## IAD Handpump Project

# THE DYNAMICAL BEHAVIOUR OF MAN DRIVEN PISTON LIFT PUMPS

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FIRST SEMESTRIAL REPORT period: 01.10.88 till 31.01.89

March 1989 IADHPP89.02

## THE IAD HANDPUMP PROJECT

This project is being carried out at the instigation of the Netherlands Minister for Development Cooperation and has for its main goal: To provide a substantial contribution to the improvement of the (communal) drinking-water supply and small scale irrigation, notably in Third World countries.
In this stage the project concentrates on the improvement of deepwell handpumps, in view of:
- reliability and easier maintenance,
- more profitable and simpler management.

Furthermore the project will support any activities:
- stimulating management of the water supply by its users,
- leading to the production of the required pump parts in the Third World.

It is thereby in line with similar projects that have been carried out in recent years under the auspices of the World Bank.

First and foremost the project concentrates on measuring and analyzing the dynamic behaviour and the stresses in the vital parts of the deepwell pump, especially in the rising main. Physical models will be developed to support the analyses. The experiences are integrated into recommandations and design rules for handpumps which will be published at regular intervals and be put for discussion. The project results are public domain.

At the instigation of the Office for Research and Technology from the Ministry of Foreign Affairs, the project partners have joined research on deepwell handpumps in the IAD Handpump Project. The partners and their contribution to the second project phase:
- DHV works out project publications ('laiting'), advices by the formulation of the project and design rules.
- IAD coordination, realization of data acquisition hard- and software, execution of measuring program and has final responsibility.
- JV manufacturer and supplier of the Volanta pump, analyzing of measuring results.
- SWNV manufacturer and supplier of the SWN81 pump, makes available part of their infrastructure on their site in Nunspeet, assists in the erection and conversion of the test-unit.
- TUE advices on the planning and execution of the program, analyses the results, physical modelling of the dynamic behaviour of the pump, fatigue analysis.

The partners cooperate by the formulation of advices and design rules and the realisation of related publications.

For carrying out the measuring program a test-unit has been arranged on the the SWNV-site, consisting of an enclosed boring having a depth of 100 metres, in which the water level can be varied as required and data acquisition hard- and software to enable the different variables on the handpumps to be measured. This infrastructure is owned by the Ministry.

Test-side situated at: SWNV, Industrieweg 47, 8071 CS Nunspeet
Telephone: 03412-54046, extension 51.

Any new orders will be welcomed by the team. For additional information, please contact the Project Coordinator: Jos Besselink
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Publication number: IADHPP89.02

"THE DYNAMICAL BEHAVIOUR OF
MAN DRIVEN PISTON LIFT PUMPS;
SEMESTERIAL REPORT"

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March 1989 R 991 D

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## 1. INTRODUCTION

Water is one of the necessaries of life. Besides that, a good water supply is necessary for hygiene. In the third world countries it is difficult to supply enough water for all people and it is getting worser because of the growing population rate and the decreasing ground water level. Structural improvements within the next 10 years are absolutely necessary.

Particularly in rural water supply posting hand pumps must be considered as (perhaps) the best way to guarantee a good water supply (lit: "Community Water Supply; The Hand pump Option",[ARL87]).

The waterpumps have to meet globally the following requirements:

i: the pump must be able to pump sufficiently deep; it must be taken account of the decreasing ground water level. (In Africa lift height up to 100 m).

ii: the discharge rate has to be high enough. A pump with a low capacity causes a lot of inconvenience because of long queues and waiting times. (Sometimes other, poluted water is available, which people will use to save time).

iii: the pump sould not break often. In Africa the pump corrodes fast because of the climat (warm) and the water quality (acid).

iv: the pump should not be expensive; economically it is favourable when the pump is produced in the third world countries themselves.

v: the pump has to be constructed in such a way that village people are able to maintain and repair the pump themselves.

The most promissing pump meeting all these requirements is the man driven piston lift pump.

The piston pump consists of a long tube from the surface till below ground water level. Below ground water level a piston is moved up and down in the pipe by a long rod. A valve mechanism is used, so the piston lifts the water column in the pipe (called rising main) during the upward stroke, while the waterlevel will not decrease during the downward stroke.

Until now all other types of pumps are less suitable. So in this report we only discuss the conventional piston pump. On the other hand it is still possible that another type of pump will be designed suiting better for third world countries.

Non of the pumps produced at the moment meets all the requirements mentioned above. That is why the manufacturers started to improve their pumps. Mostly this is restricted to detecting the weak parts of the pump by testing the durability and trying to increase the lift of the pump by reinforcing the weak parts.

Some research institutes have started to invesigate where and why the greatest loads occur in the pumps, finally causing a defect. The Dutch department of foreign affairs (development cooperation, BUZA DPO/OT) finances such an investigation, the so-called IAD hand pumps project. The project is mainly executed by IAD, a small company in Arnhem. IAD is assigned to realize experiments (see [BES89]) on two commercial available hand pumps (SWN81 and Volonta), while the Eindhoven university of technology provides the theoretical support (this report).

The idea is that most defects occur at parts of the pump which endure a heavy and, due to the pump action fluctuating load, always going with corrosion. Experiments concentrate on measuring precisely the progress of the load of the pumprod (drives the piston) and the rising main. The theoretical support contains development of physical models, explaining and predicting the load quantitatively. Research on the fatigue life of pvc rising mains and stainless steel pumprods has been done by Beekman and De Jongh (see [BEE89]). Their work will not be discussed in this report.

Our aim is to gather enough insight into phenomena occuring in hand driven piston pumps, to be able to design construction rules for piston pumps and perhaps realize some structural improvements of hand pumps using the experimental experiences and physical models.

## 2. HANDPUMPS IN THE THIRD WORLD COUNTRIES

In figure 1 the handdriven piston pump is shown schematically.

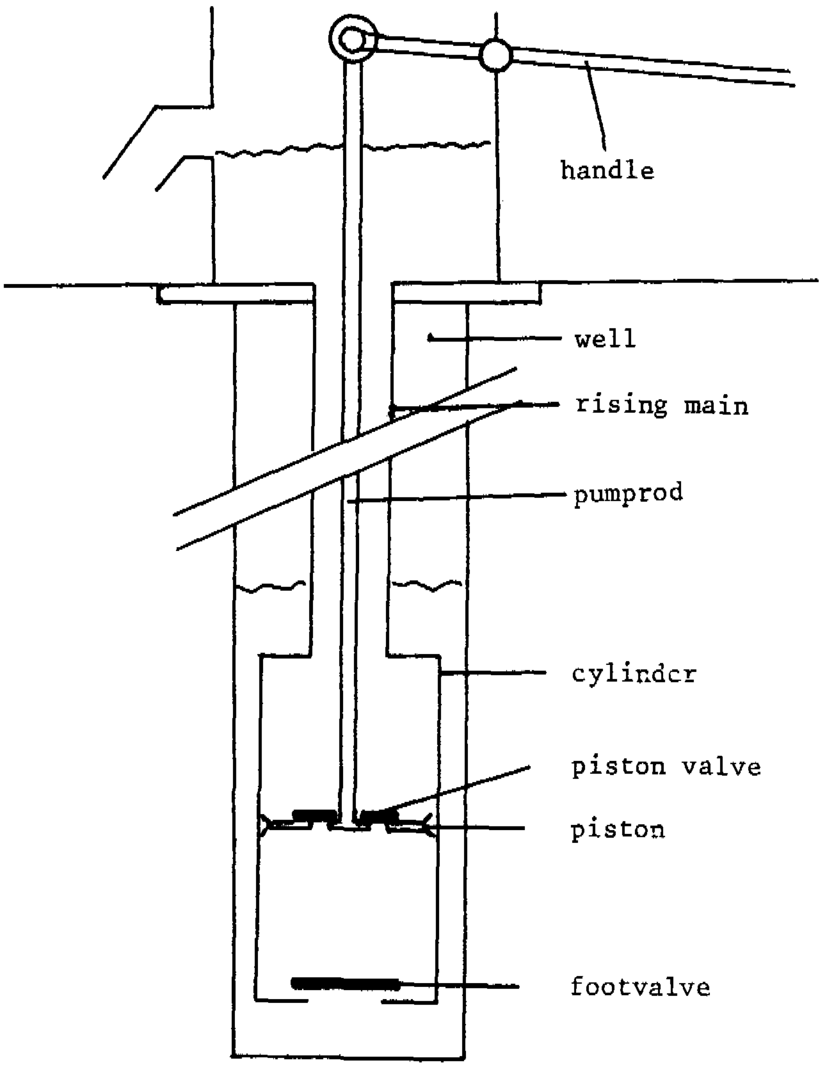


Fig. 1: The handdriven piston pump

Generally the ground water in third world countries is warm and acid (sometimes pH = 5 and less). Furtheron a small amount of sand (also larger or smaller particles) is pumped with the water sometimes. Consequently it is necessary to design a pump highly persistant to corrossion.

The rising main is made of pvc, which is cheap and persistant to corrosion. The pumprod is made of stainless steel, because of the great load. Furtheron the piston cup should not wear when sand is pumped with the water.

Using pvc has some influence on the pump action. During the downward stroke reflow is stopped by the footvalve. However, the weight of the water column causes the fairly elastic pvc to stretch, causing a decrease of the water level in the rising main. The upward stroke has to be used partly to recover the waterlevel. Only the remainder of the stroke causes a water discharge.

This effect gets important at lifting heights of about 40m and more. It causes a loss of efficiency, because lifting the water column takes energy. All this is discussed in chapter 4. Because of the oscillating motion of the piston pressure waves arise in the pipe. These pressure fluctuations cause an extra load on the pump.

Using pvc may cause amplification of these waves, due to a rather complicated mechanism, discussed in chapter 5.

The user of the pump feels the pressure waves and will adjust the pumping to the waves. The question arises in what way this phemomena will influence the duration of life of the pump.

## 3. EXPERIMENTS ON HANDPUMP SWN81

IAD has built a hand pump testing setup in Nunspeet (see [BES89]). This setup consists of a 100m deep drilled well, in which the waterlevel can be controlled. The hand pump that has been tested (SWN81) has been provided with:

= strain gauges attached at the rising main at different heights
= strain gauges attached at the pumprod at different heigths
= a measuring device that measures the location of the pump cylinder
= a measuring device that measures the movement of the piston related to the cylinder position.
= a measuring device that measures the position (angle) of the handle
= a pressure transducer to measure the pressure in the cylinder
= the possibility to measure the water discharge during pumping
= the possibility to have the handle operated by an electric motor

During the experiments the following parameters are varied:

= the length of the rising main
= the water level
= motor operated pumping: i: the pumping frequency
  ii: the piston stroke
= manual pumping: i: slow pumping with maximum stroke
  ii: fast pumping with maximum stroke
  iii: very fast pumping (2 Hz) with a small stroke

All measured data are recorded and processed by a pc using the ASYST program environment.

## 4. A COMPARISON BETWEEN DIFFERENT MEASURED QUANTITIES

It is possible to calculate the position of the cylinder from the data of most measured quantities, using only simple models to describe the relation between the cylinder position and the measured quantity. This is carried out in §4.1.

It shows that the agreements is qualitatively good, but quantitatively the results differ pretty much. We suspect that these differences are mainly caused by shortcomings in the models used.

### 4.1 Calculating the cylinder position from the pressure inside the cylinder

The cylinder movements is caused by the stretch of the rising main. This stretch is caused by the weight of the water in the rising main and also by the dynamical behaviour of the rising main and the water column.

To relate the cylinder movement to the pressure inside the cylinder we have to make some assumptions:
the velocity of sound in pvc is about 1500 m/s, thus, using a rising main of 100 m, the phase shift of movements of different parts of the pvc pipe will be at maximum about 0,07 s.
From the experiments it follows that in the rising main no events take place with frequencies higher than 10 Hz. For simplicity we assume that all parts of the pvc pipe move in phase for time-intervals greater then 0,1 s.

In first approximation we assume the pressure inside the cylinder to be equal to the static pressure. During the downward stroke the complete weight of the water column rests on the rising main footvalve. During the upward stroke the cylinder bottom is completely relieved from the static pressure. During the downward stroke the cylinder bottom is loaded by the static pressure.

So, the stretch of the rising main is:

$$\Delta l_{rm} = L \frac{\rho g L \cdot A_m}{E_{pvc} \cdot A_{pvc}}$$

where:

$\Delta l_{rm}$ = change in length of the rising main

L = length of the rising main

$\rho$ = density of water (1000 kg/m³)

$A_{rm}$ = inside intersection of the rising main (SWN81: $10^{-3}m^2$)

$E_{pvc}$ = Young's elasticity modulus ($2,4.10^9$Pa)

$A_{pvc}$ = cross section of the pipe wall (SWN81: 7,54 $10^{-4}m^2$).

The cylinder intersection of the SWN81 is greater than its rising main intersection. This causes continuously an upward directed force (see fig. 2). During the upward stroke the rising main can undergo an axial pressure, instead of an axial stroke. This axial pressure can be large enough to cause bending of the rising main.

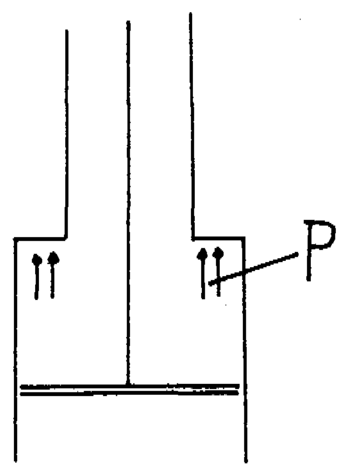


Fig. 2: During the upward stroke the cylinder will be pushed upward.

As mentioned before also pressure fluctuations appear in the rising main. We can take account of these by using the measured pressure in the cylinder instead of the static pressure ($\rho$gL).

To calculate the stretch from the pressure we have to know whether the piston is moving upward or downward relative to the cylinder movement. This can and will be deduced from the measurements.

**4.2 Comparing four ways of determining the cylinder movement**

There are four ways to determine the cylinder position:

i: using directly the data obtained with the device that measures the cylinder location relative to the firm ground (called CVO)

ii: calculating the cylinder position from the data obtained from the stretch measured with the strain gauges on the rising main (called RBA)

iii: calculating the cylinder position from the static pressure

iv: calculating the cylinder position from the stretch, due to the measured pressure in the cylinder (called P).

As an illustration the results of these methods are plotted in figure 3. The results are gathered in appendix B.

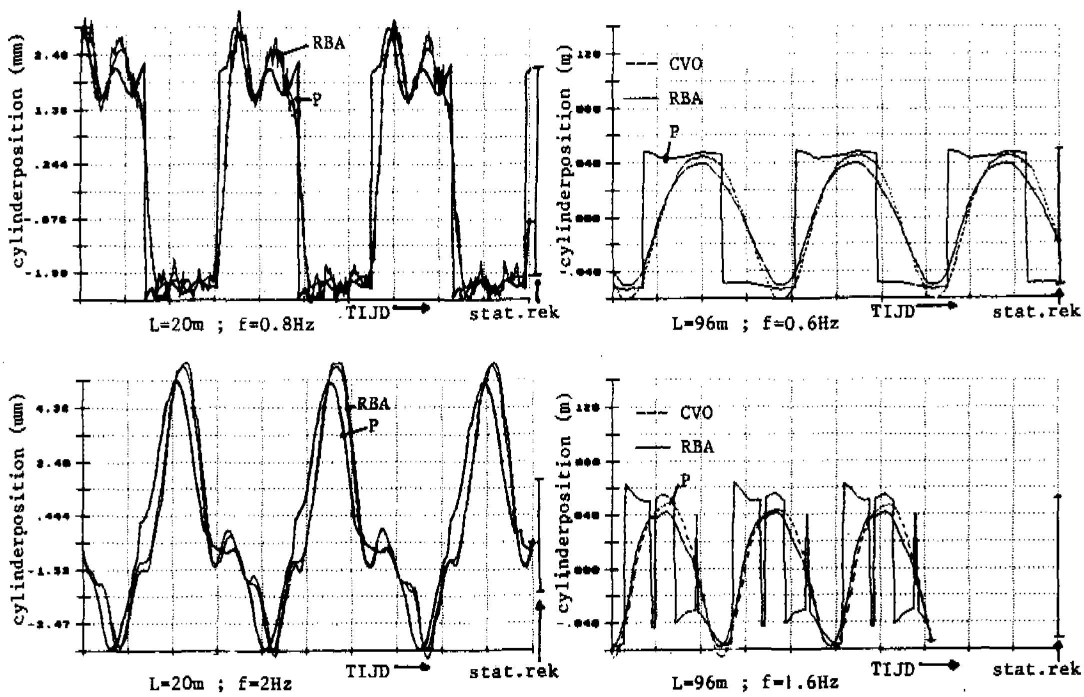


Fig. 3: Cylinder movement determined from CVO, RBA, P and $P_{statis}$
(At L = 20 m, CVO defect).

Comparison between the measured stretch (RBA) and the measured cylinder position (CVO):

Qualitatively the agreement is good, but quantitative there is a difference in magnitude and phase:

= Pumping lift 40 m and 60 m:
  The magnitude of CVO is smaller than the magnitude of RBA, also CVO has a phase delay to RBA. The differences increase with increasing pumping frequency.

= Pumping lift 80 m and 96 m:
  The magnitude of CVO is larger than the magnitude of RBA. Both signals are in phase. This is independent of the pumping frequency.

The calculated static pressure and the measured pressure agree fairly well.
Looking at the plots it can be seen that the dynamical effects (pressure waves) correspond to the measured stretch. It can be seen too that the change in stretch is not as abrupt as calculated from the pressure when the piston movement changes its direction. This can be explained by the fact that we used a simple model to describe the process of valve opening and closing.

The stretch, calculated from the pressure and RBA agree fairly well. So the measurement CVO differs more or less from RBA and the pressure. This could be explained by the fact that the rising main swings in the well. As the pipe will be bended by the swinging, the cylinder will be displaced. This displacement cannot be measured by RBA, but its axial component is still measured by CVO.

The strain gauges are located on the pvc pipe in such a way that the bending of pipe can be measured. From these measurements it follows that the pipe indeed swings.

## 5. OSCILLATING FLOW THROUGH A PIPE; THEORY

The pressure in the cylinder determines the load of the pump parts and consequently the wear of the pump parts. The pressure consists of two components:

i: The static (constant) pressure caused by the weight of the water column

ii: During the first half of the upward stroke the water is accelerated upward. This causes an increase of pressure. In fact we enforce an oscillating flow, which involves an oscillating pressure. We will see that the properties of the pipe define the pressure oscillations.

In chapter 4 the static pressure is discussed. In this chapter we will try to make a model that describes the relation between the piston movement and the pressure inside the cylinder. It shows that the pressure strongly depends on the properties of the pipe.

In water the velocity of sound is 1480 m/s. The phaseshift of the movements of the upside and downside of the water column is max. 0.075 seconds for a rising main length of 100 m. Because all phenomena in the tube have a frequency smaller then 10 Hz (measured), the whole column would move in phase. However, this is not true for water in a pvc pipe.

The relative high velocity of sound in water is caused by the fact that water is almost incompressible. However, water in a pvc pipe is much easier to compress, because the increase of pressure cause the pvc pipe to expand, involving a relative large change in volume.

Consequently the water-pvc system is much more compressible, so the velocity of sound is smaller in the water-pvc system as expressed in formula (4). In case of the SWN81 pump the velocity of sound is about 500 m/s.

Having this velocity of sound we cannot assume that the complete water column is moving in phase. Notice that at low frequencies this is advantageous, because the water column won't be accelerated as a whole, so pressure oscillations will have a smaller magnitude.

However, the low velocity of sound causes resonances in the rising main: for L = 20 m at 6 Hz and higher; for L = 100 m at 1.25 Hz and higher. During this resonance high pressure magnitudes appear, which cause a heavy load on the pump parts.

In this chapter the relation between velocity and pressure in the cylinder is derived in a strictly formal manner.

To describe this relation the so-called impedance will be introduced. Supplying a certain pressure to the cylinder will result in a flow through the rising main. The magnitude of the flow will be restricted, because the pipe and the inert mass of the water resist the flow. The impedance is very simular to this resistance.

In fact we have to deal with a periodic fluctuating (non-sinusoidal) pressure in the cylinder. This pressure signal can be developed in several harmonic componenents by Fourier transformation. From the theory we calculate a harmonic flow component due to each harmonic pressure component. From the basic flow equations it follows that we only have to summarize all the harmonic flow components (i.e. inverse Fourier transformation) to get the resulting periodic fluctuating flow in the rising main.

Notice that the last step in the calculation (the inverse Fourier transformation) is not straightforward and only allowed if the (flow-)equations meet strict requirements (the equations have to be linear).

Consequently, all we have to know to describe the flow in the pipe is the relation between the harmonic pressure component and the harmonic flow component at any frequency. This frequency dependent relation is given by the impedance.

At each frequency the pipe has a specific impedance. It can be shown that, if the equations are linear, the impedance only depends on the properties of the pipe, and not on magnitude of the pressure or flow component.

So, the main importance of the impedance is that we can calculate the flow through the pipe caused by an arbitrary pressure (and vice versa) and that the impedance is solely depending on the mechanical properties of the pump.

## 5.1 General flow equations[1]

The equation of momentum for a one-dimensional flow through a circular tube can be written as:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial x} + \frac{\partial v}{\partial t} + g + \frac{1}{\rho}\frac{4\tau_0}{D} = 0 \tag{1}$$

where the last term accounts for the transfer of momentum to the pipe wall due to the friction. The equation of continuity can be written as:

$$\frac{\partial\ (\rho A v)}{\partial x} + \frac{\partial\ (\rho A)}{\partial t} = 0 \tag{2}$$

Including the mechanical properties of the pipe from (2) the following equation can be derived:

$$\frac{1}{\rho}\frac{\partial p}{\partial t} + \frac{v}{\rho}\frac{\partial p}{\partial x} + a^2\frac{\partial v}{\partial x} = 0 \tag{3}$$

where:

$$a^2 = \frac{K/\rho}{1 + \frac{K}{E}\frac{D}{e}} \tag{4}$$

where:

- K: bulk modulus of elasticity of water (2.2 $10^9$ Pa)
- $\rho$: density of water (1000 kg/m$^3$)
- E: Young's modulus of elasticity of pipe material ($E_{pvc}$= 2.4 $10^9$ Pa)
- D: inside pipe diameter (SWN81: 0.036 m)
- e: pipe wall thickness. (SWN81: 0.00575 m)

For the SWN81 follows from (4): a = 500 m/s.

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1 see e.g. [WYL83].

## 5.2 The stationary solution

In the stationary case equations (1) and (3) simplify to:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + v\frac{\partial v}{\partial x} + g + \frac{1}{\rho}\frac{4\tau_0}{D} = 0 \tag{5}$$

$$\frac{1}{\rho}v\frac{\partial p}{\partial x} + a^2\frac{\partial v}{\partial x} = 0 \tag{6}$$

From equation (5) it follows that for an upward flow (positive x-direction) the pressure decreases in upward direction because of the gravity and the friction. Equation (6) expresses that at higher pressure the velocity decreases, due to the expansion of the pipe.

Elimination of $\partial v/\partial x$ in (5) and (6):

$$\frac{1}{\rho}\frac{\partial p}{\partial x}(1 - \frac{v^2}{a^2}) + g + \frac{1}{\rho}\frac{4\tau_0}{D} = 0 \tag{7}$$

In all cases for hand pumps $|v| << a$, so this term can be ignored in equation (7).

To solve equation (7) we need an expression for the wall shear, stress $\tau_0$. This expression depends on the kind of flow we deal with, i.e. we have to know whether the flow is laminar or turbulent.

The Reynolds-number of the flow is:

$$Re = \frac{\rho\ v\ D}{\eta} \approx \frac{1000 \times 0.1 \times 0.0055}{10^{-3}} = 550 \tag{8}$$

The chosen values are experimentally obtained data. (For D is used half the value of the inside pipe diameter (0.0036 m) minus the outside diameter of the poly-ethyleen coat of the pumrod (0.025 m)).

The flow changes from laminar to turbulent at Re $\simeq$ 2000 for stationary flows to Re $\simeq$ 5000 for oscillatory flows. So we may assume we have to deal with a laminair flow. The expression for $\tau_0$ is:

$$\tau_0 = \rho\frac{8\nu v}{D} \tag{9}$$

To estimate the contribution of the friction we estimate v by the mean velocity. The solution of (7) is finally:

$$p(x) - p(x=0) = -x \left[g + \frac{16\nu v}{D^2}\right] =$$

$$= -x\,(9.81 + 5.2\ 10^{-2}) \tag{10}$$

This shows that the contribution of friction to the stationary solution is very small. From equation (6) it follows:

$$\frac{\partial v}{\partial x} L = v \frac{g + \frac{16\nu v}{D^3}}{\rho a^2} L = v \frac{9.81 + 5.2\ 10^{-2}}{1000 x 500^2} 100 << v \tag{11}$$

Equation (11) means that the mean velocity is constant along the pipe, so the effect of the change in diameter due to the pressure is a minor effect.

## 5.3 The wave equation

We introduce a new pressure p' and velocity v':

$$p = p' + p_{stat}(x) \tag{12}$$

$$v = v' + v_{stat} = v' + v_{mean} \tag{13}$$

Substitution of (12) and (13) in (1) and (3):

$$\frac{1}{\rho}\frac{\partial p'}{\partial x} + \frac{\partial v'}{\partial t} + v\frac{\partial v'}{\partial x} = 0 \tag{14}$$

$$\frac{1}{\rho}\left[\frac{\partial p'}{\partial t} + v\frac{\partial p'}{\partial x}\right] + a^2\frac{\partial v'}{\partial x} = 0 \tag{15}$$

In this derivation we assume that the oscillatory component of the flow is frictionless. The influence of friction is discussed in section 5.5.

It will be shown that $v\frac{\partial}{\partial x}$ may be ignored compared to $\frac{\partial}{\partial t}$ when $|v| << a$. Neglecting $v\frac{\partial}{\partial x}$ the wave equation can be derived from (14) and (15):

$$\frac{\partial^2 \xi}{\partial t^2} - a^2 \frac{\partial^2 \xi}{\partial x^2} = 0 \qquad (6)$$

where: $\xi = p', v'$:
Intermezzo:
The general solution of (16) is:

$$\xi = f\left[t - \frac{x}{a}\right] + F\left[t + \frac{x}{a}\right] \qquad (17)$$

The total differential of $\xi$ is:

$$d\xi = \frac{\partial \xi}{\partial t} dt + \frac{\partial \xi}{\partial x} dx \qquad (18)$$

Consider only the wave traveling in the positive x-direction $f\left[t - \frac{x}{a}\right]$. The directional deriviative of f with $t - \frac{x}{a} =$ constant is zero:

$$\left.\frac{df}{dt}\right|_{x = at} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{d(at)}{dt} = 0 \qquad (19)$$

Thus:

$$\frac{\partial f}{\partial x} = -\frac{1}{a}\frac{\partial f}{\partial t} \qquad (20)$$

So:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = \frac{\partial f}{\partial t}\left(1 - \frac{v}{a}\right) \approx \frac{\partial f}{\partial t} \qquad (21)$$

because $v << a$.
The same goes for F. This shows that a solution of the wave equation is also a solution of equations (14) and (15).

## 5.4 Solution of the wave equation

The wave equation (16) gives:

$$\frac{\partial^2 v'}{\partial t^2} - a^2 \frac{\partial^2 v'}{\partial x^2} = 0 \tag{22}$$

The boundary conditions are:

B.C.1: p = const. holds at x=0

From eq. (15) it follows $\left.\frac{\partial v'}{\partial x}\right|_{x=0} = 0$ (23)

B.C.2: at x = – L the water velocity is determined by the piston movement:

$$v'(x = -L, t) = V(t) \tag{24}$$

Substitute the solution of the flat wave:

$$v' = \tilde{v}\, e^{i(\omega t + kx)} \tag{25}$$

in equation (22):

$$-\omega^2 + a^2 k^2 = 0 \tag{26}$$

This leads to the so-called dispersion relation:

$$k^2 = \frac{\omega^2}{a^2} \tag{27}$$

So:

$$k = \pm \frac{|\omega|}{a} \tag{28}$$

We distinguish the travelling wave travelling upwards (= pos x-direction):

$$k^r = -\frac{|\omega|}{a} \quad \text{where: } v' = \tilde{v}^r \, e^{i(\omega t - \frac{|\omega|}{a} x)} \tag{29}$$

and the wave travelling down:

$$k^\ell = \frac{|\omega|}{a} \quad \text{where: } v' = \tilde{v}^\ell \, e^{i(\omega t + \frac{|\omega|}{a} x)} \tag{30}$$

Using boundary condition 1 (eq. 23):

$$\frac{\partial v'}{\partial x} = \frac{\partial}{\partial x}\left\{\tilde{v}^\ell e^{i(\omega t + k^\ell x)} + \tilde{v}^r e^{i(\omega t + k^r x)}\right\}\Big|_{x=0} = 0$$

gives:

$$-ik^\ell \tilde{v}^\ell - ik^r \tilde{v}^r = 0$$

Thus:

$$\tilde{v}^\ell = -\frac{k^r}{k^\ell}\tilde{v}^r = \tilde{v}^r \tag{31}$$

Using boundary condition 2 (eq. 24):

$$\tilde{v}^r e^{i(\omega t - k^r L)} + \tilde{v}^\ell e^{i(\omega t - k^\ell L)} =$$

$$\tilde{v}^r\left\{e^{-ik^r L} + e^{-ik^\ell L}\right\} e^{i\omega t} = \tilde{V} e^{i\omega t} \tag{32}$$

The solution is finally:

$$v' = \tilde{v}^r e^{\,i(\omega t + k^r x)} + \tilde{v}^\ell e^{\,i(\omega t + k^\ell L)} = \frac{V\left\{e^{\,i(\omega t + k^r x)} + e^{\,(i\omega t + k^\ell x)}\right\}}{e^{-ik^r L} + e^{-ik^\ell L}} \tag{33}$$

By substituting the flat wave solution (25) in eq. (14) and (15) it follows that p' can be written as:

$$p' = \tilde{p}\, e^{\,i(\omega t + kx)}$$

Eq. (15) shows:

$$\frac{1}{\rho} i\omega\, \tilde{p}^{\ell,r} + ik^{\ell,r} a^2\ \tilde{v}^{\ell,r} = 0$$

Thus:

$$\tilde{p}^{\ell,r} = -\,\rho a \frac{a\ k^{\ell,r}}{\omega}\ \tilde{v}^{\ell,r} \tag{34}$$

Notice: when $\omega > 0$: $\tilde{p}^\ell = -\rho a \tilde{v}^\ell = -\rho a \tilde{v}^r$; $\tilde{p}^r = -\rho a \tilde{v}^r$ (35)

At this point we introduce the so–called impedance of the pipe. This quantity describes the way the system (pvc–pipe and water column) responds to an applied pressure or flow. The impedance is completely determined by system parameters.
The impedance is defined as:

$$Z(\omega, x = -L) = \frac{\tilde{p}(\omega, x = -L)}{\tilde{v}(\omega, x = -L)} \tag{36}$$

We derive the following expression for $Z(\omega)$ when $\omega > 0$ (using (31), (32), (34) and (35)):

$$Z(\omega > 0, x = -L) = \frac{\rho a \tilde{v}^{r}\left[e^{\,i(\omega t - k^{r}L)} - e^{\,i(\omega t - k^{\ell}L)}\right]}{\tilde{V}\, e^{\,i\omega t}} =$$

$$= \rho a \frac{e^{-ik^{r}L} - e^{-ik^{\ell}L}}{e^{-ik^{r}L} + e^{-ik^{\ell}L}}$$

where:

$$k^{r} = -\frac{|\omega|}{a}$$

$$k^{\ell} = \frac{|\omega|}{a} \qquad (37)$$

The same goes for $\omega < 0$, resulting in (note that eq. (35) changes for $\omega < 0$):

$$Z(-\omega) = Z^{*}(\omega) \qquad (38)$$

For $\omega > 0$, eq. (37) can be written as:

$$Z(\omega) = i\rho a \tan\left[\frac{\omega L}{a}\right] \qquad (39)$$

At $\omega L/a = n\pi/2$ with $n = \ldots.\ -2, -1, 0, 1, 2, \ldots$ $|Z(\omega)|$ becomes zero or infinite. This implies infinite magnitudes of $\tilde{p}$ or $\tilde{v}$, which are unphysical.

In reality the waves are damped. This results in finite values of $Z(\omega)$. One of these damping mechanisms is friction.

## 5.5 Solution of the damped wave equation

Experimental observations (Fourier transformation of the pressure in the cylinder) lead to the conclusion that frequencies higher than 10 Hz are damped out. However, including friction in the model results in a much smaller damping, in fact only frequencies greater than 1000 Hz are damped out.

That is the reason why the main derivation of damping by friction is put in appendix C. Here we only describe the method of including damping in our model.

The damped wave equation is written as:

$$\frac{\partial^2 v'}{\partial t^2} + K\frac{\partial v'}{\partial t} - a^2\frac{\partial^2 v'}{\partial x^2} = 0 \tag{40}$$

The parameter K is the damping constant. In appendix C an expression for K due to friction is derived that holds for $\omega > 1.25\ s^{-1}$.

Substituting the flat wave solution in (40) results in a dispersion relation. For K sufficiently small (that is true for friction) this results in:

$$k^{\ell} = \frac{|\omega|}{a}\left[1 - \frac{i}{D}\sqrt{\left(\frac{2\nu}{|\omega|}\right)}\right] \tag{41}$$

and:

$$k^{r} = \frac{|\omega|}{a}\left[1 - \frac{i}{D}\sqrt{\left(\frac{2\nu}{|\omega|}\right)}\right] \tag{42}$$

In appendix C is also shown that, for K sufficiently small, relation (37) still holds. $Z(\omega)$ is computed for the SWN81 pump and plotted in figure 4. First some general features of $Z(\omega)$ are derived:

= For $1.25 < \omega << 2\pi a/4L$ from (41) and (42):

$$k^{\ell,r} = \pm\frac{|\omega|}{a}$$

thus from (39):

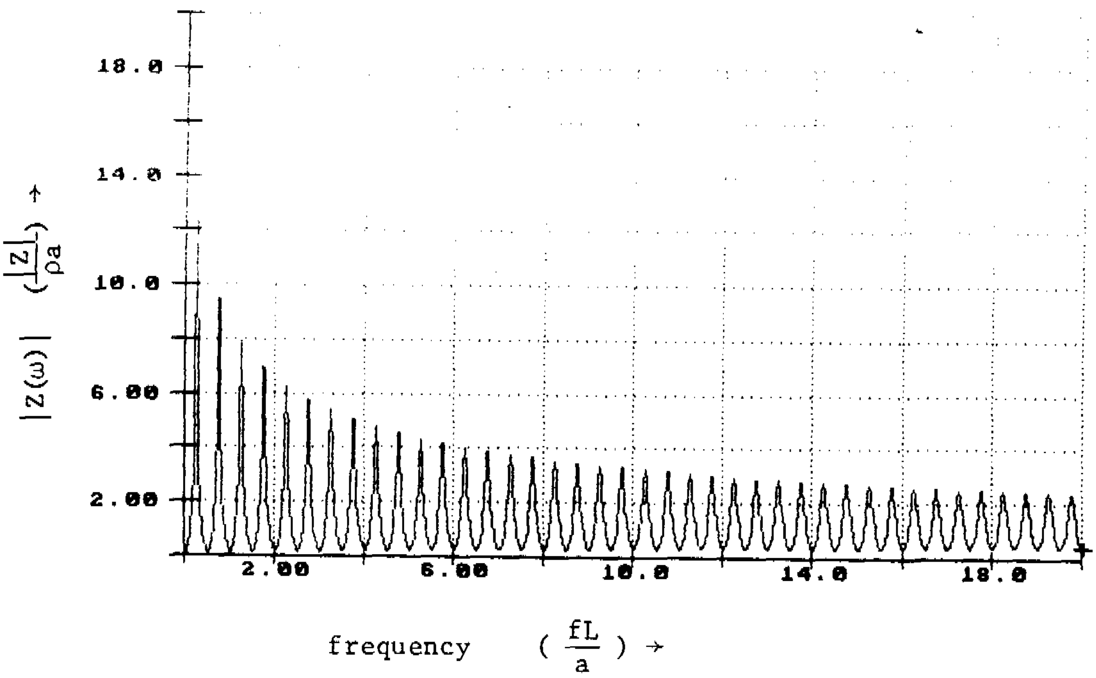
$$Z(\omega) = i\rho a \tan\left[\frac{|\omega| L}{a}\right] = i\rho L|\omega| \tag{43}$$

So at these frequencies $Z(\omega)$ is only determined by L.

= For $\omega \rightarrow \infty$:

$$\lim_{\omega \rightarrow \infty} Z(\omega) = \rho a \tag{44}$$

So at high frequencies the piston only "sees" upward travelling waves. The reflected waves are fully damped out.
In figure 4 $Z(\omega)$ is shown. We see that friction causes damping for $fL/a > 100$. However, in reality all phenomenons with $f > 10$ Hz are damped out.
Consequently it must be concluded that friction is not the most important damping mechanism.



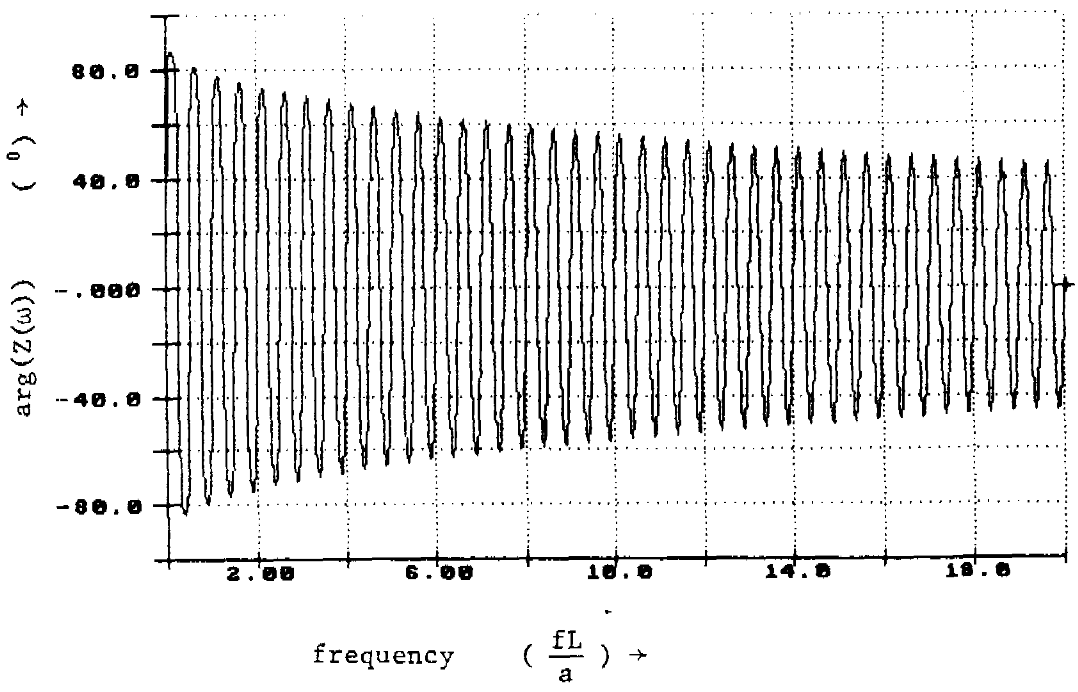




Fig.4: The impedance $Z(\omega)$, damped by friction.

$\nu = 10^{-6}\ m^2\ s^{-1}$

$D = 0,036$ m

## 6. OSCILLATING FLOW THROUGH A PIPE; EXPERIMENTS

### 6.1 Measurement of pressure and water velocity

To measure the pipe's impedance we have to know the pressure and the water velocity inside the cylinder. The pressure is known through direct measurement; determination of the water velocity is difficult.

Until now the following method has been used to determine the water velocity: first the piston movement and the cylinder movement are determined. When the piston is moving down compared to the cylinder, the water velocity is equal to the cylinder velocity. When the piston is moving up compared to the cylinder, the water velocity is determined by both the piston velocity and the cylinder velocity (see fig. 2).

The water velocity can be written as:

$$v_{piston} < v_{cylinder} : v_{water} = v_{cylinder}$$

$$v_{piston} > v_{cylinder} : v_{water} = \frac{A_{piston}}{A_{rising\ main}} (v_{piston} - v_{cylinder}) + v_{cylinder}$$

Some problems when using this method are:

i: the calculated water velocity changes abruptly when the relative piston movement changes direction, i.e. the acceleration will be infinite

ii: in chapter 4 it shows out that we cannot determine the cylinder movement uniquely. In the calculations here the RBA measurement is used, because it seemed most reliable.

### 6.2 Fourier transformation of pressure and water velocity

To determine the impedance the water velocity and the pressure have to be Fourier transformed. Within the Asyst program environment a FFT (fast fourier transform) is available. However, to get the frequency spectrum with FFT the signal has to meet some requirements.

We need to remember we deal with periodical signals, so we will find a discrete frequency spectrum. The usual procedure is taking at least 20 periods of the signal, and putting a so-called window on the signal, which makes sure that the signal is zero at the ends of the interval. Using FFT on the signal prepared this way gives a continuous frequency spectrum, where the peakes are broadened. This (usually small) broadening is fully determined by the window.

However, because of the limited memory capacity the offered signals consist of 3 to 10 periods, which is not enough when using the usual method.

So we have developed another approach. First the basic frequency (= pumping frequency) is determined accurately by counting the number and distance of the zero readings of a reliable reference signal, usually from the handle position. Next an interval is chosen, in which a whole number of periods is exactly present.

The FFT algorithm requires $2^n$ samples as input. Therefore $2^n$ (in our case 512) values are calculated in the choosen interval from the measured data. A value is calculated by linear interpolation between the two neighbouring samples.

When for example 3 periods with basic frequency $f_0$ are transformed this way, we get fourier coefficients at $1/3\ f_0$, $2/3\ f_0$, $f_0$, $4/3\ f_0$, ..., $512/3\ f_0$. Only coefficients at the basic frequency and higher harmonics ($f_0$, $2f_0$, $3f_0$, ...) should give non-zero values. However, in practise the other coefficients are not quite zero, because of non-periodicities in the signal and noise, but they can be neglected compared to the "harmonic" coefficients.

To calculate the frequency spectrum from the Fourier spectrum, the Fourier spectrum has to be normalized, i.e. the coefficients have to be divided by the number of transformed values (in our case 512).

The frequency spectrum contains noise. The noise is filtered by putting all coefficients, smaller than a choosen noise level, equal to zero. An appropriate noise level filters all high-frequency noise and non-periodicities. On the basis of the noise level the accuracy of the measurement of the impedance is determined.

## 6.3 Experimental determination of the impedance

By dividing the Fourier coefficients of pressure and water velocity we find the impedance at the concerning frequency. The accuracy is determined by the noise level.

In practice the accuracy is determined almost completely by the noise level of the water velocity (the measurement of the pressure is very accurate). Consequently only relative large coefficients exceed the noise level, so we experimentally mainly find measurements of impedance at frequencies where the impedance is low.

In figure 5 some typical results are plotted.

Almost all experiments (except L = 96 m) show that the results at frequencies higher than the lowest resonance frequency are rather chaotic. Therefore we now only discuss results at frequencies lower than the lowest resonance frequency.

Looking at the magnitude we conclude that at L = 20 m the measured impedance is higher than the theoretical impedance. At L = 40 m, 60 m, 80 m and 96 m the measured impedance is smaller than theoretically expected. Remember that theoretically the impedance at low frequencies only depends on the length of the rising main L (see chapter 5.5).

Looking at the phase we conclude that at L = 20 m, 40 m and 60 m the phase starts at low frequencies at $90^{O}$, what is in agreement with the theory, be decreases much further. At L = 80 m and 96 m the phase starts at approximately $120^{O}$ and decreases faster than theoretically expected too.

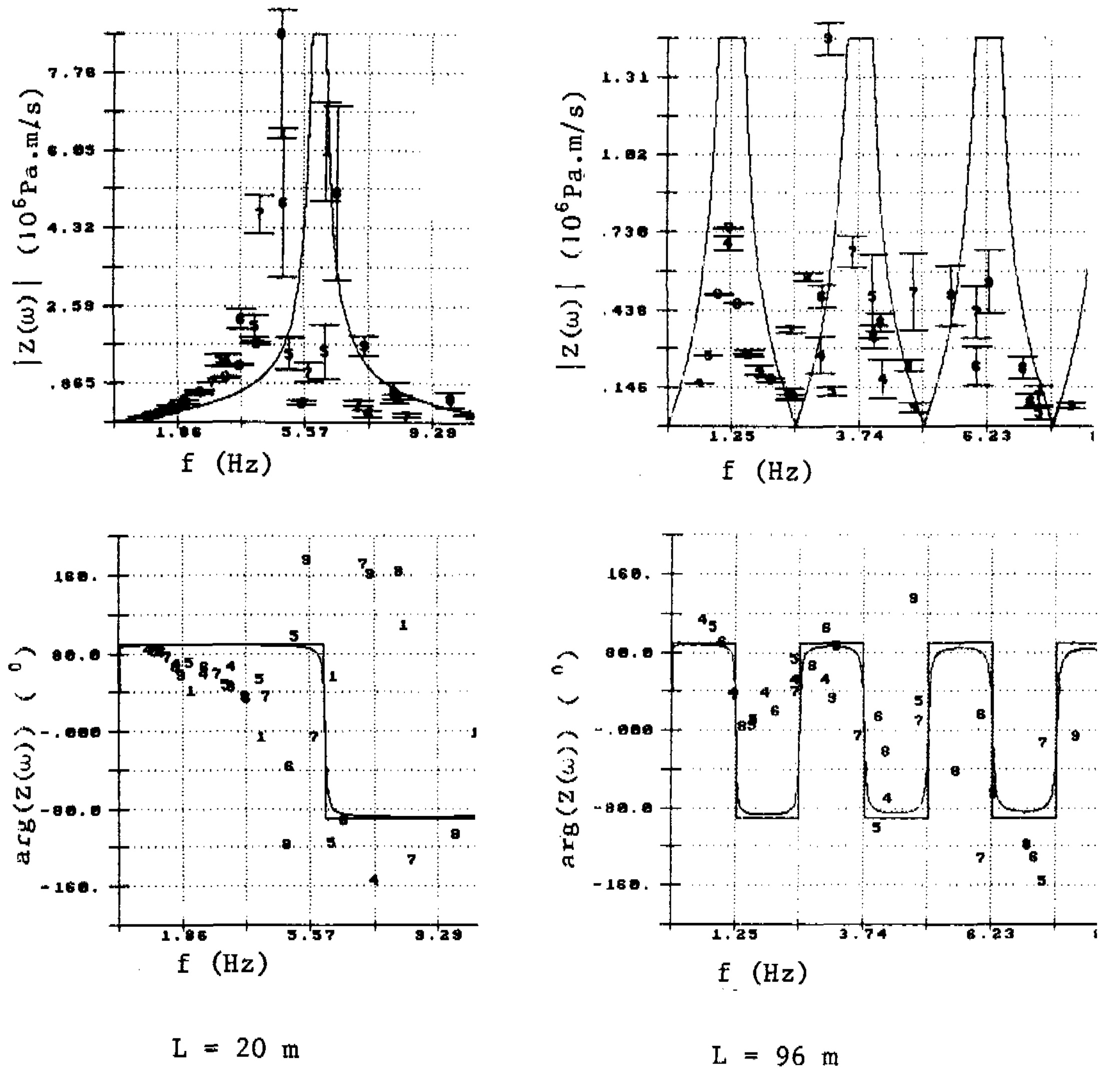



Fig.5: The impedance of the rising main at L = 20 m and L = 96 m.
Both the damped (friction) and undamped impedance are drawn. At these frequencies the only visible difference is the phase of the impedance. (The damped ones are closer to zero.)
The figures in the plot are the experimental results.
Each figure refers to impedances obtained at a specific pumping frequency.
Other results can be found in appendix d.

## 7. CONCLUDING REMARKS

During the first semester of the research program 3 models have been developed each describing a part of the pump action of hand driven piston lift pumps:

Model 1: a relation between the pressure in the cylinder and the water motion in the cylinder, due to the pipe geometry.

Model 2: a relation between the pressure in the cylinder and the cylinder motion.

Model 3: a relation between the water motion and both the piston motion and the cylinder motion.

Note that the equations derived from the models describe the complete pump action. When the equations are solved (which is not a straight forward task) it is possible to predict the dynamic behaviour of the pump at arbitrary geometries.

However, before making predictions we have to check whether the models are working satisfactory and whether we indeed included all phenomena important to the dynamic behaviour of the pump.

First we have checked the models using experimental data. We discuss briefly the results:

Model 1: the model predicts the resonance frequencies well. However, the amplitudes of the pressure waves in the pipe are overestimated.
Obviously the waves are damped by an until now unknown damping mechanism. The model could be used for prediction when the damping coefficient would be increased adhoc. This method is rather unsatisfactory.

Model 2: generally this model works well. Only when the piston valve opens or closes, the model predicts to abrupt cylinder movements. This is caused by the way the model describs the valve opening and closing mechanism.
Comparing different methods in measuring the cylinder motion, we see that results, obtained by detecting the strain of the pipe, sometimes differ from direct measurements of the cylinder position. This could be explained by the fact that the rising main swings in the well. This will be discussed later in this chapter.

Model 3: this model hasn't been checked until now. The most important parameter in this model is the valve position (open or closed). We will check the model using measured valve positions. It should be mentioned that the valve position measuring device isn't working satifactorily.

It is mentioned before that the rising main swings in the well. However, we didn't describe the swinging in a model, because this is rather difficult. Consequently, we cannot estimate whether the swinging is important to the dynamical behaviour of the pump. Research will be done on this subject.

Until now only experiments are studied where the pump is driven by an electric motor. In reality the user of the pump feels the pressure waves and will adjust his pumping. If we want to predict the dynamical behaviour of the pump we also have to predict in what way a user will pump. Also on this subject research has to be done.

## Literature

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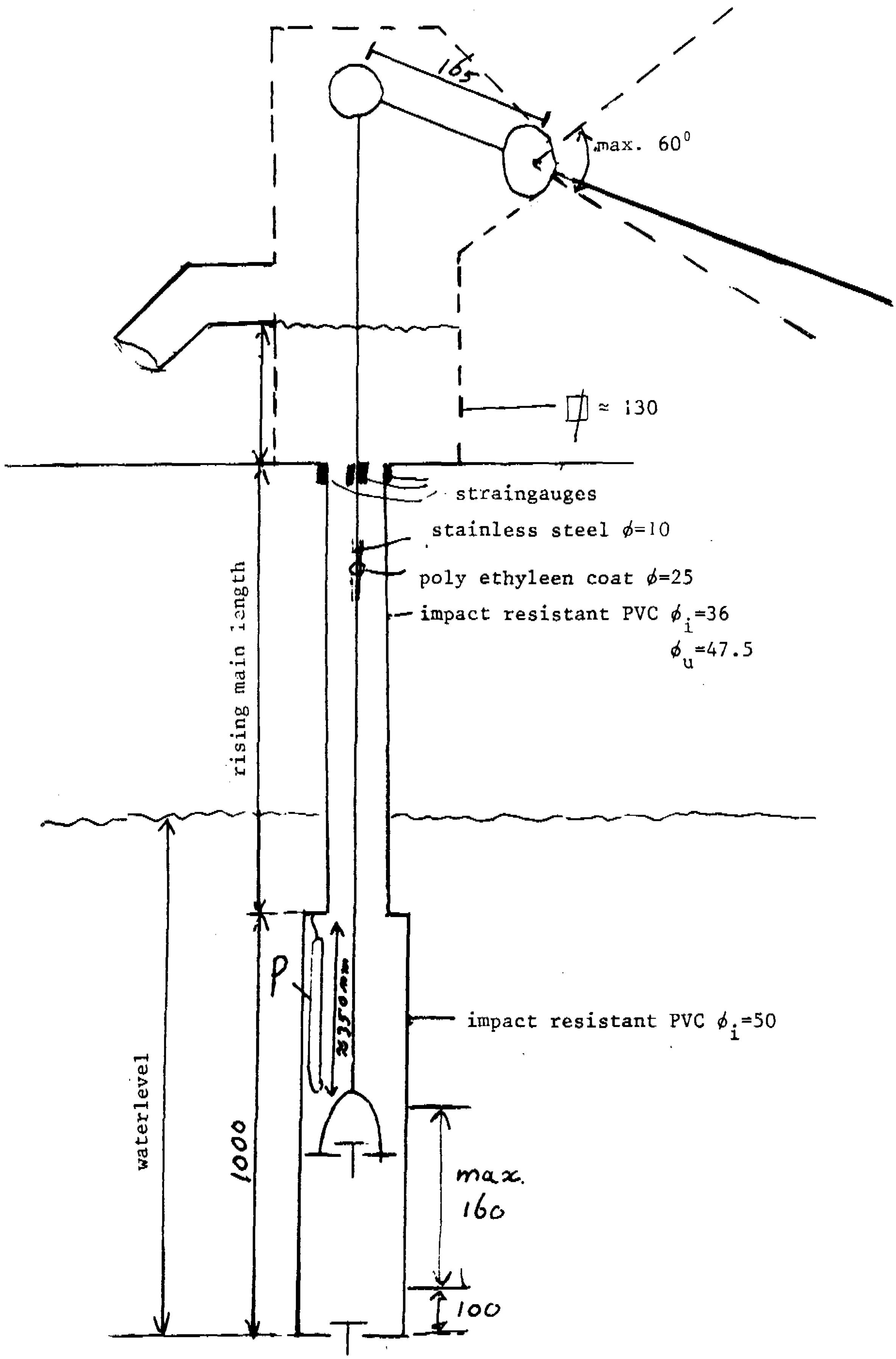
Appendix A: SWN81 in experimental setup



Appendix B : Plots of the cylindermovemoment (SWN81)

In the following plots the motion of the cylinder is shown ( location versus time ). Plotted are (see section 4.2):

= CVO: dashes line (not plotted for L=20m).

= RBA: solid line.

= P: solid line; can be distinguished from RBA by the vertical line segments at the time where the piston changes direction.

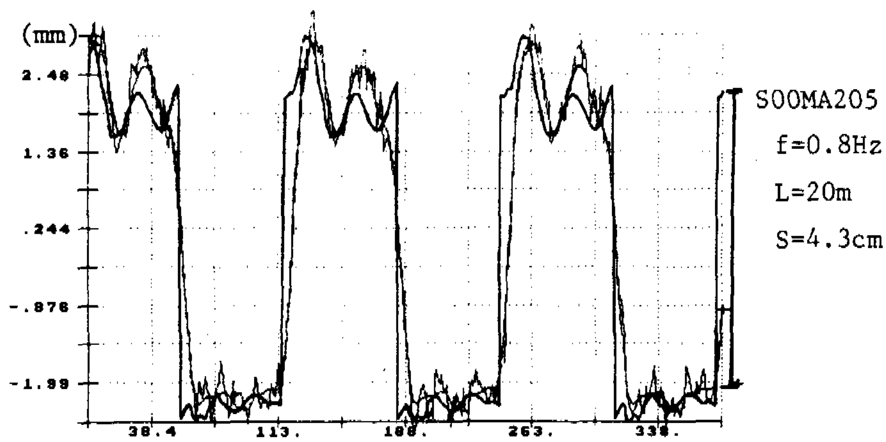
= the stretch calculated from the static pressure is shown at the right of the plot by the symbol: **I**

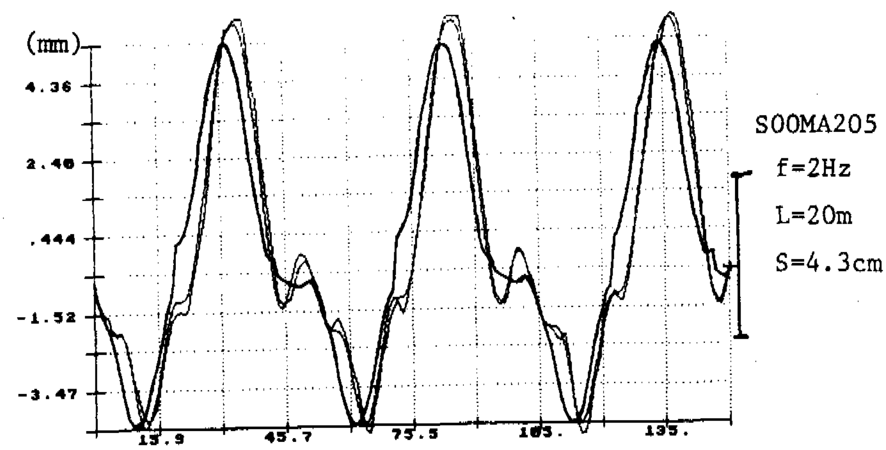
Notation: f = pumping frequency (Hz)

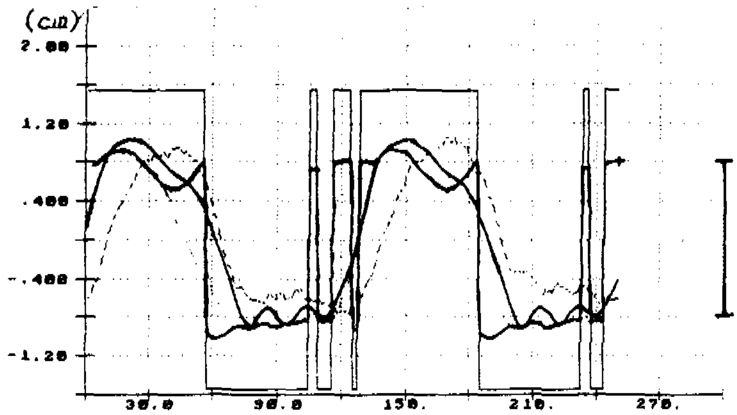
L = pumping depth (m)

S = piston stroke, refering to firm ground (m)

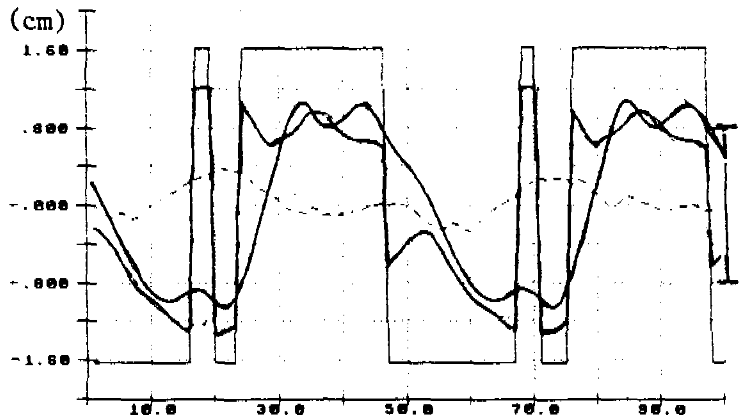
Note: the square wave (not shown in all plots) gives the direction of the piston motion.



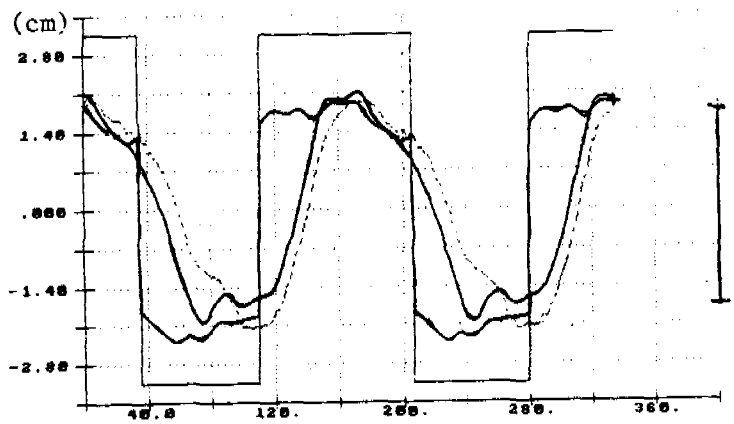





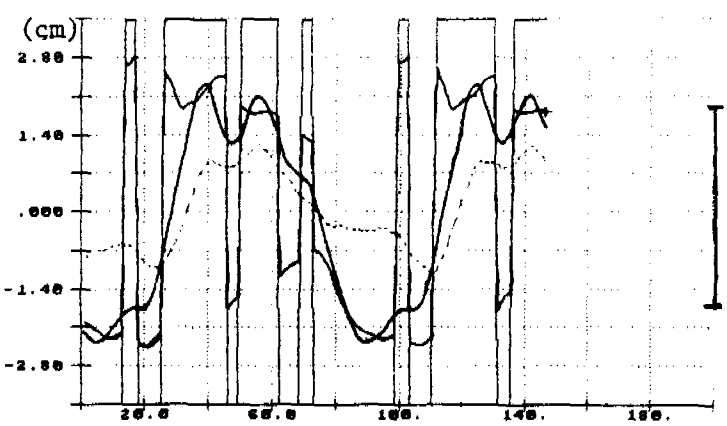

S00MA404
f≒.8Hz
L=40m
S=4.3cm



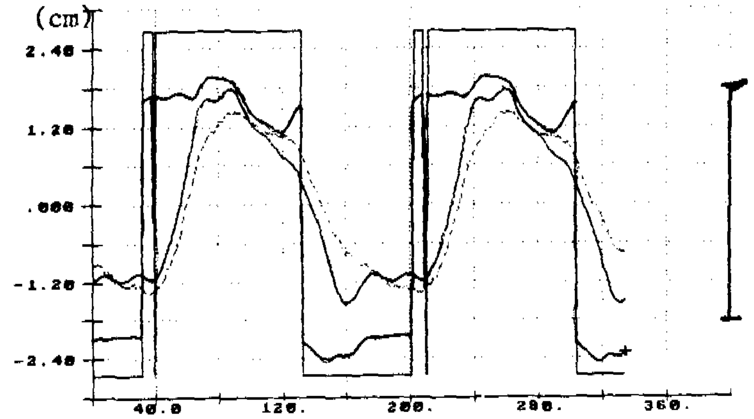

S00MA404
f=2Hz
L=40m
S=4.3cm



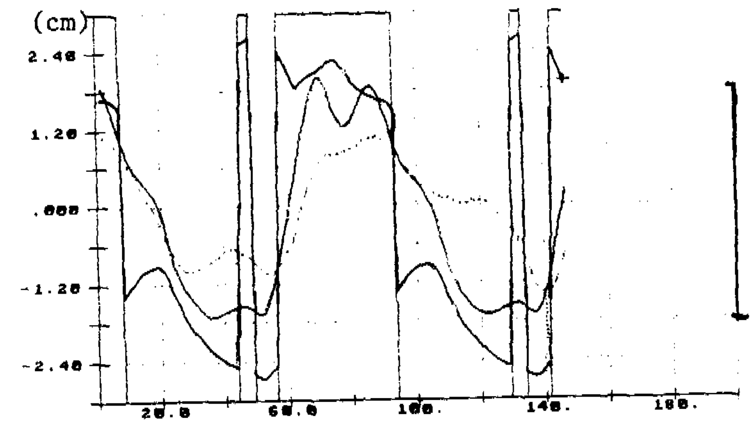

S00MA601
f=0.6Hz
L=60m
S=7.3cm



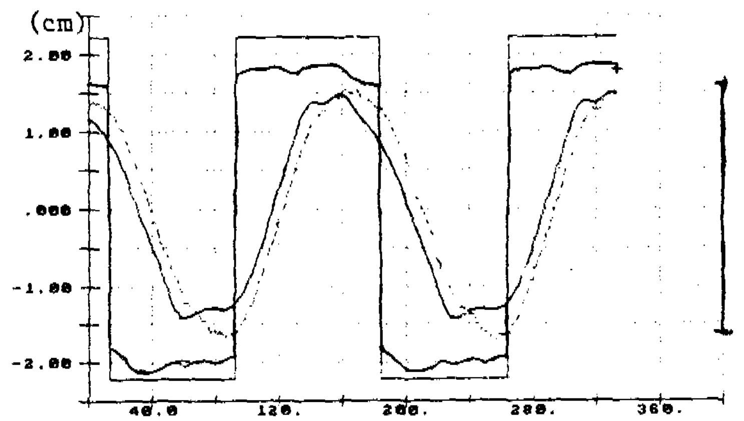

S00MA601
f=1.4Hz
L=60m
S=7.3cm




S01MA601
f=0.6Hz
L=60m
S=7.3cm




S01MA601
f=1.4Hz
L=60m
S=7.3cm

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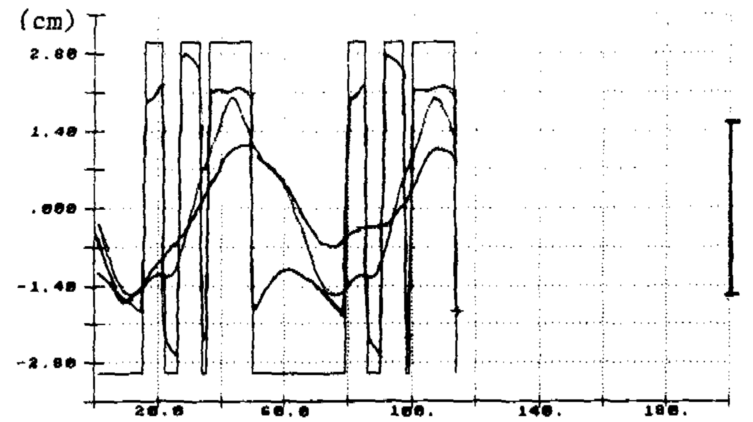
L=60m
S=4.3cm

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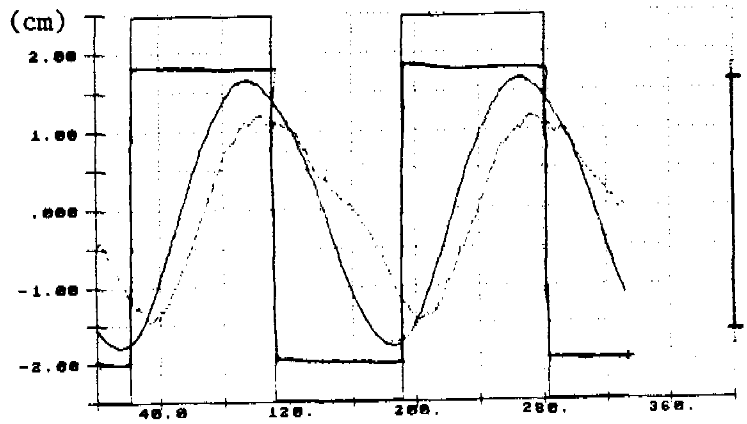
S=4.3cm

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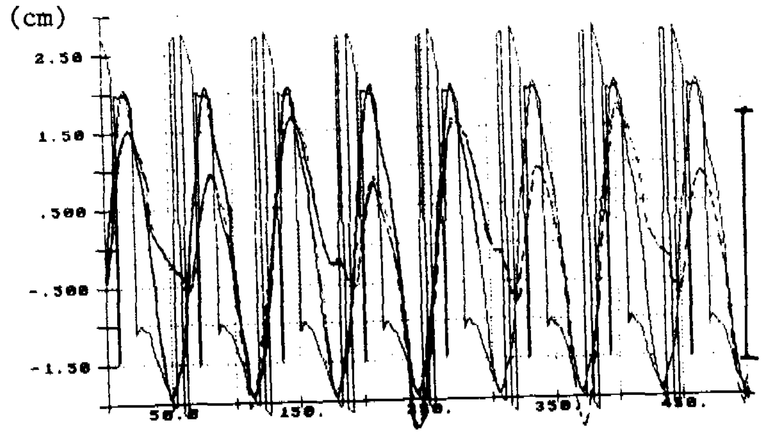
S=4.3cm

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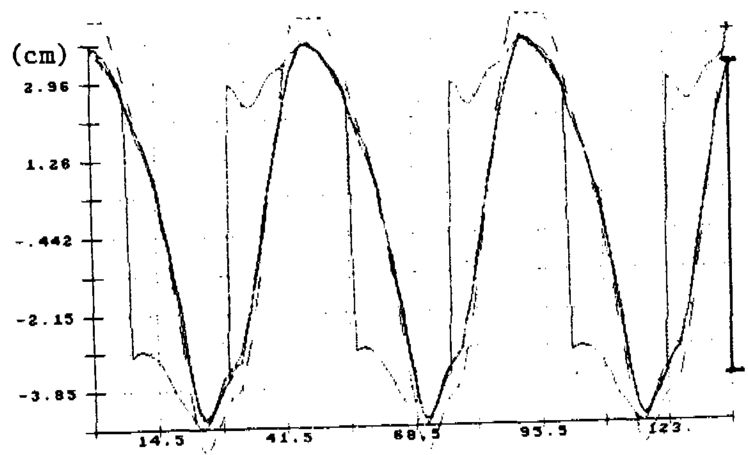
f=1.8Hz
L=60m
S=4.3cm

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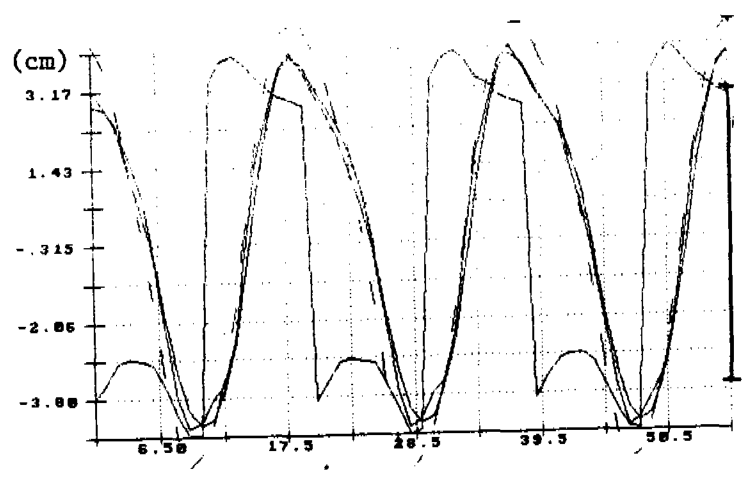
L=80m
S=10.81cm

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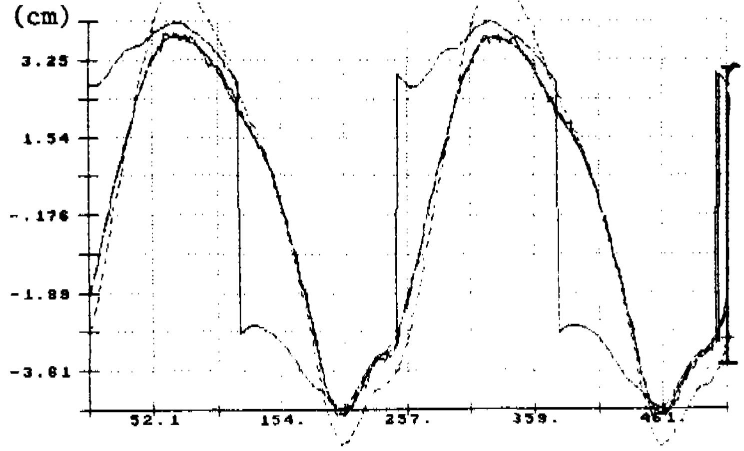
L=80m
S=10.81cm

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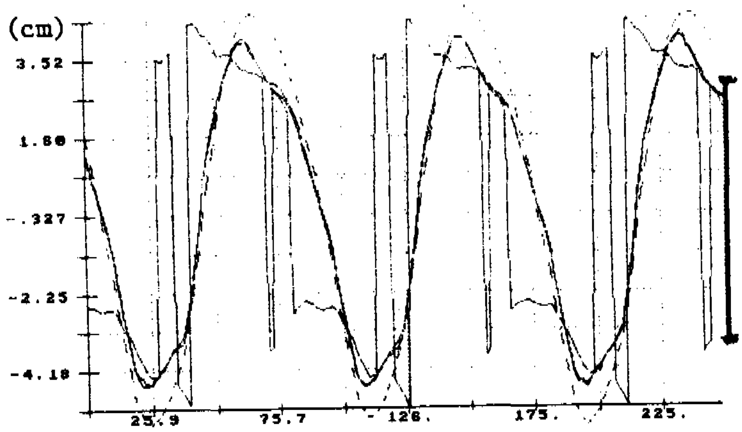
L=80m
S=11cm

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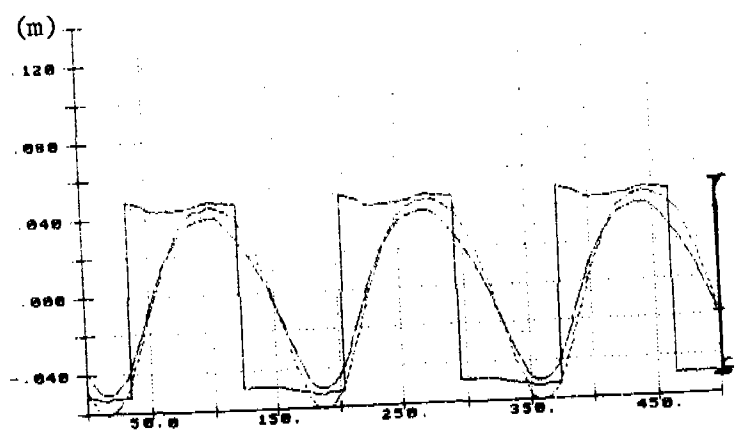
L=80m
S=11cm

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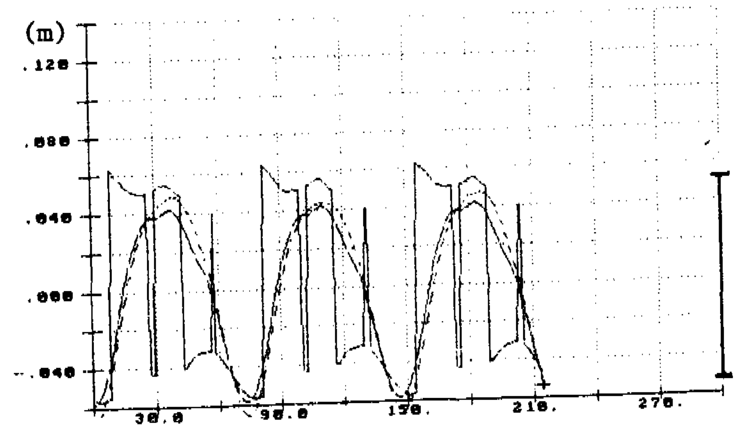
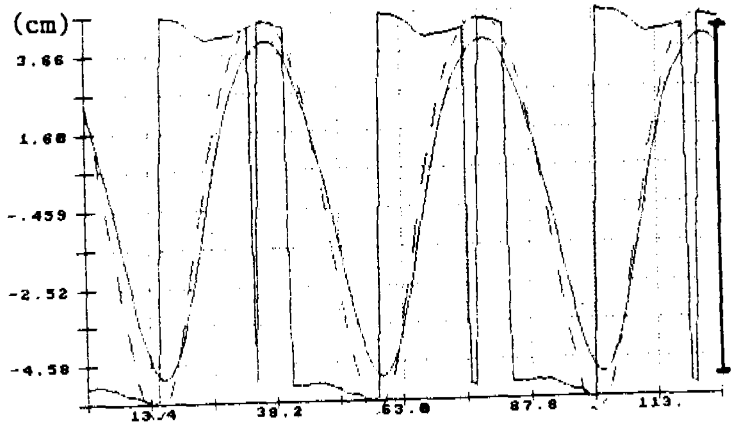
S=11cm




S01MA901
f=1.6Hz
L=96m
S=11cm

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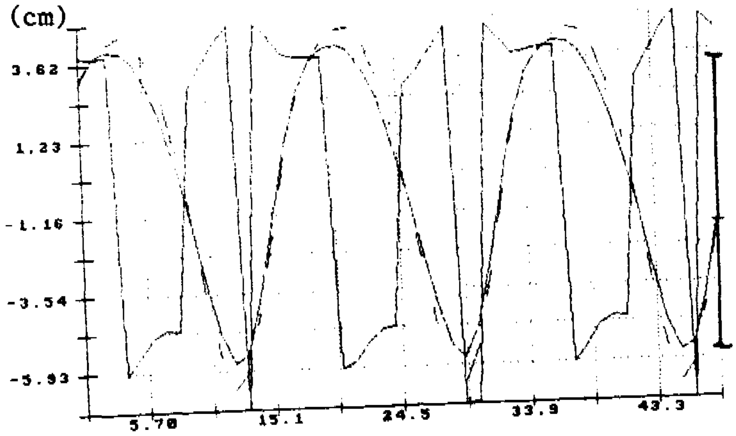


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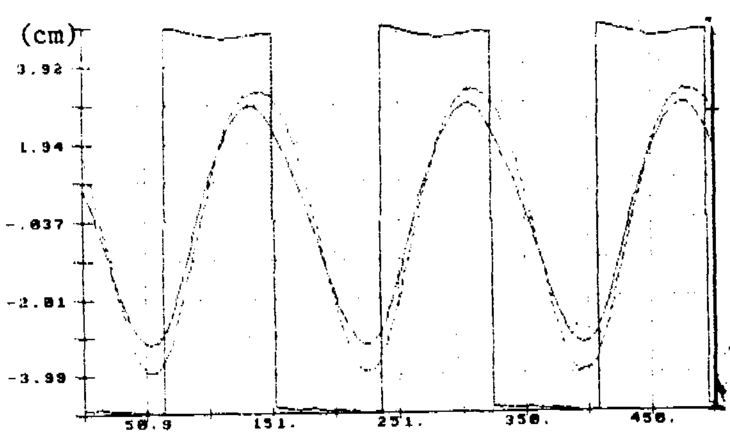


S001MA903
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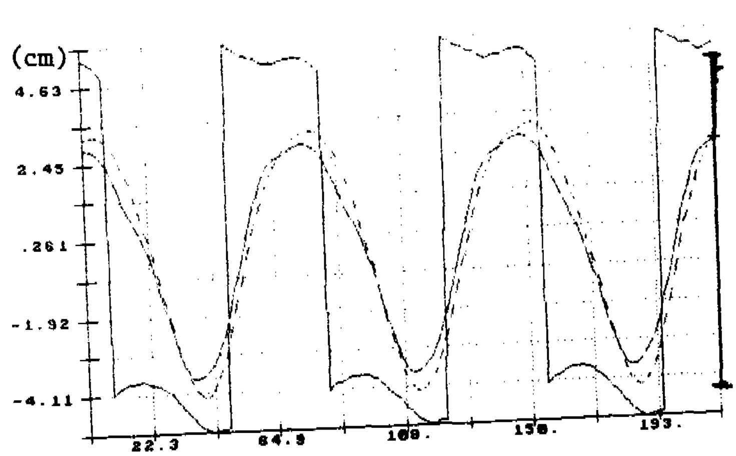
L=96m
S=7.3cm




S01MA903
f=1.6Hz
L=96m
S=7.3cm

Appendix C: Damping by friction

1: Wall shear stress in an oscillating flow (see [ZIE68])

The Navier-Stokes equation for an oscillating flow through a circular pipe is:

$$\frac{1}{\rho}\cdot\frac{\partial p}{\partial x}+\frac{\partial u}{\partial t}-\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial u}{\partial r}\right)=0 \qquad (1)$$

Suppose an uniform pressure in radial direction:

$$\frac{\partial p}{\partial x}=-\rho A\cdot e^{i\omega t} \qquad (2)$$

The solution of (1) and (2) is:

$$u(r,t)=-i\frac{A}{\omega}\cdot e^{i\omega t}\left\{1-\frac{J_0\left(i^{3/2}\cdot\alpha\cdot\frac{r}{R}\right)}{J_0\left(i^{3/2}\alpha\right)}\right\} \qquad (3)$$

using the so-called Woomersley parameter $\alpha$:

$$\alpha=\frac{R}{\sqrt{\nu/\omega}} \qquad (4)$$

Analysing (3) shows that all effects due to viscosity are located in a thin film along the wall. The so-called viscous penetration depth is $\sqrt{(\nu/\omega)}$ . In our case (SWN81):

$$R=\frac{1}{2}\cdot\left\{\frac{1}{2}(0{,}036-0{,}025)\right\}=0{,}00275\ m$$
$$\nu=10^{-6}\ m^2s^{-1}$$
$$5<\omega<60\ s^{-1}$$

So the viscous penetration depth is: $10^{-4}<\sqrt{\frac{\nu}{\omega}}<4.10^{-4}\ m$

The Woomersley parameter is: $6{,}9<\alpha<27$

Simplification of (3) for $\alpha \gg 1$ ($\alpha > 40$) results in:

$$u(r,t) = -\frac{iA}{\omega} \cdot e^{i\omega t} \left\{ 1 - \sqrt{\frac{R}{r}} \cdot e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}(R-r)} \right\} \qquad (5)$$

At $\alpha = 6.9$ the error in (5) is: magnitude max. 20%
phase $14^0$

The wall shear stress can be calculated from:

$$\tau_0 = \rho\nu \left.\frac{\partial u}{\partial y}\right|_{y=0} \qquad (6)$$

where y is the distance from the wall.

So:

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = -\left.\frac{\partial u}{\partial r}\right|_{r=R} = \frac{iA}{\omega} \cdot e^{i\omega t} \left\{ \frac{1}{2r}\sqrt{\frac{R}{r}} \cdot e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}(R-r)} + \right.$$

$$\left.\left. - \sqrt{\frac{R}{r}}(1+i)\sqrt{\frac{\omega}{2\nu}} \cdot e^{-(1+i)\sqrt{\frac{\omega}{2\nu}}(R-r)} \right\}\right|_{r=R} =$$

$$= \frac{iA}{\omega} e^{i\omega t} \left\{ \frac{1}{2R} - (1+i)\sqrt{\frac{\omega}{2\nu}} \right\} =$$

$$= \underbrace{-\frac{iA}{\omega} \cdot e^{i\omega t}}_{\approx u(r,t)} \cdot \frac{1}{R\sqrt{2}} \left\{ (1+i)\alpha - \underbrace{\frac{1}{\sqrt{2}}}_{\text{neglected}} \right\} \qquad (7)$$

So: $$\tau_{0,dyn} = \rho\nu \cdot u \frac{R/\sqrt{\nu/\omega}}{R\sqrt{2}}(1+i) =$$

$$= u\rho\sqrt{\frac{\nu\omega}{2}}(1+i) \qquad (8)$$

2: Impedance corrected for wall shear stress

The equation of momentum and the continuity equation are (see section 5.3):

$$\frac{1}{\rho}\cdot\frac{\partial p'}{\partial x}+\left\{\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right\}v'+\frac{1}{\rho}\cdot\frac{4\,\tau_{0,dyn}}{D}=0 \qquad (9)$$

$$\frac{1}{\rho}\cdot\left\{\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right\}p'+a^2\frac{\partial v'}{\partial x}=0 \qquad (10)$$

The wall shear stress can be written as:

$$\tau_{0,dyn}=\begin{cases} v'\rho\sqrt{\frac{\nu\omega}{2}}(1+i) \text{ when } \omega>0 & (11)\\ v'\rho\sqrt{\frac{\nu|\omega|}{2}}(1-i) \text{ when } \omega<0 & (12)\end{cases}$$

Differentiate (9) to t and (10) to x; eleminate $\frac{\partial^2 p'}{\partial t\partial x}$ :

$$\frac{\partial^2 v'}{\partial t}+K\frac{\partial v'}{\partial t}-a^2\frac{\partial^2 v'}{\partial x^2}=0 \qquad (13)$$

where: $K=\frac{4}{D}\sqrt{\frac{\nu\omega}{2}}(1+i)$ when $\omega>0$

$K=\frac{4}{D}\sqrt{\frac{\nu|\omega|}{2}}(1-i)$ when $\omega<0$

B.C.1: $x=0$; $p=const$; $\left.\frac{\partial p'}{\partial t}\right|_{x=0}=0 \Rightarrow \left.\frac{\partial v'}{\partial x}\right|_{x=0}=0$ (14)

B.C.2: $x=-L$; $v'(x=-L)=\bar{V}$ (imposed) (15)

Substitute the flat wave solution:

$$V'=\tilde{V}e^{i(\omega t+kx)} \qquad (16)$$

in (13) results in:

$$-\omega^2-i\omega K+a^2k^2=0 \qquad (17)$$

Giving the dispersion relation:

$$k^2=\frac{\omega^2}{a^2}\left(1+i\frac{K}{\omega}\right) \qquad (18)$$

In our case $\frac{|K|}{|\omega|} << 1$ is valid, so:

$$k = \pm \frac{|\omega|}{a}\left(1 + i\,\frac{K}{2\omega}\right) \qquad (19)$$

Resulting in: $$k^r = -\frac{|\omega|}{a}\left(1 - i\,\frac{1}{D}\sqrt{\frac{2\nu}{|\omega|}}\right) \qquad (20)$$

$$k^l = \frac{|\omega|}{a}\left(1 - i\,\frac{1}{D}\sqrt{\frac{2\nu}{|\omega|}}\right) \qquad (21)$$

Using b.c.1:

$$\left.\frac{\partial v'}{\partial x}\right|_{x=0} = \frac{\partial}{\partial x}\left(\tilde{v}^l e^{i(\omega t + k^l x)} + \tilde{v}^r e^{i(\omega t + k^r x)}\right)\Big|_{x=0} = 0 \qquad (22)$$

results in: $$i k^l \tilde{v}^l + i k^r \tilde{v}^r = 0 \Rightarrow \tilde{v}^l = -\frac{k^r}{k^l}\tilde{v}^r \approx \tilde{v}^r \qquad (23)$$

Using b.c.2: $$\tilde{v}^r e^{i(\omega t - k^r L)} + \tilde{v}^l e^{i(\omega t - k^l L)} = \tilde{V} e^{i\omega t} \qquad (24)$$

So: $$\tilde{v}^r = \frac{\tilde{V}}{e^{-ik^r L} + e^{-ik^l L}} \qquad (25)$$

From equation (9), (10) and (16) it follwos that:

$$p' = \tilde{p}\, e^{i(\omega t + kx)} \qquad (26)$$

having the same disperion relation as (18).

From equation (10) it follows that:

$$-i\omega \cdot \frac{1}{\rho} \cdot \tilde{p}^{l,r} - i k^{l,r} a^2 \tilde{v}^{l,r} = 0$$

$$\tilde{p}^{l,r} = -\rho a \cdot \frac{a k^{l,r}}{\omega}\, \tilde{v}^{l,r} \qquad (27)$$

When $\omega>0$ and $k^{\ell} \simeq |\omega|/a$ ; $k^{r} \simeq -|\omega|/a$ :

$$\tilde{p}^{\ell} = -\rho a \tilde{v}^{\ell} = -\rho a \tilde{v}^{r} ; \quad \tilde{p}^{r} = \rho a \tilde{v}^{r} \tag{28}$$

So finally:

$$Z(\omega>0) = \frac{\rho a \tilde{v}^{r}\left(-e^{i(\omega t - k^{\ell} L)} + e^{i(\omega t - k^{r} L)}\right)}{\tilde{V} \cdot e^{i\omega t}} =$$

$$= \rho a \frac{e^{-ik^{r}L} - e^{-ik^{\ell}L}}{e^{-ik^{r}L} + e^{-ik^{\ell}L}} \tag{29}$$

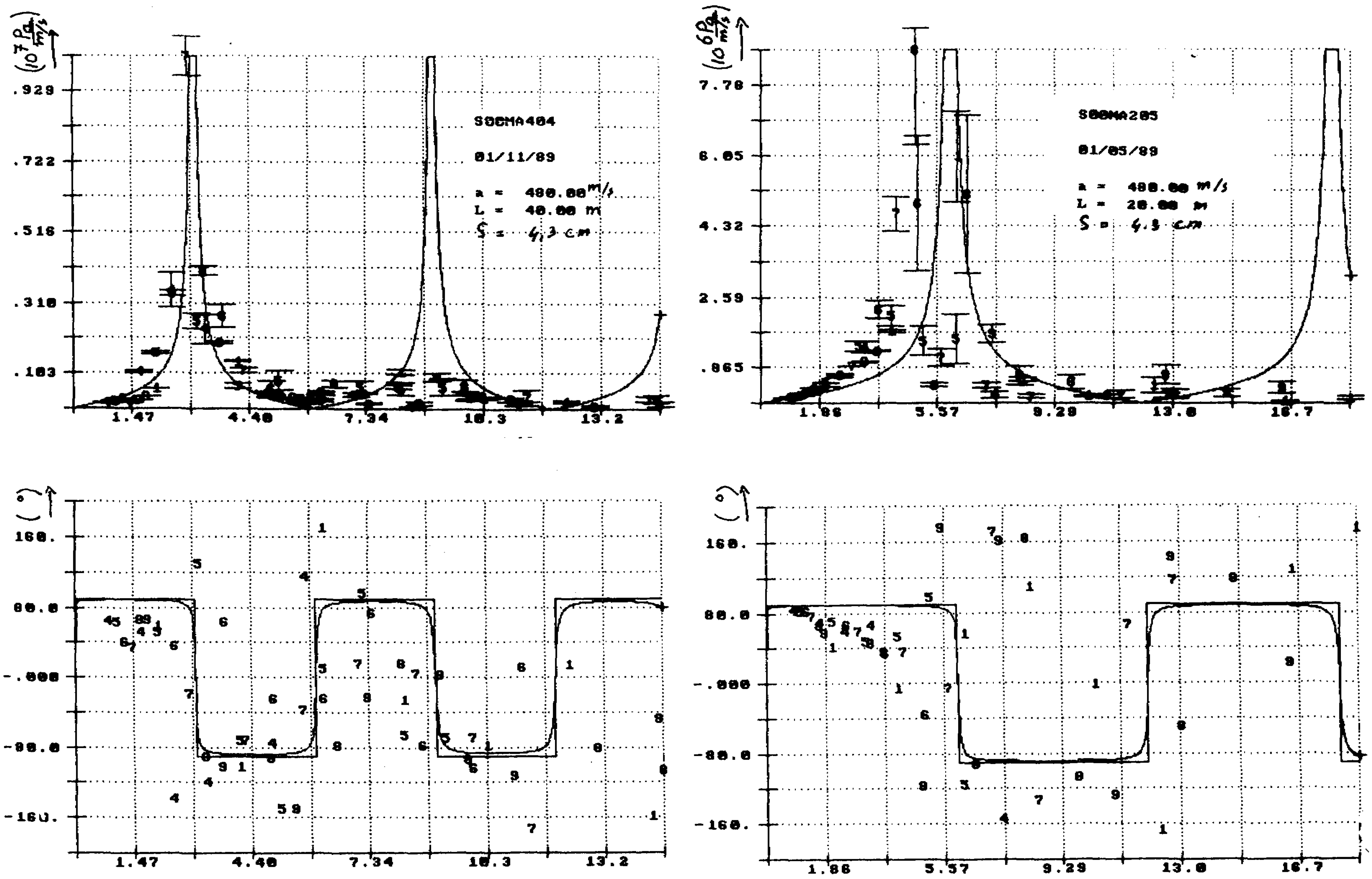
where: $k^{\ell} = \frac{|\omega|}{a}\left(1 - \frac{i}{D}\sqrt{\frac{2\nu}{|\omega|}}\right)$

$k^{r} = -\frac{|\omega|}{a}\left(1 - \frac{i}{D}\sqrt{\frac{2\nu}{|\omega|}}\right)$

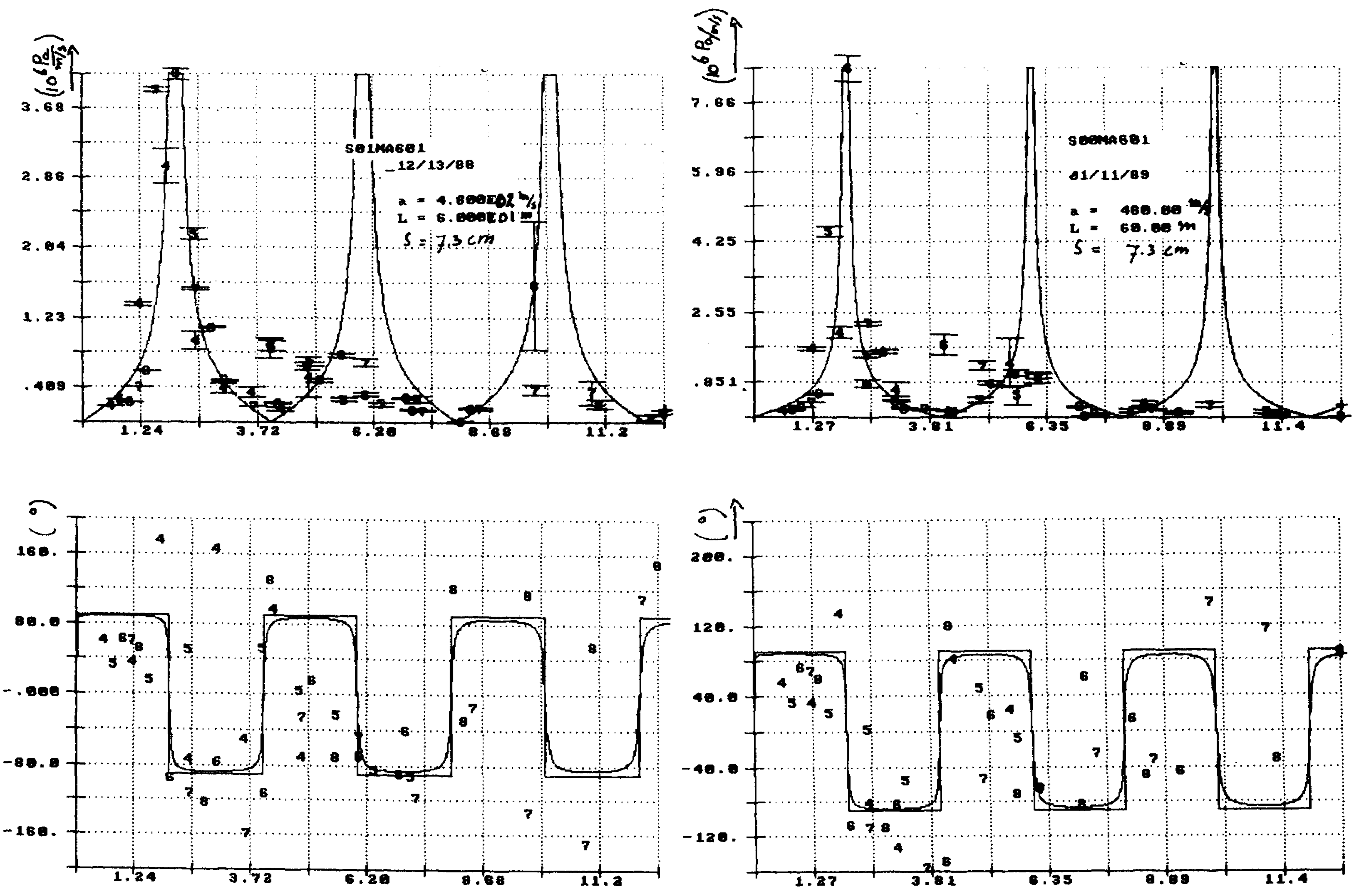
Appendix D: Plots of the impedance (SWN81)

In the following figures the magnitude of the impedance ($\frac{Pa}{m/s}$) and the argument of the impedance ( $^0$) are plotted versus the frequency (Hz). The experimental data that is processed is listed in the table.

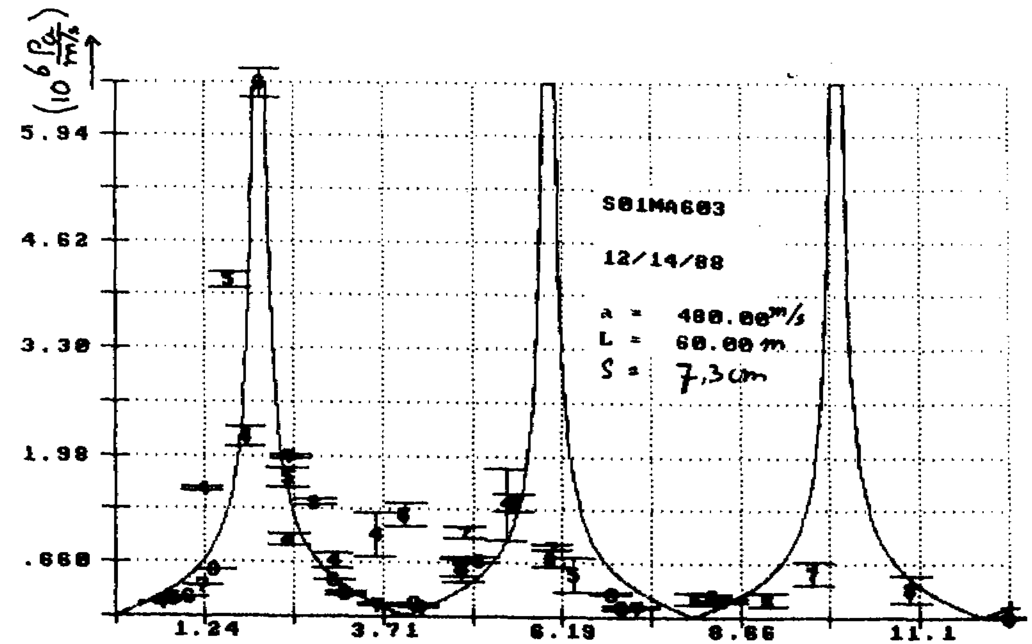
| exp.no. | pumping depth L | piston stroke | pumping frequency range |
|---|---|---|---|
| | (m) | (cm) | (Hz) |
| S00MA205 | 20 | 4.3 | 0.8 - 2 |
| S00MA404 | 40 | 4.3 | 0.8 - 2 |
| S00MA601 | 60 | 7.3 | 0.6 - 1.4 |
| S01MA601 | 60 | 7.3 | 0.6 - 1.4 |
| S01MA602 | 60 | 4.3 | 0.6 - 1.8 |
| S01MA603 | 60 | 7.3 | 0.6 - 1.4 |
| S01MA604 | 60 | 4.3 | 0.6 - 1.8 |
| S00MA803 | 80 | 7.3 | 0.6 - 1.6 |
| S00MA805 | 80 | 10.8 | 0.6 - 1.4 |
| S00MA806 | 80 | 11 | 0.6 - 1.4 |
| S01MA901 | 96 | 11 | 0.6 - 1.6 |
| S01MA902 | 96 | 11 | 0.6 - 1.6 |
| S01MA903 | 96 | 7.3 | 0.6 - 1.6 |

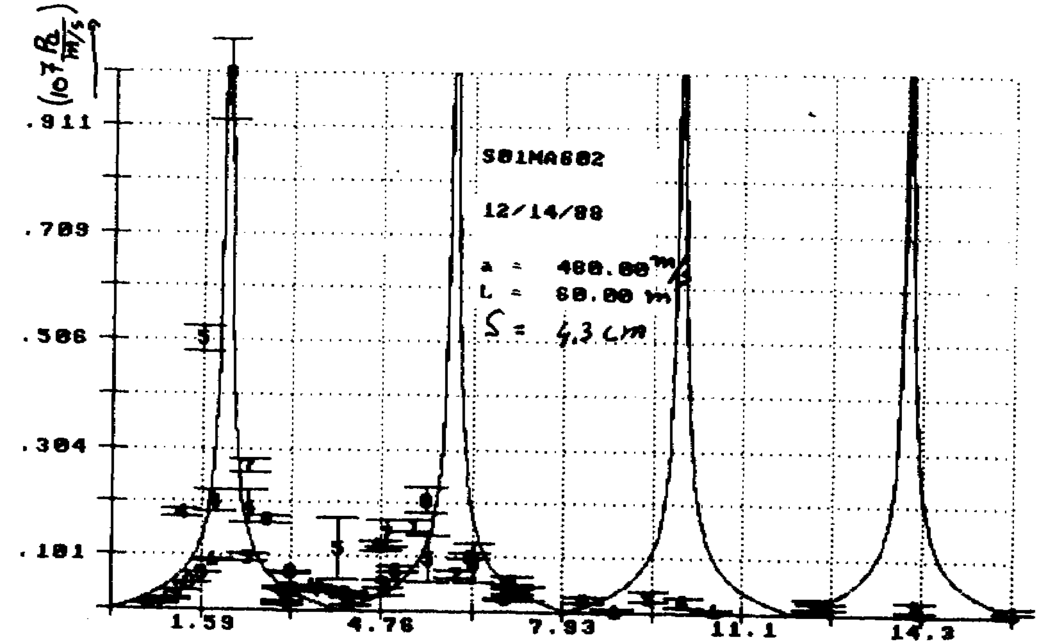


SBCMA404
01/11/89
a = 480.00 m/s
L = 40.00 m
ς = 4,3 cm
.929
.722
.516
.310
.103
1.47
4.40
7.34
10.3
13.2
SBCMA205
01/05/89
a = 480.00 m/s
L = 20.00 m
ς = 4.3 cm
7.78
6.05
4.32
2.59
.865
1.86
5.57
9.29
13.0
16.7
160.
80.0
-.000
-80.0
-160.

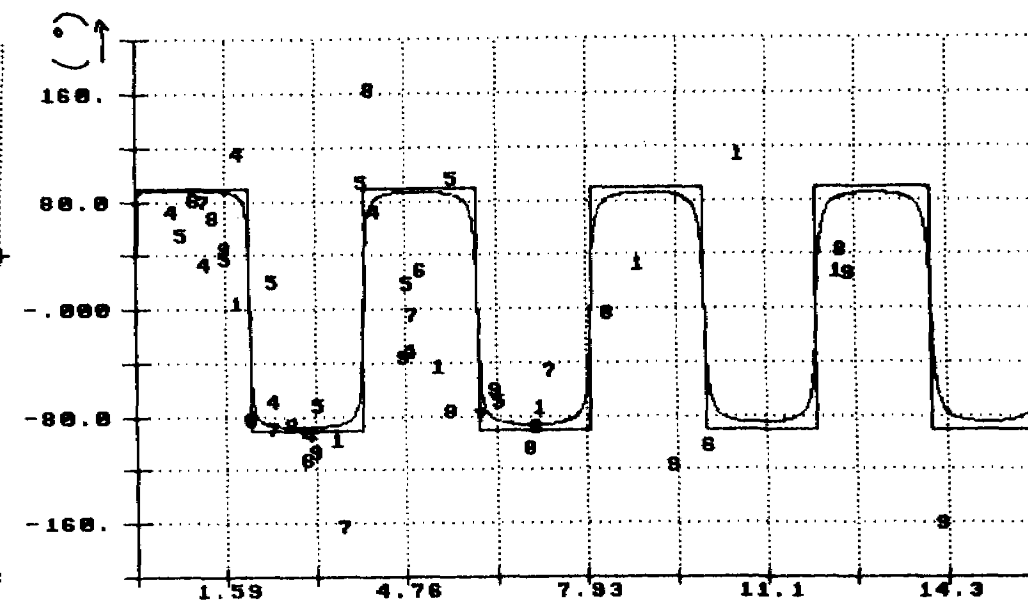


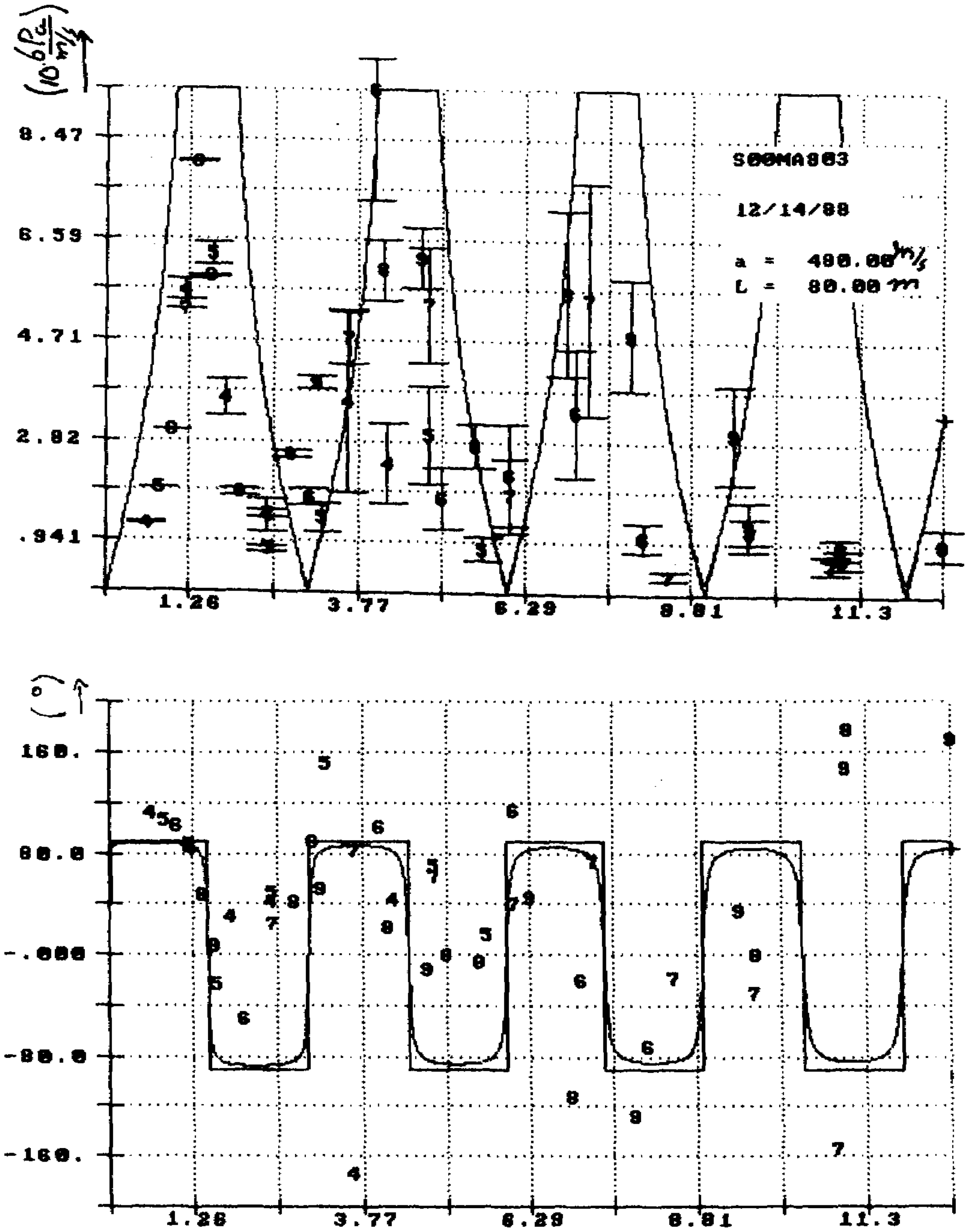
S81MA681
12/13/88
a = 4.000E02 m/s
L = 6.000E01 m
S = 7.3 cm
3.68
2.86
2.04
1.23
.409
1.24
3.72
6.20
8.68
11.2
160.
80.0
-.000
-80.0
-160.
S80MA681
01/11/89
a = 400.00 m/s
L = 60.00 m
S = 7.3 cm
7.66
5.96
4.25
2.55
.851
1.27
3.81
6.35
8.89
11.4
200.
120.
40.0
-40.0
-120.


$(10^6 \frac{Pa}{m/s})$
5.94
4.62
3.30
1.98
.660
1.24
3.71
6.19
8.66
11.1
S01MA603
12/14/88
a = 400.00 m/s
L = 60.00 m
S = 7,3 cm

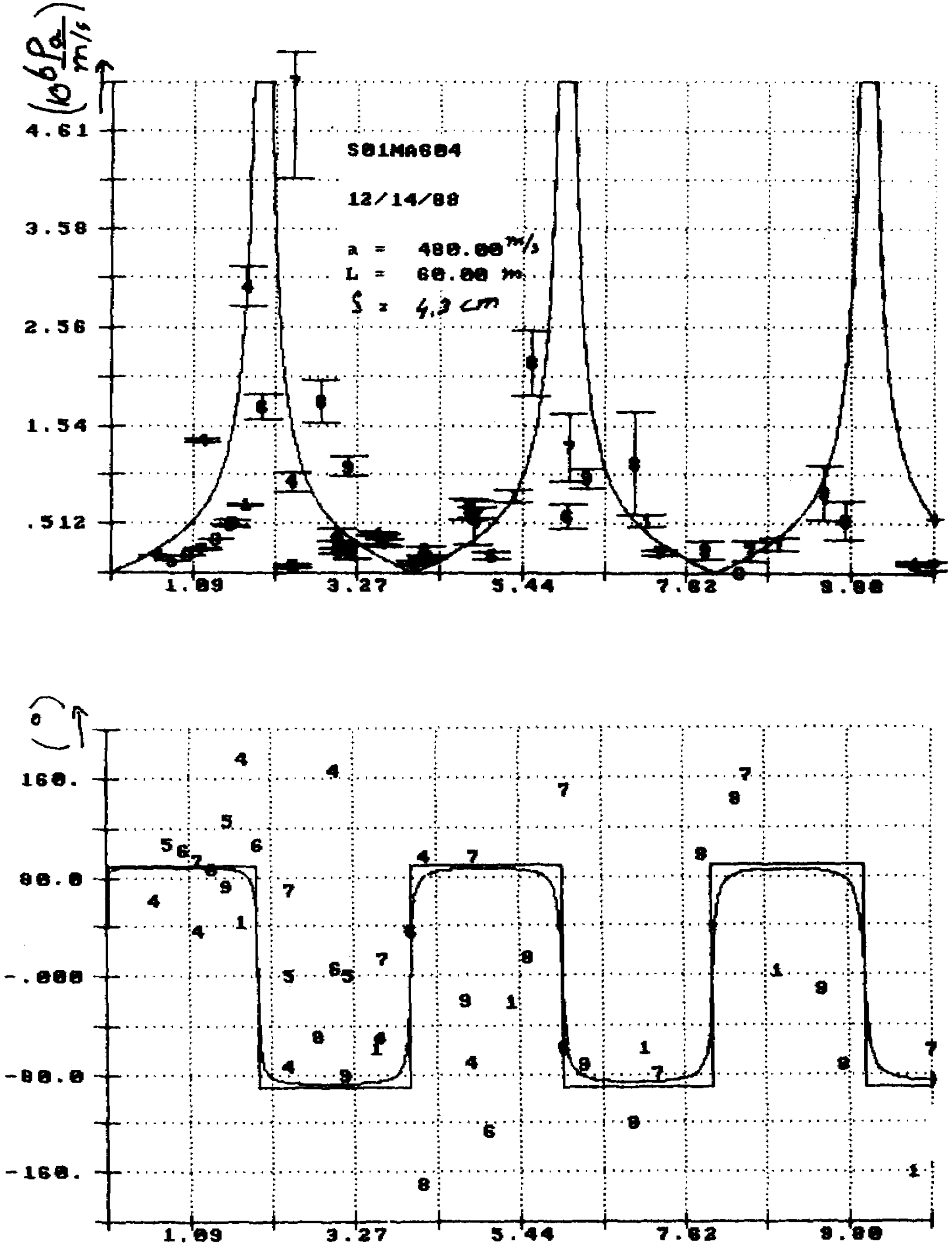

$(10^7 \frac{Pa}{m/s})$
.911
.709
.506
.304
.101
1.59
4.76
7.93
11.1
14.3
S01MA602
12/14/88
a = 400.00 m/s
L = 60.00 m
S = 4,3 cm


(°)
200.
120.
40.0
-40.0
-120.
1.24
3.71
6.19
8.66
11.1

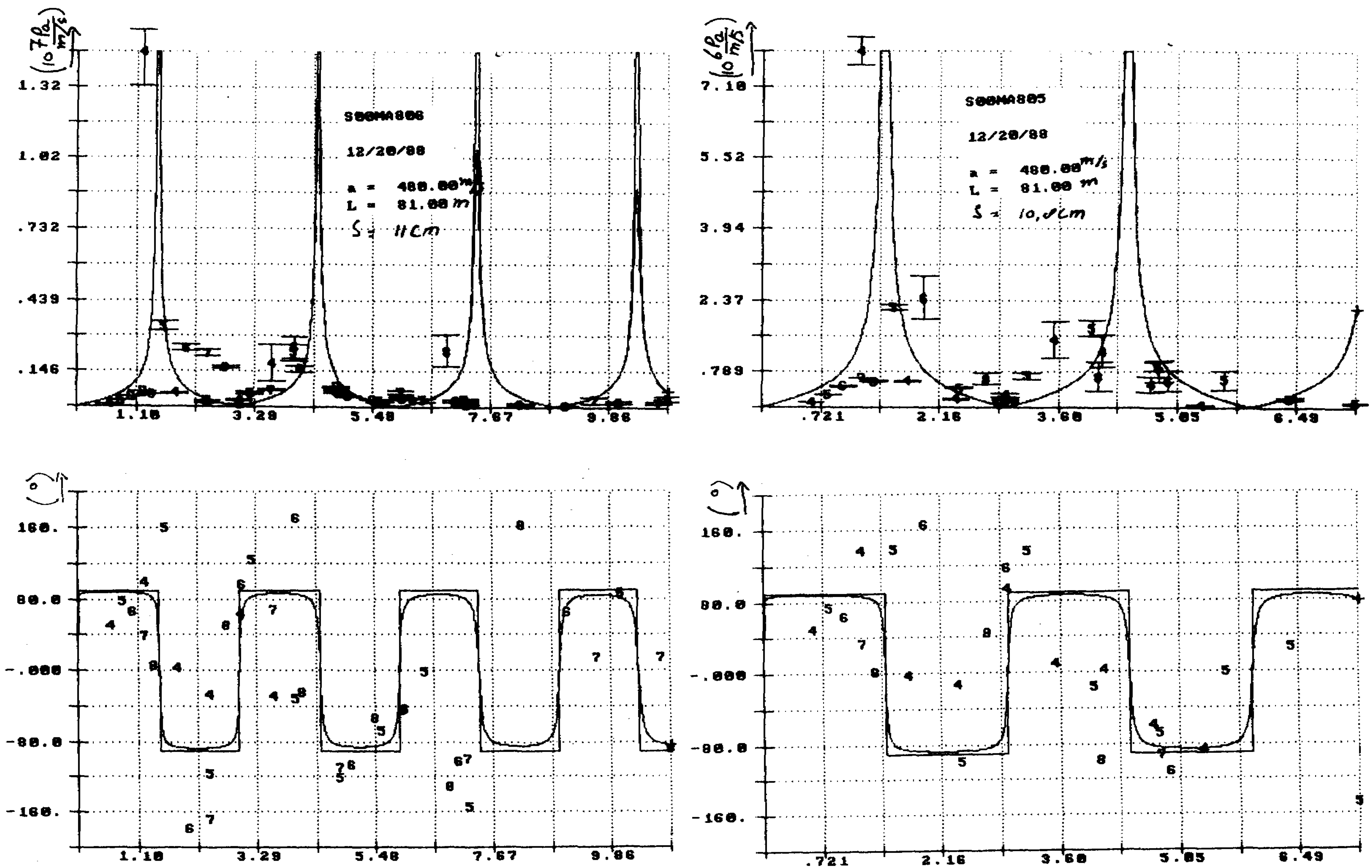

(°)
160.
80.0
-.000
-80.0
-160.
1.59
4.76
7.93
11.1
14.3



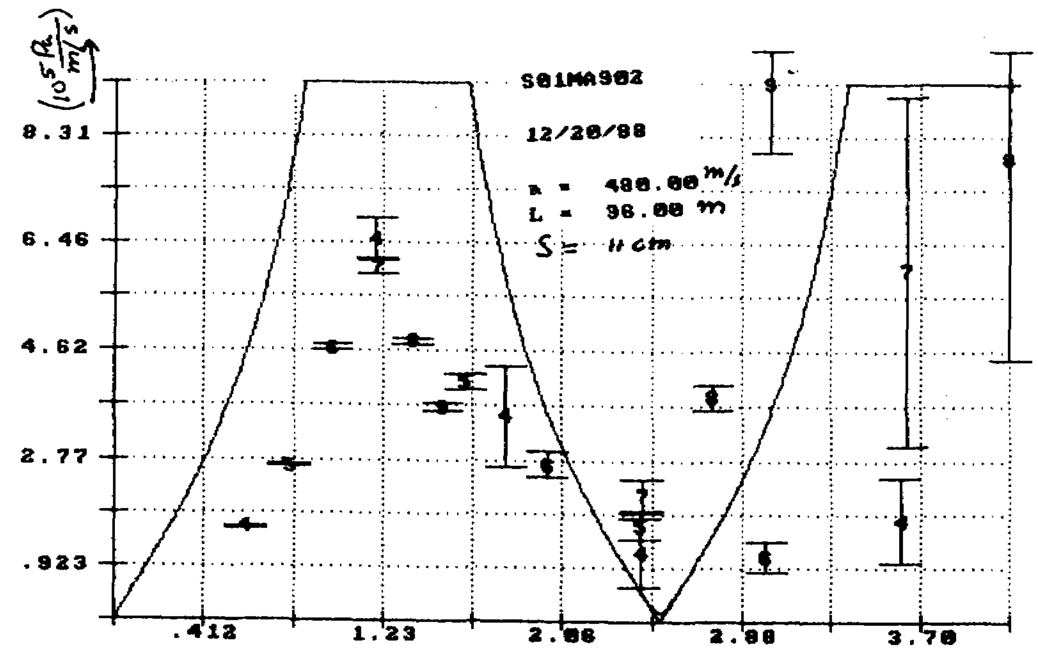
S00MA803
12/14/88
a = 400.00 m/s
L = 80.00 m
8.47
6.59
4.71
2.82
.941
1.26
3.77
6.29
8.81
11.3
160.
80.0
-.000
-80.0
-160.

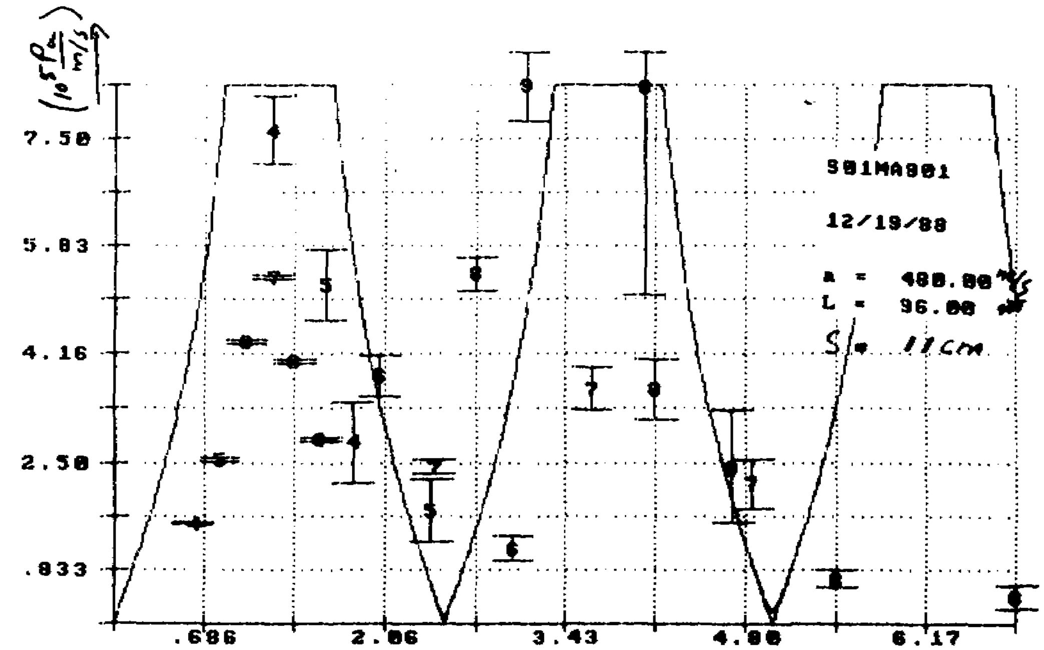



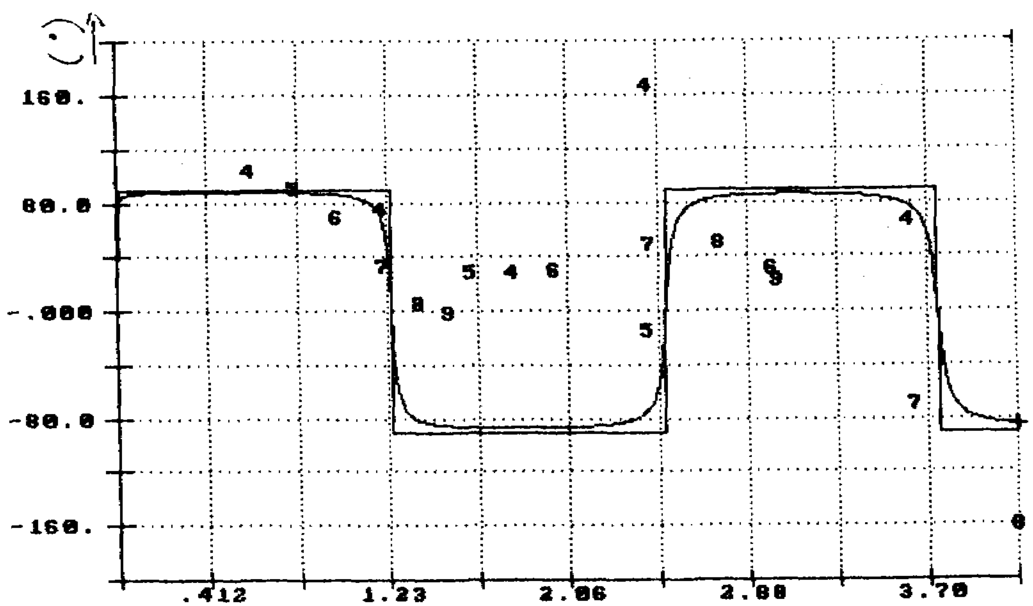
S01MA604
12/14/88
a = 400.00 m/s
L = 60.00 m
δ = 4,3 cm
4.61
3.58
2.56
1.54
.512
1.09
3.27
5.44
7.62
9.80
160.
80.0
-.000
-80.0
-160.

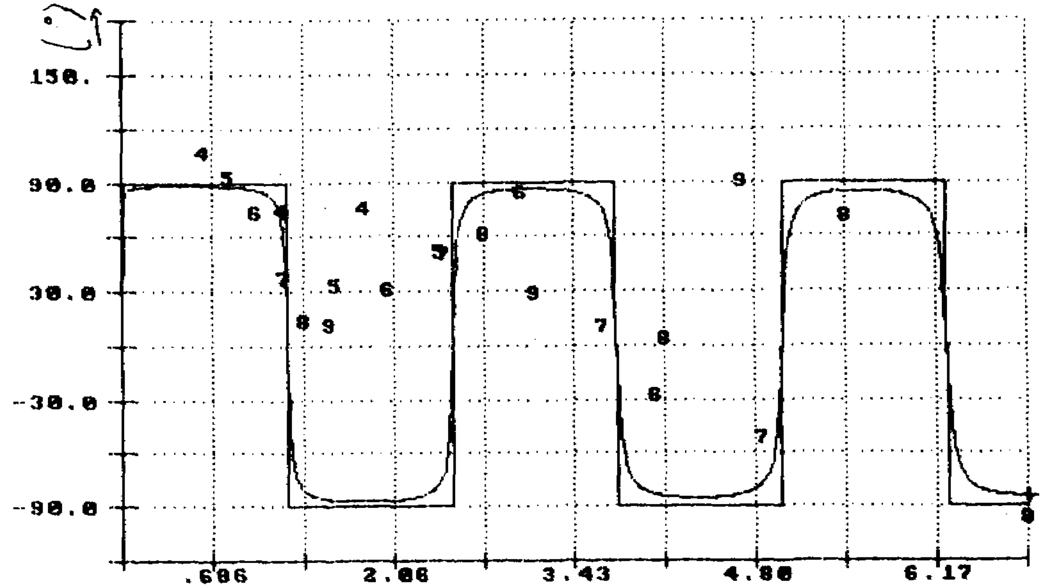


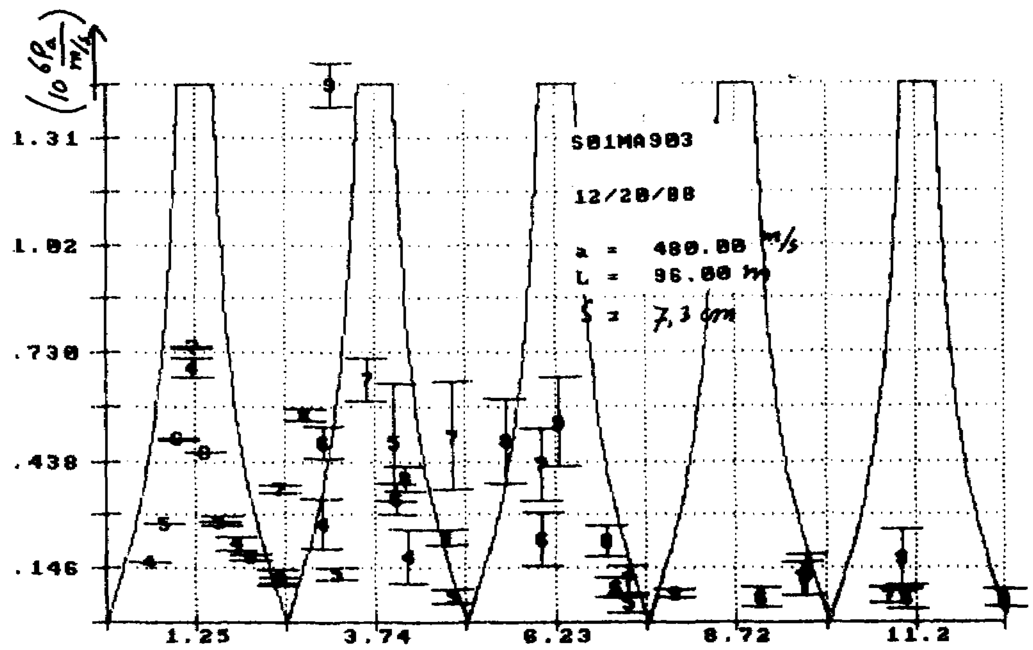
SOOMA806
12/20/88
a = 400.00 m/s
L = 81.00 m
S = 11 cm
1.32
1.02
.732
.439
.146
1.10
3.29
5.48
7.67
9.86
SOOMA805
12/20/88
a = 400.00 m/s
L = 81.00 m
S = 10,0 cm
7.10
5.52
3.94
2.37
.789
.721
2.16
3.60
5.05
6.49
160.
80.0
-.000
-80.0
-160.

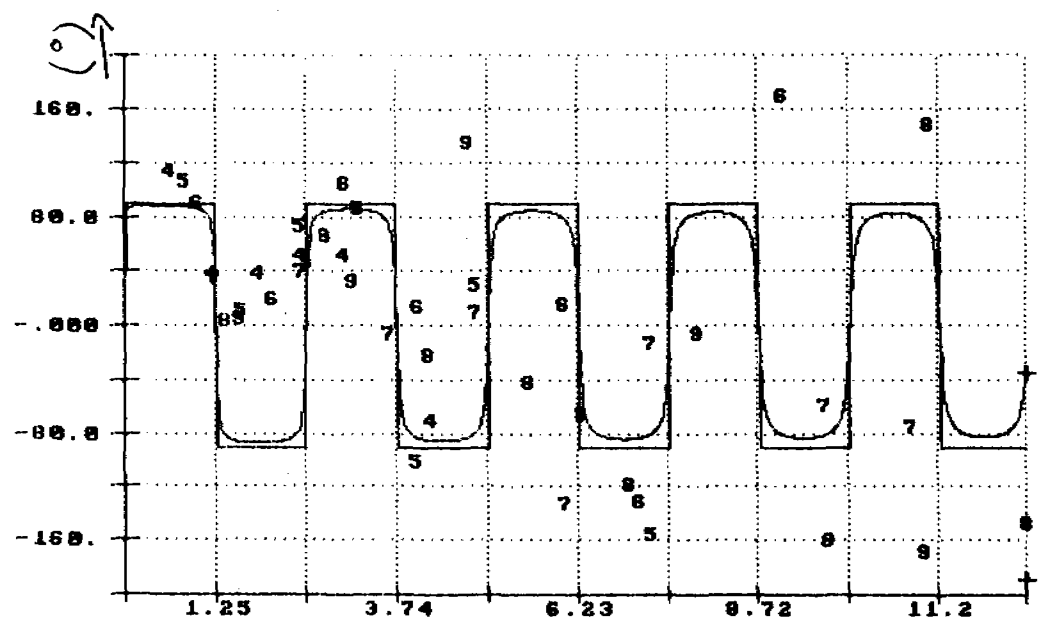

(10^5 Pa/(m/s))
S01MA902
12/20/88
a = 480.00 m/s
L = 96.00 m
S = 11 cm
8.31
6.46
4.62
2.77
.923
.412
1.23
2.06
2.88
3.70


(10^5 Pa/(m/s))
S01MA901
12/19/88
a = 480.00 m/s
L = 96.00 m
S = 11 cm
7.50
5.83
4.16
2.50
.833
.686
2.06
3.43
4.80
6.17


(°)
160.
80.0
-.000
-80.0
-160.
.412
1.23
2.06
2.88
3.70


(°)
150.
90.0
30.0
-30.0
-90.0
.686
2.06
3.43
4.80
6.17


$\left(10^6 \frac{Pa}{m/s}\right)$
1.31
1.02
.730
.438
.146
1.25
3.74
6.23
8.72
11.2
S01MA903
12/20/88
a = 480.00 m/s
L = 96.00 m
ξ = 7,3 cm


(°)
160.
80.0
-.000
-80.0
-160.
1.25
3.74
6.23
8.72
11.2