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**INTELLIGENT PROGRAMS FOR PUBLIC INVESTMENTS
IN DEVELOPING COUNTRIES**

by

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This paper is primarily concerned with public investments in community water supply and sanitation systems. Many of the concepts herein apply to other fields of investment as well. In most developing countries, water and sanitation planning is done by a central agency of the national government which needs to decide such things as: when to initially construct and subsequently expand community systems, how large to make them, what level of service to provide, whether to make individual house connections to the water and sanitation systems or construct public taps and latrines, what size pipe diameters to use in networks, whether to construct separate community systems or regional facilities, and what prices to charge the users.

To answer these and similar questions, typical practice is to rely heavily on the experience, recommendations, and standards of the industrialized countries. For example, system capacity is often sufficient to meet demands for 20 or more years into the future, as is done in the U.S. and Europe; house connections are preferred to public taps, as in the high-income countries; the design average daily per capita flow is usually 150 liters or more; and minimum pipe sizes are seldom less than 2 inches in water networks or 6 inches in sewers.

The standards of the industrialized countries are often unsuitable for developing countries because the conditions between the two are so different. In the industrialized countries, labor is typically very expensive compared to capital;

the social rate of discount is comparatively low, as is population growth and associated rates of demand; industrialized countries are more concerned with capacity expansions of existing facilities than with entirely new construction; the high income countries have greater fiscal autonomy and ability to pay, making them less dependent on government subsidies; and with investment at the local rather than national level of government, capital budgeting in industrialized countries is less restrictive.

It is widely accepted that use of planning and design standards from the industrialized countries by the developing ones is basically unwise and can lead to serious misallocation of scarce resources. Instead, it is seen that investment decisions must be tailored to local economic, social and other conditions. Without question, for a very large class of public investments, the most appropriate method for obtaining such appropriate solutions is through use of mathematical models and intelligent computer programs.

Intelligent programs are computer codes that can assist the planning, design, and operation of systems; they assist decision making, which involves selecting the best course of action from among alternatives. The main advantage of intelligent programs is that they facilitate the rapid and efficient screening of options.

The main categories of intelligent programs of interest in this paper are optimization and simulation codes. Numerous textbooks describe the characteristics of these programs; hence, the following discussion touches only the highlights.

An optimization program begins with a mathematical model in which the user identifies a set of decision variables (i.e., the things about which decisions are to be made). Next, a mathematical objective function is written in terms of them, which is to be optimized. In the field of public investment, the objective function is usually a statement of project cost, which is to be minimized. Additional mathematical statements in terms of the decision variables are written for the scarce resources and other constraints on the problem, thereby completing the model, which is then read into the computer.

It is unlikely that an optimization program would ever be developed with the intention of obtaining only a single solution. Rather, even if the model were to be applied to one specific situation, the user would want to solve it several times and with different parameter values, if for no other reason than to account for uncertainty. The next phase of the optimization program therefore consists of the user reading into the computer appropriate numerical values for the parameters in the model. This is followed by use of a mathematical optimization algorithm to solve the problem, which depends on the mathematical form of the objective function and constraints. Solution consists of

obtaining optimal numerical values for the decision variables followed by backsubstituting them into the objective and constraints to obtain values for these functions. The search for the best solution using the algorithm is automatic, and the user rarely sees the different alternatives during the screening process, only the final result.

A simple example illustrates the optimization program described above. Suppose the user needs to determine the optimal width x and height y that maximize the area A of a rectangle subject to a constraint that the perimeter equals P . The first step is to develop the model in parametric terms.

$$\text{Maximize } A = xy \quad (1)$$

$$\text{Subject to } 2x + 2y = P \quad (2)$$

Next, the user supplies numerical values for the parameters; in this case $P = 100$, which is read into the program. The computer then converts the problem to appropriate form for optimization. Replacing P by 100 in (2) and solving for x results in an expression in terms of y that can be substituted into (1). The final resulting optimization problem is:

$$\text{Maximize } 50y - y^2 \quad (3)$$

This is an unconstrained nonlinear maximization problem for which various standard solution algorithms exist. Using one of them, the optimal value of y is found by the computer to be 25. Backsubstituting into (2), x is found to be 25, and from (1), the calculated maximum area is 625.

A simulation program similarly starts with identification of decision variables and mathematical statements of the objective function and constraints, which constitute the model and which is read into the computer. However, instead of using an automatic optimization technique to solve the problem, the user selects trial values of the decision variables and then lets the simulation program calculate the resulting values of objective function and constraints. Successive values are tried by the user until a satisfactory solution emerges (which may or may not be optimal). The approach, then, is for the user to ask "what if?" kinds of questions. Unlike optimization, the user investigates numerous alternatives in the search for the best solution. While optimization problems can always be formulated for solution by simulation, the reverse is not true because it is not always possible to find an automatic problem solver for any simulation problem.

Solving the previous rectangle problem by simulation, the first task is to select values for the decision variables width and height; assume the user selects $x = 10$ and $y = 20$. Substituting into (2), P is found to be 60, and from (1), $A =$

200. The user can immediately see that x and/or y need to be larger to satisfy the constraint on perimeter; the next trial is, say, $x = 20$ and $y = 20$ for which $P = 80$ and $A = 400$. Proceeding in this fashion with trial values of x and y , numerous solutions can be obtained, some of which may be close to the optimum, perhaps without ever finding the best values $x^* = 25$ and $y^* = 25$.

Optimization and Simulation Programs

The purpose of this section is to describe a few different optimization and simulation programs that have been written for microcomputers and which are being used by water and sewer agencies in developing countries and elsewhere. Some of these programs are being distributed by the World Bank and other institutions as a service to national, regional and local government agencies. Within the environmental engineering field, it would be possible to describe hundreds of programs that have been developed to assist investment planning. The following is only a small sample to provide an overview of the kinds of such programs that are available, how they are structured, and what they can do. While these models are specific to environmental engineering investments, they are typical of a much broader class of programs and therefore illustrate a general approach to programming and problem solving. All of the following programs have been written in BASIC or FORTRAN language for solution on the IBM/PC or its compatibles.

Branched Water Networks

It is not uncommon for 60% or more of a community's water system cost to be tied up in the pipe network. This component is not only expensive but is also hard to design. If selected pipe diameters are too large, network costs will be too high, making it difficult or impossible for the beneficiaries to pay and depriving other communities from obtaining water systems because of capital budgeting constraints. If diameters are too small (which is the most common situation), it will be impossible for users to obtain target quantities of water from the network because pressures will be too low. For these reasons, proper network design is extremely important.¹

Water networks are of two kinds, either with or without closed circuits. A network without closed circuits is branched, an example of which is shown in Fig. 1. In this network, water from the source enters at node 1 and is taken out at nodes 2 and 4 to meet demands. In branched networks, the flow in any link of the system can be easily determined by inspection or simple

¹ The proper design of sewers is similarly important. They are expensive, hard to design, and require that both over and under design be avoided. Optimization and simulation programs for sewers (similar to those for water networks) have been developed and are in use; they are not described herein. However, they are available from the World Bank (1985).

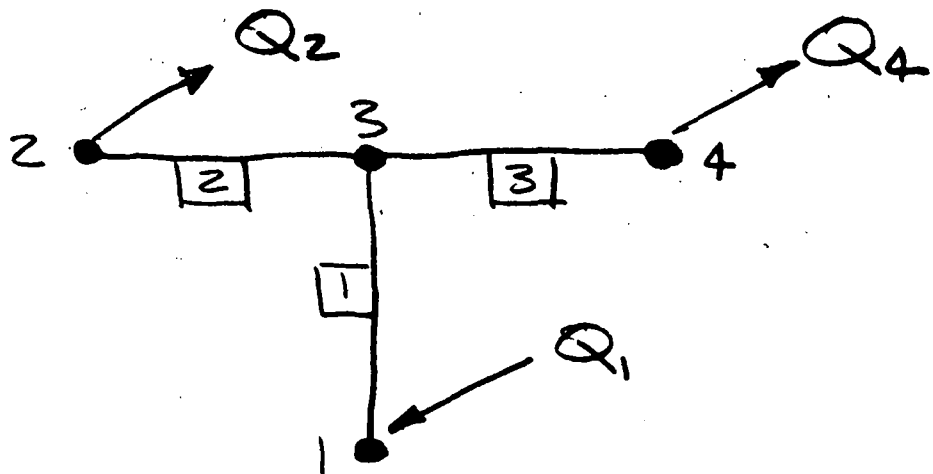


FIG 1 - BRANCHED NETWORK

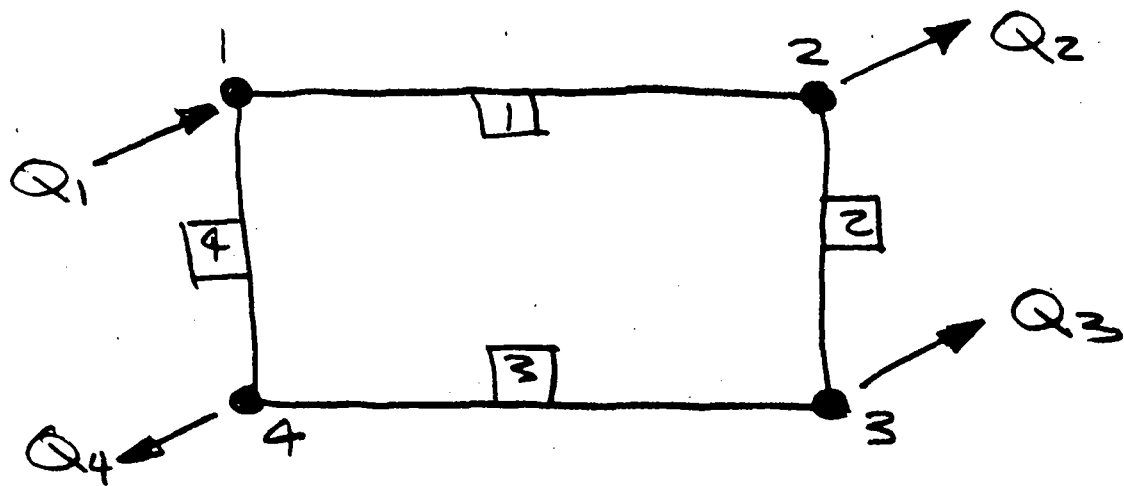


FIG 2 - LOOPED NETWORK

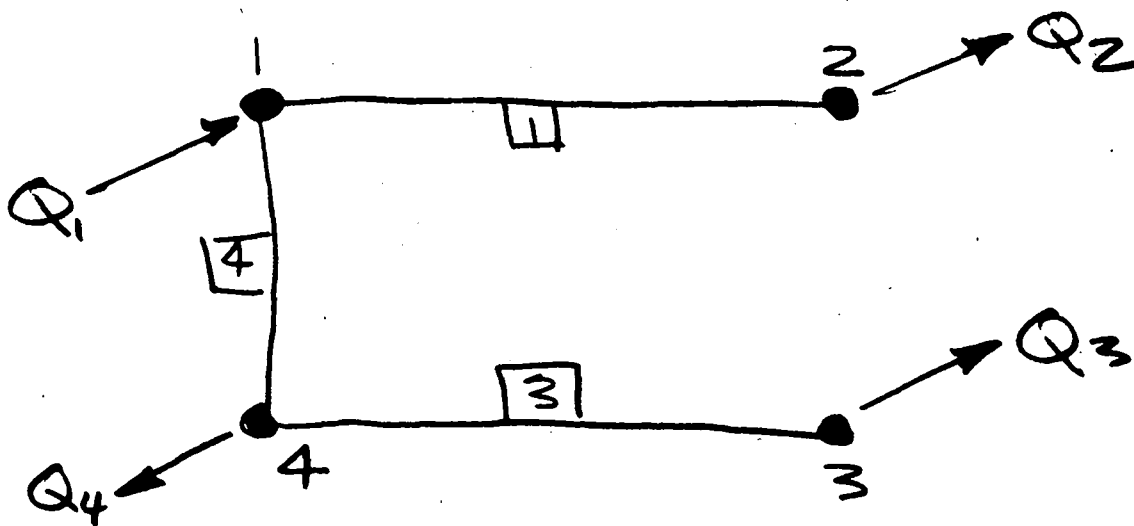


FIG 3 - PRIMARY (BRANCHED) NETWORK

calculation; e.g., the flow in link 1 is $Q_2 + Q_4$, and the flows in links 2 and 4 are Q_2 and Q_4 respectively.

Water flows from points of higher pressure to points of lower pressure; the highest pressure in Fig. 1 is at node 1. Pressure is reduced as water flows along a pipe due to friction. If the network in Fig. 1 were flat, the lowest pressures would be at nodes 2 and 4. There are well known equations that describe how the pressure in a pipe decreases as a function of the flow, pipe diameter, length, and pipe material (i.e., pipe roughness).

The basic problem of network design once the system has been laid out is to select the lengths and diameters of pipes in the system. About fifteen years ago, a mathematical model was developed showing how this could be done optimally using linear programming (LP) (Robinson, et al., 1976). In Fig. 1, assume that each link consists of pipes with different diameters laid end to end in series; the user must select the candidate diameters for these pipes (rules of thumb are available for this). The task of design is to determine the optimal length of each different diameter pipe in each link (lengths are the decision variables).

The objective function is an expression of total construction cost, which depends on the unknown lengths of the pipes with different diameters; the optimization problem is to find the pipe lengths that minimize cost. Two sets of

constraints complete the model. The first requires that the total length of all the proposed pipes in each link equal the length of the link; a separate equation is needed for each link (3 in the case of Fig. 1). The second set of constraints imposes hydraulic restrictions on the design. Assuming that the available pressure at source node 1 is known and that minimum allowable pressures have been specified for demand nodes 2 and 4, the first hydraulic constraint requires that the pressure at node 1 less the friction losses in pipes 1 and 2 result in a pressure at node 2 equal or greater than the specified minimum. A similar constraint is written for the hydraulic pathway between nodes 1 and 4; i.e., the pressure at node 1 less friction losses in pipes 1 and 3 must result in minimum specified pressure at node 4.

Because the network is branched and because the decision variables are pipe lengths, the objective function and constraints are all linear expressions.² This makes it possible to use LP as the optimization algorithm for solution. While the above description covers the most rudimentary considerations of network design, it is possible to expand the model to include such realistic elements as multiple sources of water supply, uneven terrain, constraints on maximum pressure (for steep

² This problem could have been formulated assuming the diameter of each link for its entire length is constant. In this case, pipe diameters would be the decision variables, and solution would require nonlinear programming.

elevations) as well as minimum pressure, and use of parallel pipes in links (to enable expansion of networks).

A computer program has been written for this model using the IBM/PC, which is being distributed by the World Bank (1985). With this program, the user has only to specify such things as network configuration, flows at demand nodes, ground elevations at nodes, target pressures at source and demand nodes, link lengths, candidate diameters, and the cost per unit length for each different diameter, in order to obtain an optimal solution (i.e., the pipe lengths that minimize cost). After reading these data into the computer, the program formulates the LP problem and then solves it using an LP algorithm. The user is informed of the final solution but does not see any of the intermediate solutions that are investigated in the search for optimality.

Looped Water Networks

The other kind of piped water networks has closed circuits; a system with a single circuit is shown in Fig. 2. Input flow Q_1 is at node 1, and demand flows are at nodes 2, 3 and 4. While it is clear that $Q_1 = Q_2 + Q_3 + Q_4$, it is impossible to easily determine the flows in the pipes, certainly not by inspection as was possible in the case of Fig. 1. This is a major difference between branched and looped networks, and it accounts for an entirely different mathematical modeling and computer programming approach to design.

Although various optimization models have been published for the design of looped networks, they are all nonlinear, which makes them difficult to solve and somewhat impractical for real world applications, especially for large complicated networks. The preferred approach is to use computer simulation, which is briefly described as follows.

Assume the designer knows the pressure at source node 1 and has minimum allowable values for the other demand nodes. The supply and demand flows are known, as are pipe lengths and ground elevations. Assume further that the designer selects trial diameters for all the pipes in the network. Using the empirical flow equations mentioned in the previous section (which are nonlinear), the computer calculates the pressures at the demand nodes. These pressures constitute a simulation of the system.

If the node pressures are far above the allowable minimum, the trial diameters are too large (i.e., friction losses are too small), but if the pressures are too low, the trial diameters are too small. It is not uncommon to obtain some pressures that are too high and others that are too low. Based on judgement, the user selects a new set of pipe diameters, reducing those that seem to be too large and enlarging those that are too small. Another submission to the computer will reveal whether the diameter adjustments have resulted in satisfactory pressures.

This process of selecting diameters and simulating pressures is continued until the user is satisfied. When to stop the trials is a matter of judgement; a mathematically optimal solution will probably never be found, but a very good solution with cost close to the minimum is likely. In this process of simulation, it is interesting to note that network cost is not usually considered after each trial, despite the fact that the engineering goal is to minimize cost. The supposition is that node pressures close to the minimum will result in a cost that is similarly near the minimum. Computer programs for simulating looped networks are available to national water supply and other agencies from the World Bank (1985).

Combined Optimization and Simulation

In the case of looped network design as described above, it follows that the better the judgement of the user in selecting initial pipe diameters, the fewer will be the trials needed to obtain a satisfactory solution. A significant problem in developing countries, however, is that the engineers frequently have little or no experience in designing networks and therefore do not have very good judgement in selecting trial diameters. This can result in the need for numerous simulations plus the risk that even when the final solution is selected, it might be far from optimal.

In order to overcome this problem, it is possible to use a combination of optimization and simulation to obtain good network designs. The approach is to first ignore some of the links in the system in order to convert the looped network to one with branches. This network can be optimally designed using the LP program described above for which little experience and judgement are needed. This solution results in a set of trial diameters for the primary (branched) pipes of the looped system. The next task is to select diameters for the links that close the circuits of the looped network (i.e., the ones ignored in converting the looped system to one with branches); simple rules to aid judgement are available for this. Finally, the looped system with initial diameters for all the pipes can be simulated to see if it performs satisfactorily (i.e. whether node pressures are acceptable). If pressures are considered unsatisfactory, some diameters may have to be changed, but the number of changes is usually far fewer than if initial trial diameters are selected entirely by judgement, and convergence to solution can be quite rapid.

This approach to design can be illustrated by referring to the looped network in Fig. 2. The first thing to notice is that a branched instead of a looped network would be quite capable of delivering the input flow Q_1 to demand nodes 2, 3 and 4. For example, pipe 2 could be discarded, which would result in the branched system shown in Fig. 3; this branched network is called the primary system because these pipes are the principal ones for

delivering flows. The first task, then is to decide which pipe to discard in forming the primary (branched) network, no. 1, 2, 3 or 4. The general principle is to retain those pipes in the branched network that have shortest total length in connecting the source node (no. 1) with the demand nodes (no. 2, 3 and 4). To find this branched network, it is possible to use an optimization technique called the minimum spanning tree algorithm, which is particularly useful for large complicated looped systems with many circuits.³

Once the primary branched network is identified, the optimal sizes of its pipes can be found using the LP program described above. The next step is to select diameter(s) for the pipe(s) omitted from the branched system. Since these pipes are mainly needed to close the circuits and not to deliver demand flows to nodes, they are called the secondary links and can often be arbitrarily selected using minimum allowable pipe diameters. By adding these pipes to the branched network, they change its hydraulic characteristics, and it does not follow that node pressures will be satisfactory, even though the primary network has been optimally designed. For this reason, it is necessary to submit the entire looped network to the computer for simulation. Generally, node pressures will be satisfactory or nearly so. If, however, adjustments in pipe sizes are found to be needed, they

³ Such an algorithm is included in the package of programs distributed by the World Bank.

can usually be easily made using trial and judgement. Fig. 4 shows a flowchart of the optimization and simulation process.

Reservoir Programs

The first of the above three sections describes an optimization approach to design, the next describes simulation, and the last describes a combination in which optimization is initially used to obtain an approximate solution which is then fine tuned using simulation. Although the above descriptions are for water networks, similar programs and approaches to design are used for a wide variety of public investments. This section and the next describe just two of them.

Water storage reservoirs are typically very expensive and therefore benefit from being designed using intelligent computer programs. A large literature exists on optimization approaches to reservoir design.⁴ A classic and powerful model was developed by Revelle et al. (1969) that uses LP for solution. The heart of the model is a set of mass balance equations for the amount of water in the reservoir for each month of the planning period. Another set of equations describes the operating rule for the reservoir, which is assumed to be a linear expression. The objective is to minimize reservoir size subject to meeting

⁴ See, for example, the journal of Water Resources Research and Loucks et al. (1981).

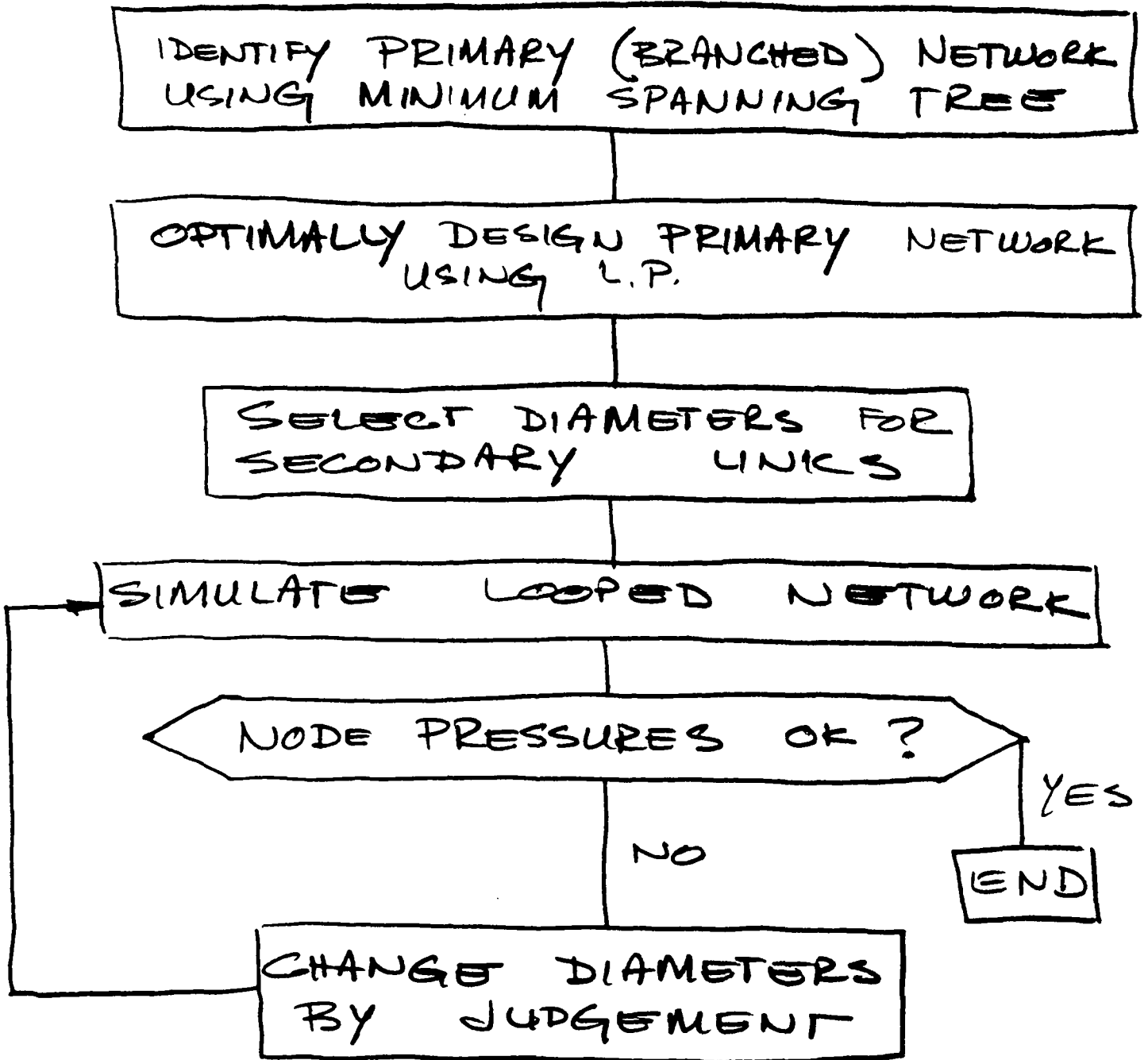


FIG 4 - LOOPED NETWORK DESIGN USING OPTIMIZATION & SIMULATION

demands, maintaining water volume within upper and lower bounds, etc. Solution is easily obtained by LP.

Just as the LP model for networks was applicable to only a certain class of systems (viz. ones without closed circuits), so too the LP model for reservoirs pertains only to systems where the operating rule is linear. In cases where a more complicated nonlinear operating rule is used, a simulation approach to reservoir design is used. As in the case of looped networks, the user must select trial values of the decision variables, which in this case are reservoir size and key parameters of the operating rule. The computer is then used to simulate reservoir performance, with specific attention paid to monthly water shortages in meeting target demands. If shortages are judged to be unsatisfactory, new trial values of size and operating parameters are chosen, and the simulation process is repeated until an acceptable solution is obtained.

In the case of looped networks, it was shown that a combination of optimization and simulation can overcome some of the problems of poor judgement in selecting initial trial values of the decision variables. A similar approach is possible with reservoirs, the LP model being used to initially estimate optimal system size and operation which can then be fine tuned by simulation.

Pricing Programs

A large literature exists on capital budgeting models that can assist water and other agencies with determining when and how much to borrow to implement capital improvement programs and how to change prices over time to meet target revenue requirements.⁵ In many respects, these models are similar to those for reservoirs.

Optimization models have been developed from capital budgeting that use LP. As in the case of reservoirs, the core of the model is a set of inventory equations that describes cash flow for each year of the planning period; the balance at the end of any year depends on the amount at the start plus income from such sources as revenues, grants, loans, and interest less expenditures for such things as construction, operation, maintenance and debt service. Various constraints can be added to the model such as minimum annual cash reserves and limits on fluctuations in prices. The basic set of decision variables are annual prices, which in the case of water is the amount charged per unit volume in each year of the planning period. While at least a few different objectives can be formulated for the model, all include some expression of total prices or total revenue, which is to be minimized.

⁵ See, for example, Clark et al. (1979) and Wilkes (1977).

As in the case of LP models for networks and reservoirs, optimization can be used for capital budgeting only under certain simplistic and restrictive assumptions. For networks, they had to be branched; for reservoirs, they had to employ a linear operating rule; for capital budgeting, the source of revenue is water sales, for which only a single price that remains constant in each period of the model can apply. In real situations, however, there are usually several sources of revenue such as initial connection charges to the system, fixed monthly service fees, and monthly commodity charges that depend on sales. Furthermore, commodity charges may include increasing or declining blocks of prices rather than a single constant price, and they may be different for residential, commercial and industrial customers as well as for places within and outside city limits.

To handle these complexities, simulation programs are used. They are extremely common in the industrialized countries and almost without exception make use of an electronic spread sheet. The user first selects trial fees and prices (and possibly other decision variables such as the amounts, timing and interest rates for loans), and then uses the program to calculate the flow of revenue shortages and surpluses. Variations of these programs used by such international finance institutions as the World Bank can additionally calculate indicators of project viability such as the internal rate of return.

As before, the combination of optimization and simulation holds the promise of identifying near optimal solutions for complicated capital budgeting problems. An LP model can first be solved to obtain an initial set of commodity prices. These can then be disaggregated by judgement to account for the complexities of the real tariff structure, and a simulation can then be made to test for revenue sufficiency and other indicators of fiscal viability.

Discussion

The above descriptions of intelligent models and programs suggest that many of the ones for public investments have certain universal characteristics, despite the fact that they apply to quite different situations. Both optimization and simulation approaches have their own advantages and disadvantages, which are briefly described herein.

In the case of optimization models, one of their main advantages, at least in theory, is that they can identify the globally optimal solution; i.e., the unique set of decision variable values that optimize the objective function while satisfying the set of problem constraints. Simulation cannot do this; at best it can identify a near optimal solution, assuming that the user has proposed one for investigation. The real question here is, how important is it to know the globally optimal solution? While the mathematically inclined can get

excited about optimization, the realists might argue that with so much uncertainty in the world, deterministic problem formulation and solution are not all that meaningful. They might say (correctly, I believe) that the mathematical optimum is a fiction. Rather, the goal is to find one or a few very good solutions, ones that are hopefully better than if an intelligent program were not used.

Optimization programs have the advantage that they are easy to use. They can be treated like a black box, where the user has little more to do than read in the parameter values, and out pops the solution. Accordingly, the user needs (or at least seems to need) little expertise or understanding of the program. Simulation programs, on the other hand, require the intelligent interaction of the user, who must scrutinize results at each step and decide what values should be tried next for the decision variables.

Both of these programs have a downside when placed in the hands of uninformed users. The risk with optimization is that the inexperienced user can apply the program to situations for which it was never designed. Meaningless results can unknowingly be believed and applied, with disastrous consequences. Simulation, on the other hand, forces the user to become at least somewhat knowledgeable about the problem of concern. The risk, however, is that good solutions may be completely overlooked. If

only poor alternatives are proposed, a near-optimal solution will never be found.

The time and effort required for problem solution can be enormously different between optimization and simulation programs. In optimization, the alternative problem solutions are internally generated and automatically evaluated by the computer, whereas the user must perform this function interactively with the computer in the case of simulation. Simulation takes a long time, and if the sole concern is with the final solution, this would appear to be time unproductively spent. However, the positive aspect of simulation is that the long time for solution is actually devoted to educating the user, where he/she gets vicarious learning experience that in the long run enables improved decision making.

A related issue with respect to solution time is the matter of sensitivity analysis. With both optimization and simulation, it is possible for the user to change parameter values to account for uncertainty. However, in the case of optimization, it is generally necessary to resolve the entire problem de novo each time parameters are changed, requiring the internal computerized screening of perhaps hundreds or even thousands of alternatives. With simulation, once the general region of a good solution has been found, sensitivity analysis can be rapid and straightforward.

Discussion of a few other characteristics of optimization and simulation will complete this section. Optimization models can be formulated for very large and complicated problems. However, the tendency is to structure them as linear problems since LP is by far the most powerful optimization technique. This in fact limits the kinds of problems that can be handled by optimization, and it also leads to the risk that nonlinear problems may be inappropriately modified to linear form so as to enable quick and easy solution.

Simulation has the advantage of being nearly independent of the mathematical form of the problem, whether linear or nonlinear. For this reason, it can handle complexities that cannot be addressed by LP. As was shown in the presentation of the sample programs, simulation is often reserved for the fine tuning of approximate LP solutions. Although a bit difficult to generalize, simulation programs frequently have more modest computer requirements than optimization, and they are usually not dependent on proprietary or sophisticated computer codes as is sometimes the case with optimization. However, simulation codes are in general more difficult to write than ones using optimization (especially LP).

An important theoretical and often real advantage of optimization programs, especially in the case of LP, is that they supply economic information and insights into the planning problem which are simply lacking in the case of simulation. By

far, the most important of these insights are shadow prices which indicate the amount by which the objective function would change for marginal changes in the levels of the constraints. Indeed, shadow prices are sometimes more important for investment planning than optimal values of the decision variables.

Conclusions

It is always difficult as well as dangerous to generalize. Nevertheless, everything considered, simulation programs for public investments in developing countries seem to hold the edge over optimization.

Perhaps the major advantage of simulation is that it requires users to understand their systems; this results from having to review computer output after each iteration and make decisions for the next trial. As a consequence, users quickly gain "experience;" although vicarious it is still an excellent teacher. This is accomplished without the agony of having to actually spend scarce resources, only to learn some months or years later that the project is a failure. Simulation programs can be strong aids to judgement and can make it possible to avoid absurd solutions, which is all too common in the case of optimization.

Simulation programs can handle essentially any kind of problem, linear or otherwise, constrained or not, with single or

multiple objectives, and they can handle any level of detail or sophistication. Such is not the case for optimization. In addition, simulation programs can often be easily changed to handle new or unusual circumstances plus new planning considerations not foreseen at the time of program development.

Electronic spread sheets are an important advancement in simulation modeling. They are ubiquitous and greatly facilitate program development, reducing the requirements for user expertise in programming to a minimum. Finally, simulation programs can often fit more easily onto microcomputers than optimization codes, and they generally lend themselves to rapid sensitivity analysis, which is so important in developing countries because of uncertainty in data.

As indicated in this paper, optimization programs have an important role to play in developing countries and should not be ignored or discounted on the basis of the remarks in this section. However, when in doubt about model development, simulation may be the most appropriate approach to investment planning.

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