed in the settlers. In practical installations, the settlers probably are connected to

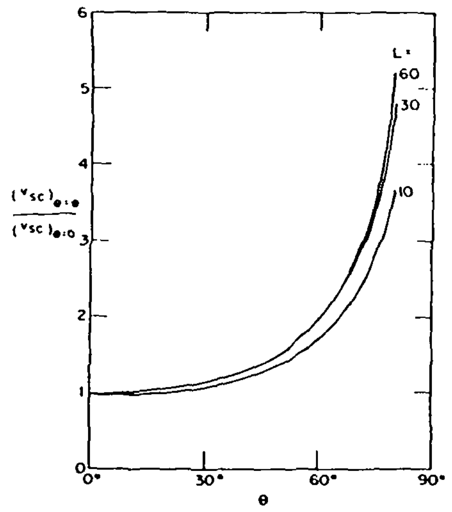

FIGURE 6.—Angle of inclination vs. performance.

an inlet chamber having a relatively large section. At the entrance of a settler, there exists a transition region in which uniform flow is changed gradually into fully developed laminar flow due to the influence of the solid boundary. The relative length, L', for this transition region in a circular tube can be estimated from the following equation (5):

$$L' = 0.058 \frac{v_0 d}{\nu} \quad \ldots\ldots\ldots 37$$

where ν is the kinematic viscosity of the fluid. As an approximation, Equation 37 is assumed to be applicable for other types of settlers. The significance and treatment of L' is discussed in the following section.

Design Consideration

Since the performance of a high-rate settling system varies with the configuration of the settler, it is interesting to know the relative economics of the various types of settlers. For the sake of simplicity, only horizontal settlers are considered.

For given values of overflow rate, v_0 and d and assuming $\theta = 0$, Equation 26 can be written as follows:

$$\frac{S_c}{l} = \text{Constant} \quad \ldots\ldots\ldots 38$$

Equation 38 indicates that the higher the value of S_c, the longer will be the required settler length to achieve the same theoretical performance. Using this as the criterion, the order of preference should be: open shallow trays or parallel plates, circular tubes, and square conduits. The open shallow tray is impracticable due to the large area required. Wide, narrow passages formed by parallel plates could be a better choice. However, flow tends to

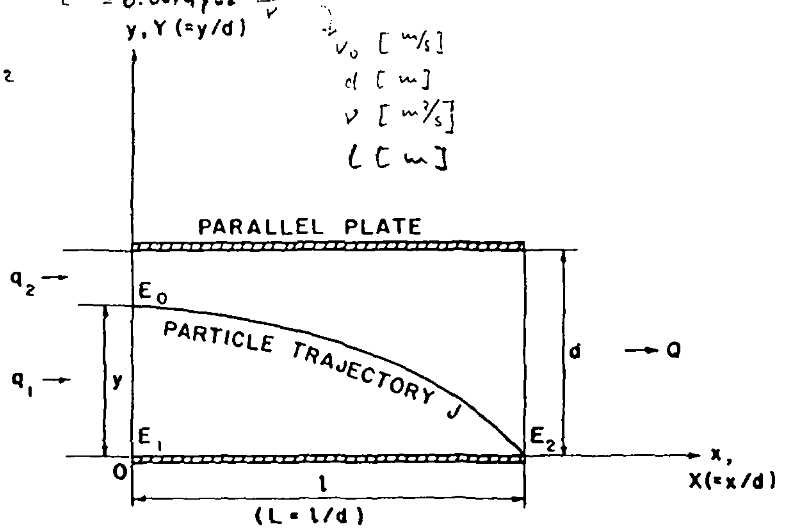

FIGURE 7. —Sketch for studying fractional removal efficiency.

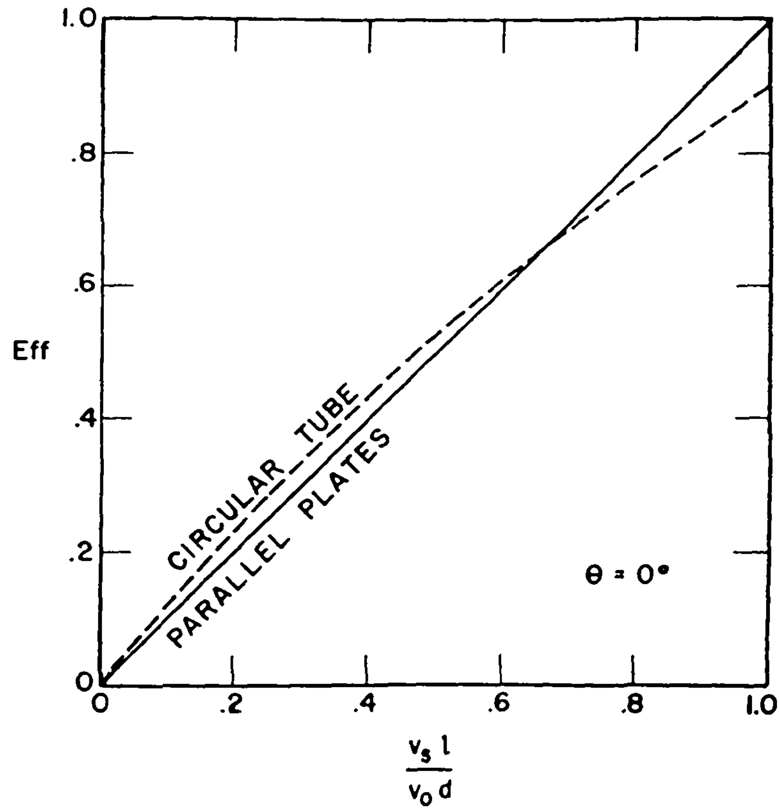

FIGURE 8.—Fractional removal efficiency (Eff) vs. S-value (θ = 0°).

be unstable in a wide channel, and there are also difficulties in maintaining a uniform inlet flow distribution. A possible compromise is to use a width-to-depth ratio between, say, 2 to 5. Circular tubes are slightly more economical than square conduits. This may be balanced out easily by the fact that square sections utilize space much more efficiently than circular ones.

The existence of a transition region at the entrance of a settler certainly complicates the situation. Presumably, the flow in the transition region should be a mixture of uniform and laminar flow. Since, as indicated by the S_c-values, the performance of a high-rate settling system with uniform flow is either comparable or better than that of a similar system with laminar flow, the existence of the transition region should not affect significantly the removal efficiency of the system.

For practical purposes, it is suggested that L' be added to L, the design relative length based on fully developed laminar flow. This would provide a safety factor in the design. It is further suggested that, in cases where L' is greater than L, a total relative length of $2L$ be used instead of the sum of L' and L. Experimental investigations are needed to test the soundness of these suggestions.

The most difficult problem in a high-rate settling system is the removal of the settled sludge from the settler. One method already in use is to install the settlers at a fairly steep angle so that the sludge can slide down on its own weight. This could mean a certain amount of sacrifice in system performance. Research is needed greatly in this area for developing efficient yet simple sludge removal methods.

In the present study, the suspended particles are assumed to behave as discrete particles. In wastewater treatment, flocculation may take place in the course of settling. This could improve the performance of the settler.

Numerical Examples

Following are numerical examples for illustrating the applications of the equations developed in the present study.

Assume that overflow rate = 800 gpd/sq ft (32.6 cu m/day/sq m), $v_0 = 0.5$ fpm (15 cm/min), $d = 2$ in. (5.08 cm), and $\nu = 10^{-5}$ sq ft/sec. Design the high-rate settlers, using (a) horizontal circular tubes, (b) horizontal parallel plates, and (c) square conduits at $\theta = 40°$. Calculate the respective detention times.

Solution (a)

By Equation 26 with $C = 6.54 \times 10^5$, $S_c = \frac{4}{3}$, and $\theta = 0$,

$$L = \frac{CS_c v_0}{800} = \frac{6.54 \times 10^5 \times 4 \times 0.5}{3 \times 800 \times 60} = 9.1$$

By Equation 37

$$L' = 0.058 \times \frac{0.5 \times 1}{60 \times 10^{-5} \times 6} = 8.1$$

$$\text{Total } L = 8.1 + 9.1 = 17.2$$

$$l = 17.2 \times \tfrac{1}{6} = 2.9 \text{ ft (88 cm)}$$

$$\text{detention time} = \frac{2.9}{0.5} = 5.8 \text{ min}$$

Solution (b)

By Equation 26 with $C = 6.54 \times 10^5$, $S_c = 1$ and $\theta = 0$,

$$L = \frac{6.54 \times 10^5 \times 1 \times 0.5}{800 \times 60} = 6.8$$

$$L' = 8.1 > L, \text{ use } 2L$$

$$\text{Total } L = 2 \times 6.8 = 13.6$$

$$l = 13.6 \times \tfrac{1}{6} = 2.3 \text{ ft (70 cm)}$$

$$\text{detention time} = \frac{2.3}{0.5} = 4.6 \text{ min}$$

Solution (c)

By Equation 26 with $C = 6.54 \times 10^5$, $S_c = \frac{11}{8}$ and $\theta = 40°$,

$$L = \frac{1}{0.766}\left(6.54 \times 10^5 \times \frac{11}{8} \times \frac{0.5}{60} \times \frac{1}{800} - 0.643\right) = 11.5$$

$$L' = 8.1$$

$$\text{Total } L = 8.1 + 11.5 = 19.6$$

$$l = 19.6 \times \tfrac{1}{6} = 3.3 \text{ ft (100 cm)}$$

$$\text{detention time} = \frac{3.3}{0.5} = 6.6 \text{ min}$$

Summary

The following parameter characterizes the theoretical performance of a high-rate settling system:

$$S = \frac{v_s}{v_0}(\sin\theta + L\cos\theta)$$

For a given type of settler, there is a critical S-value, S_c.

Theoretically, suspended particles having S-values greater than or equal to S_c of the system are removed completely in the settler.

Overflow rate can be used as the basis for designing high-rate settlers by applying the equation developed in the study.

The best relative settler length is about 20. Settler performance deteriorates rapidly with the increase in the angle of inclination of the settler after the angle reaches 40°.

A detailed discussion of design considerations and several numerical examples are presented.

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APPENDIX

Notation

- C = Constant for dimensional adjustment of the overflow rate.
- C_0, C_1 = Constants of integration.
- d = Depth of a settler (L).
- d_p = Hydraulic diameter of a suspended particle (L).
- Eff = Fractional removal efficiency.
- F = Fluid resisting force, (ML/T^2).
- g = Gravitational acceleration, (L/T^2).
- K = $S_c\left(\dfrac{L}{\sin\theta + L\cos\theta}\right)$.
- L = Relative settler length = l/d.
- L' = Relative length of the flow transition region.
- l = Length of the settler (L).
- m = Mass of a suspended particle (M).
- Q = Total flow through a settler (L^3/T).
- q_1 = Portion of Q having its suspended particles completely removed in the settler (L^3/T).
- q_2 = Portion of Q having its suspended particles all remaining in the flow (L^3/T).
- S = $\dfrac{v_s}{v_0}(\sin\theta + L\cos\theta)$.
- S_c = Critical S.
- t = Time (T)
- u = Local flow velocity in the x-direction (L/T).
- V = Volume of a suspended particle (L^3).
- v = Local flow velocity in the y-direction (L/T).
- v_0 = Average velocity of flow through a settler (L/T).
- v_p = Velocity of a suspended particle (L/T).
- v_{px}, v_{py} = x and y components of $\mathbf{v_p}$, respectively (L/T).
- v_s = Fall velocity of a suspended particle (L/T).
- v_{sc} = Critical v_{sc} (L/T).
- X = x/d.
- x, y = coordinates (L).
- Y = y/d.
- α = $\left(\dfrac{3v_sL}{4v_0}\right)^{1/2}$
- β = $\sqrt{1-\alpha^2}$
- θ = Angle of inclination of the settler.
- μ = Dynamic viscosity of the fluid (M/TL).
- ν = Kinematic viscosity of the fluid (L^2/T).
- ρ, ρ_p = Densities of the fluid and suspended particle, respectively (M/L^3).

Quantities in boldface letters are vectors.

References

1. Hazen, A., "On Sedimentation." *Trans. Amer. Soc. Civil Engr.*, **53**, 45 (1904).
2. Camp, T. R., "Sedimentation and the Design of Settling Tanks." *Trans. Amer. Soc. Civil Engr.*, **111**, 895 (1946).
3. Hansen, S. P., and Culp, G. L., "Applying Shallow Depth Sedimentation Theory." *Jour. Amer. Water Works Assn.*, **59**, 1134 (1967).
4. Culp G., Hansen, S., and Richardson, G. "High Rate Sedimentation in Water Treatment Works." *Jour. Amer. Water Works Assn.*, **60**, 681 (1968).
5. Streeter, V. L., "Fluid Mechanics." 3rd Edition, McGraw-Hill, New York, N. Y., (1962).
6. Rouse, H., (Ed.), "Advanced Mechanics of Fluids." John Wiley & Sons, Inc., New York, N. Y. (1959).
7. Thomas, J. W., "Gravity Settling of Particles in a Horizontal Tube." *Jour. Air Poll. Control Assn.*, **8**, 32 (1958).